

NAME: Surface Areas and Ratios from Elevation Grid, v. 1.2

Aka: surfgrids.avx

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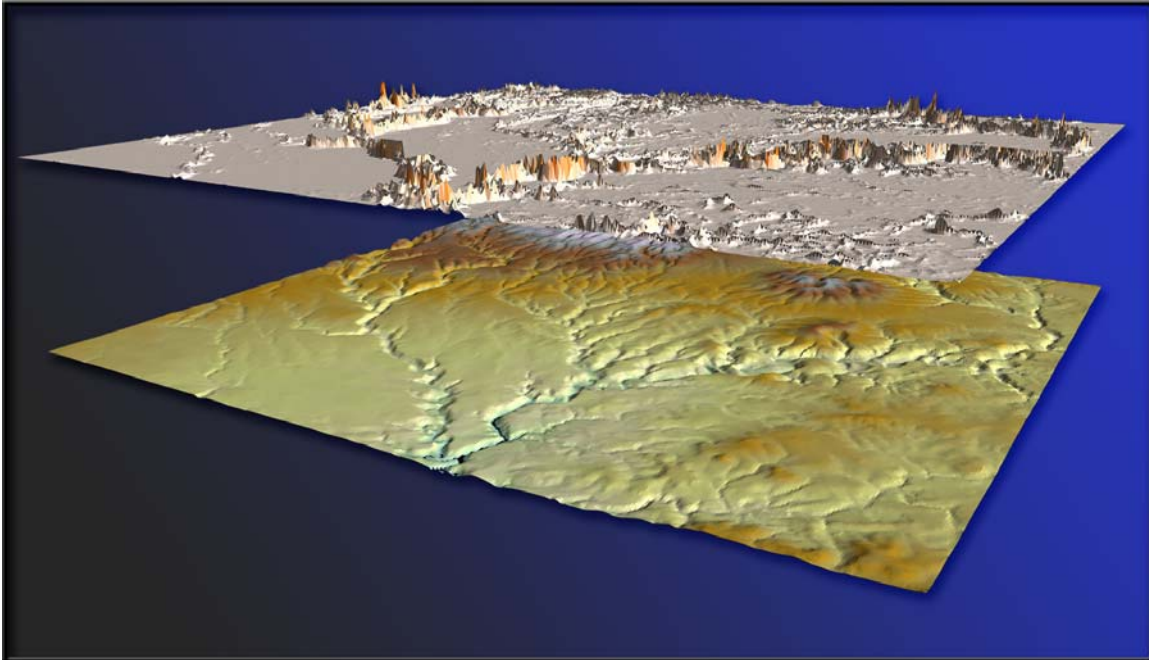
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DESCRIPTION:

This extension allows you to generate *Surface Area* and *Surface Ratio* grids from an existing Elevation grid. The cell values for these grids reflect the surface area and (surface area) / (planimetric area) ratio for the land area contained within that cell's boundaries. Both *Surface Area* and *Surface Ratio* provide useful indices of topographic roughness and convolutedness, and can give a more realistic estimate of the land area available than you can get from the simple planimetric area.



BACKGROUND: Landscape area is almost always presented in terms of planimetric area, as if a square kilometer in the Himalayas represents the same amount of land area as a square kilometer in the Nebraska. Predicted home ranges for wildlife species generally use planimetric area even when describing mountain-dwelling species such as mountain goats (*Oreamnos americanus*) and puma (*Felis concolor*). But if a species' behavior and population dynamics are a function of available resources, and if those resources are spatially limited, then these resources might be better assessed using surface area of the landscape.

Surface area also is a basis for a useful measure of landscape topographic roughness. The surface area ratio of any particular region on the landscape can be calculated by dividing the surface area of that region by the planimetric area. For example, Bowden et al. (2003) found that ratio estimators of Mexican spotted owl (*Strix occidentalis lucida*) population size were more precise using a version of this surface area ratio than with planimetric area.

There are a variety of methods in the literature for measuring terrain irregularity. Hobson (1972) described some early computational methods for estimating surface area and discussed the concept of surface area ratios. Beasom (1983) described a method for estimating land surface ruggedness based on the intersections of sample points and contour lines on a contour map, and Jenness (2000) described a similar method based on measuring the density of contour lines in an area. Mandelbrot (1983:29, 112–115) described the concept of a “fractal dimension” in which the dimension of an irregular surface lies between 2 (representing a flat plain) and 3 (representing a surface that goes through every point within a volume). Calculating this fractal dimension can be very challenging computationally, and Polidori et al. (1991), Lam and De Cola (1993) and Lorimer et al. (1994) discussed a variety of methods for estimating the fractal dimension for a landscape. An estimate of surface area also could be derived from slope and aspect within a cell (Berry 2002), although Hodgson (1995) demonstrated how most slope-aspect

algorithms generate values reflecting an area 1.6–2 times the size of the actual cell. Surface area values derived with this method would, therefore, be unduly influenced by adjacent cells.

REQUIRES ONLY SPATIAL ANALYST: This extension uses only ESRI's [Spatial Analyst](#) functions and does not require you to have a license for the [3D Analyst](#) extension. This extension also calculates surface areas much faster than 3D Analyst, and with as much accuracy. However, because the output is in Raster format rather than Vector, surface areas within polygons cannot be determined as precisely as you can do with a TIN. Surface areas within a polygon are calculated as the sum of the surface area cell values of all those cells whose cell centers lie within the polygon boundaries.

This method offers the advantage of allowing you to conduct neighborhood analyses on Surface Area grids, such that you can calculate the surface area within a particular range of all the cells in the grid. This is useful if you want to conduct analyses at different spatial scales.

GRID STATISTICS: A separate button allows you to calculate statistics for cell values that lie within polygons. These statistics include the Polygon ID, planimetric area, cell count, mean, minimum, maximum, range, standard deviation and sum for floating point grids, and also include the median, minority, majority and variety for integer grids. This extension offers two alternatives for calculating these statistics: the standard "ZonalStatsTable" method used by Spatial Analyst's "Summarize Zones..." menu item, and a modified "ZonalStatsTable" method that corrects for potential inaccuracies in the standard method (please see ['Problems with ZonalStatsTable request'](#) for an explanation).

REQUIRES: This extension requires the Spatial Analyst extension and an Elevation grid to generate *Surface Area* and *Surface Ratio* grids. It requires a polygon theme that overlays the grid to generate polygon statistics. This extension also requires that the file "avdlog.dll" be present in the ArcView/BIN32 directory (or \$AVBIN/avdlog.dll) and that the Dialog Designer extension be available in the ArcView/ext32 directory, which they almost certainly are if you're running AV3.1 or higher. You don't have to load the Dialog Designer; it just has to be available. If you are running ArcView 3.0a, you can download the appropriate files for free from ESRI, at:

<http://www.esri.com/software/arcview/extensions/dialog/index.html>

IMPORTANT: This extension also requires an elevation grid with elevation values in the same units as the X- and Y-coordinate values. This means that the extension will NOT work on unprojected elevation grids. If your grids are in geographic, unprojected coordinates, with X- and Y-values measured in degrees, then the author recommends that you project your grids into a meter- or foot-based projection (depending on the elevation units of your data) prior to using this extension. ArcInfo or ArcEditor can be used to project grids, or you can use the author's "Grid Tools" extension (see http://www.jennessent.com/arcview/grid_tools.htm).

UPDATES: September 3, 2002: The 1.2 update addresses more "ZonalStatsTable" problems and further modifies the "[Modified Method](#)" to increase its accuracy.

METHODS USED BY THIS EXTENSION are described in:

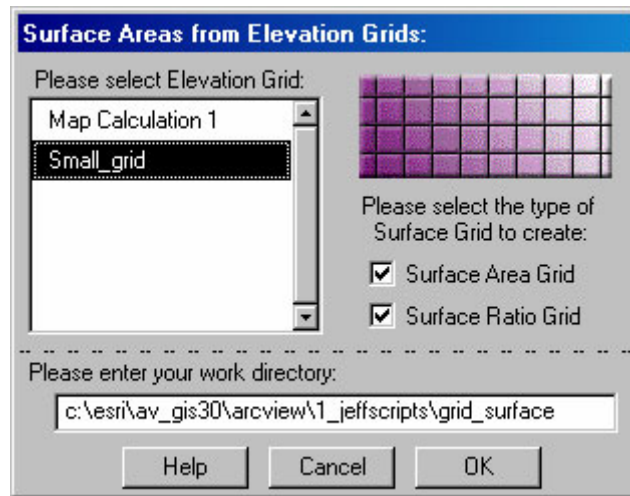
Jenness, J. S. 2004. Calculating landscape surface area from digital elevation models. *Wildlife Society Bulletin*. 32(3):829-839

General Instructions:

- 1) Begin by placing the "surfgrids.avx" file into the ArcView extensions directory (../Av_gis30/Arcview/ext32/).
- 2) After starting ArcView, load the extension by clicking on **File --> Extensions...** , scrolling down through the list of available extensions, and then clicking on the checkbox next to the extension called "Surface Areas From Elevation Grids"

3a) To Generate Surface Area and/or Surface Ratio Grids:

From your View toolbar, click on the  icon. This brings up the **Surface Areas from Elevation Grids** dialog box:



The **Please Select Elevation Grid:** list shows all of the Grid Themes available in the view. Select the one that contains elevation values. Next, select one or both of the Surface Grids to create. Finally, consider what work directory to use to create these grids.

The **Surface Area Grid** will be a new, floating point grid whose cell values reflect the surface area of the land surface contained in that cell's boundary. The **Surface Ratio Grid** will be a new, floating point grid whose cell values reflect the cell's surface area divided by the planimetric area of that cell. Surface areas are always greater than or equal to the planimetric area, so surface ratios will always be greater than or equal to 1. Please see [Appendix A](#) if you would like to review the calculation methods for determining surface areas and surface ratios. The author has also tested the accuracy of the output by comparing it to surface areas derived from TINs using 3D Analyst, and you may review the results in [Appendix B](#).

Work Directory Space: Make sure that your work directory contains plenty of room because this process can eat up a tremendous amount of hard drive space as it calculates data. As an example, the author used an integer elevation grid to generate a Surface Area and Surface Ratio grid. The Elevation grid measured 4,912 rows by 8,159 columns, contained slightly over 40 million cells, and took up about 60 megs on the hard drive. At one point during the calculation, this process was using over 1.5 gigs of hard drive space. This example produced two floating point grids that took up about 150 megs each.

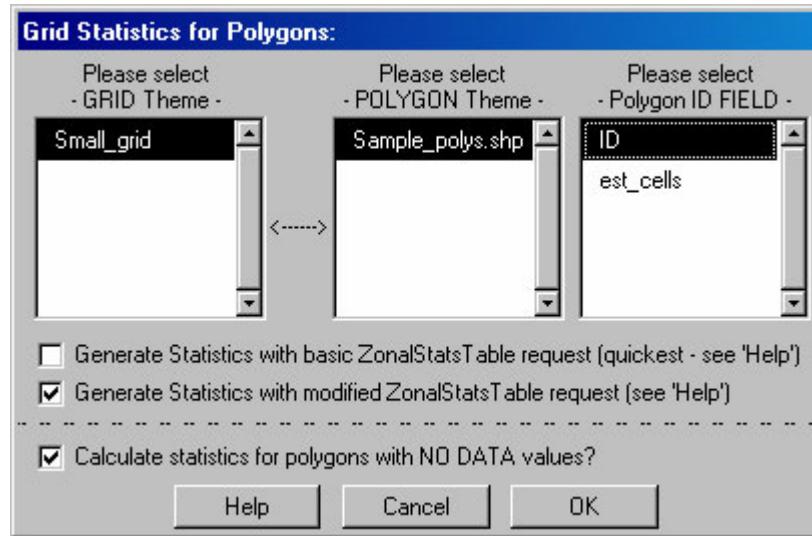
Based on this experience, the author recommends that you look at the size of your Elevation grid, then make sure you have around 25-30 times that amount of space available in your work directory to use during the calculations. All the intermediate grids are deleted at the end of the

process so most of the space is returned to you, but that space needs to be available for the calculations.

After you click the "OK" button, the extension will go to work and generate the new grids. The grids will automatically be added to your view.

3b) To Generate Statistics for Grid Cells Enclosed by Polygons:

From your View toolbar, click on the  icon. This brings up the **Grid Statistics for Polygons** dialog box:



Polygon Statistics: This operation performs a Zonal Statistics Analysis on the selected grid, with the polygons representing the zones. The statistics reflect all those cells whose cell centers are located within the polygon boundary. For each polygon, this operation will calculate the following statistics:

- ID Value of Polygon, taken from the ID Field you select.
- Count of Cells within polygon.
- Planimetric area of cells within polygon.
- Minimum cell value within polygon.
- Maximum cell value within polygon.
- Range of cell values within polygon.
- Mean cell value within polygon.
- Standard Deviation of cell values within polygon.
- Sum of cell values within polygon.

Integer grids will also get:

- Minority (value with least number of occurrences)
- Majority (value with greatest number of occurrences)
- Median
- Variety (number of unique values)

BASIC vs. MODIFIED ZonalStatsTable REQUEST: Spatial Analyst's 'ZonalStatsTable' request is a quick and simple way to derive statistics for polygons that overlay a grid. Users can access this function by clicking the 'Analysis' menu item, then 'Summarize Zones...'. The 'Basic ZonalStatsTable Request' in this dialog does the exact same thing as the 'Summarize Zones' menu item. However, the author has occasionally seen this request produce odd results,

apparently using cells that were outside the polygon boundary (see ['Problems with ZonalStatsTable Request'](#) for details), so I offer an alternative, modified version of the 'ZonalStatsTable' request.

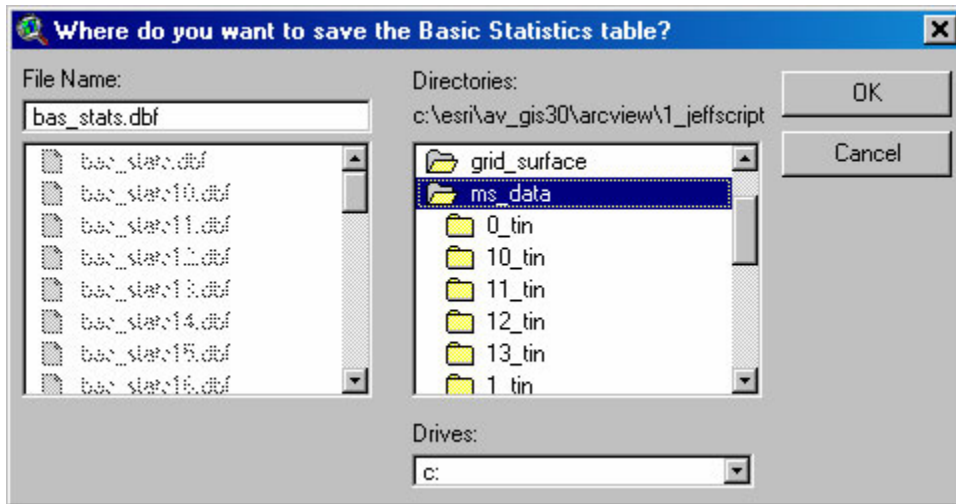
The Modified version takes each polygon in the theme and converts it into a grid with the same cell size as the Input Grid, producing a grid of "1" values in the shape of the polygon. It then multiplies that grid by the Input Grid, thereby converting all cells outside of the polygon into "No Data" cells while retaining the original cell values inside the polygon. Finally, this extension uses the 'ZonalStatsTable' request on the modified input grid to generate statistics for that single polygon. This process eliminates the possibility that the 'ZonalStatsTable' request will select cells outside of the polygon boundary by converting all cells outside of the polygon boundary into "No Data" values. I believe this process produces more reliable results, but it does take considerably longer to calculate than the basic 'ZonalStatsTable' request.

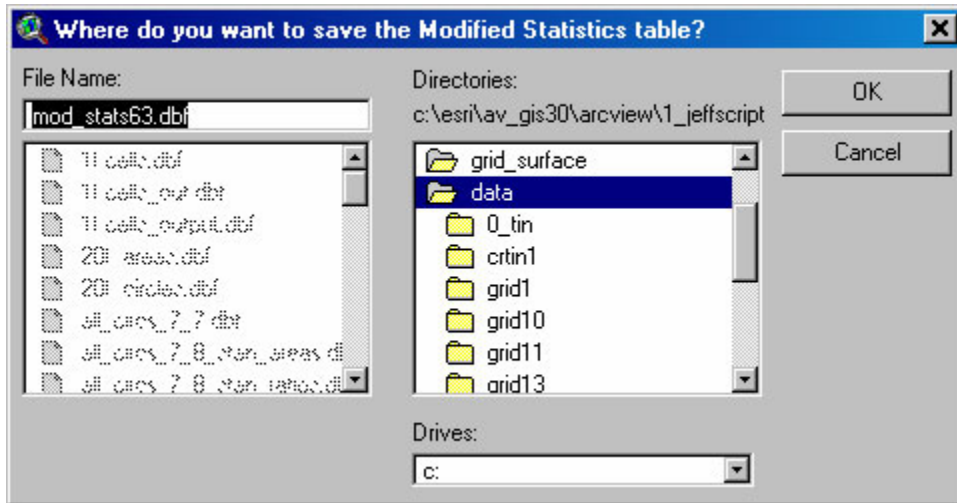
IMPORTANT: An important difference between the Basic 'ZonalStatsTable' request and the modified version presented here is that the basic version will combine polygons with identical Field ID values and provide you with statistics representing the combined areas, such that each record in the Statistics Table will represent a unique zone. The modified version will offer statistics on each polygon separately regardless of whether they have identical Field ID values.

Calculate Statistics for Polygons with 'No Data' values: If you select this option, then this extension will generate statistics for all polygons that include at least 1 grid cell with data. Statistics will reflect ONLY those grid cells with data. If a polygon has only one cell with actual data in it, then these statistics will all be based on that single cell. If you do not select this option, then this extension will not generate statistics for any polygons that include any cells with "NO DATA" values.

Once you click the "OK" button, you will be prompted to specify the name and location to save your output table(s). These are standard ArcView Dialog Boxes and should be familiar to most users. The Statistics tables are permanent tables and will not be deleted when ArcView is shut down. These tables will also appear in your Project and in your project's List of Tables with the names:

- Zonal Stats: 'Polygon theme name' by 'Grid theme name'
- Zonal Stats (modified): 'Polygon theme name' by 'Grid theme name'





An on-line version of this manual may be viewed at:

http://www.jennessent.com/arcview/surface_areas.htm

Enjoy! Please contact the author if you have problems or find bugs.

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Please visit *Jenness Enterprises* [ArcView Extensions](http://www.jennessent.com/arcview/extensions) site for more ArcView Extensions and other software by the author. We also offer customized ArcView-based [GIS consultation](http://www.jennessent.com/gis_consultation) services to help you meet your specific data analysis and application development needs.



Appendix A: Methods for Calculating Surface Area Grids

In summary, this extension calculates surface areas using the following strategy:

- For each cell in the grid, surface areas are based on triangle areas derived from eight triangles
- Each triangle connects the center point of the central cell with the center points of two adjacent cells. These triangles are located in three-dimensional space, so that the area of the triangle represents the true surface area of the space bounded by the three points.
- The triangle area is adjusted so that it only represents the portion of the triangle that overlays the central cell.
- The areas of the eight triangles are summed to produce the total surface area of that cell.
- The surface ratio of the cell is calculated by dividing the surface area of the cell with the planimetric area of the cell.

Triangulating the Surface: This extension derives surface areas for a cell by using information from the eight cells adjacent to the center cell. For example, given a sample elevation grid:

210	190	170	155	140	135
204	183	165	145	125	120
200	175	160	122	110	100
208	187	165	150	126	120

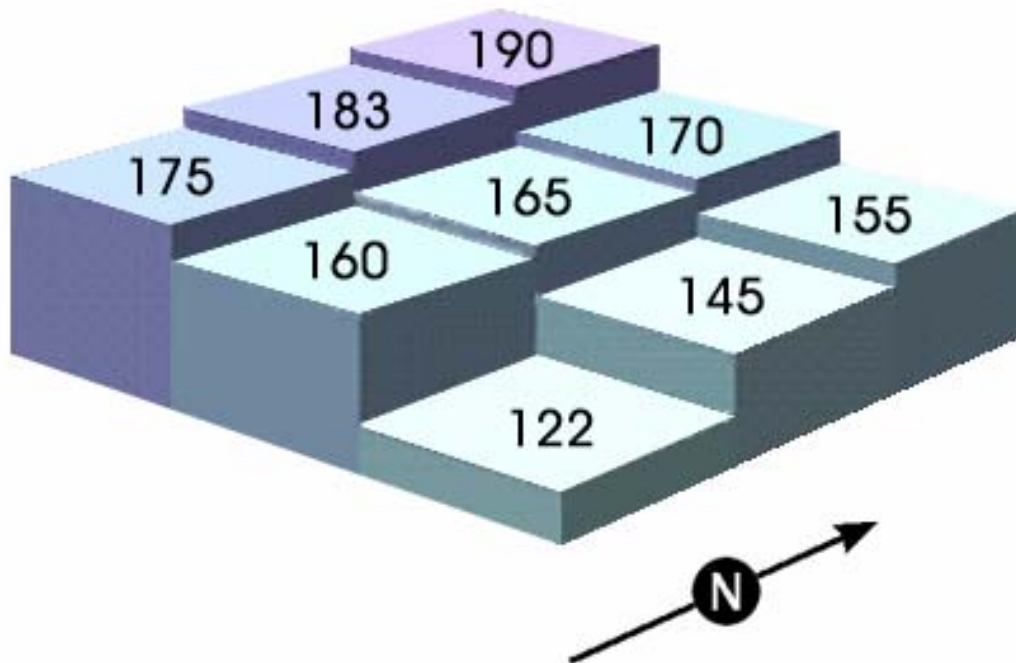
this extension would calculate the surface area for the cell with elevation value "183" based on the elevation values of the eight cells surrounding it,

210	190	170	155	140	135
204	183	165	145	125	120
200	175	160	122	110	100
208	187	165	150	126	120

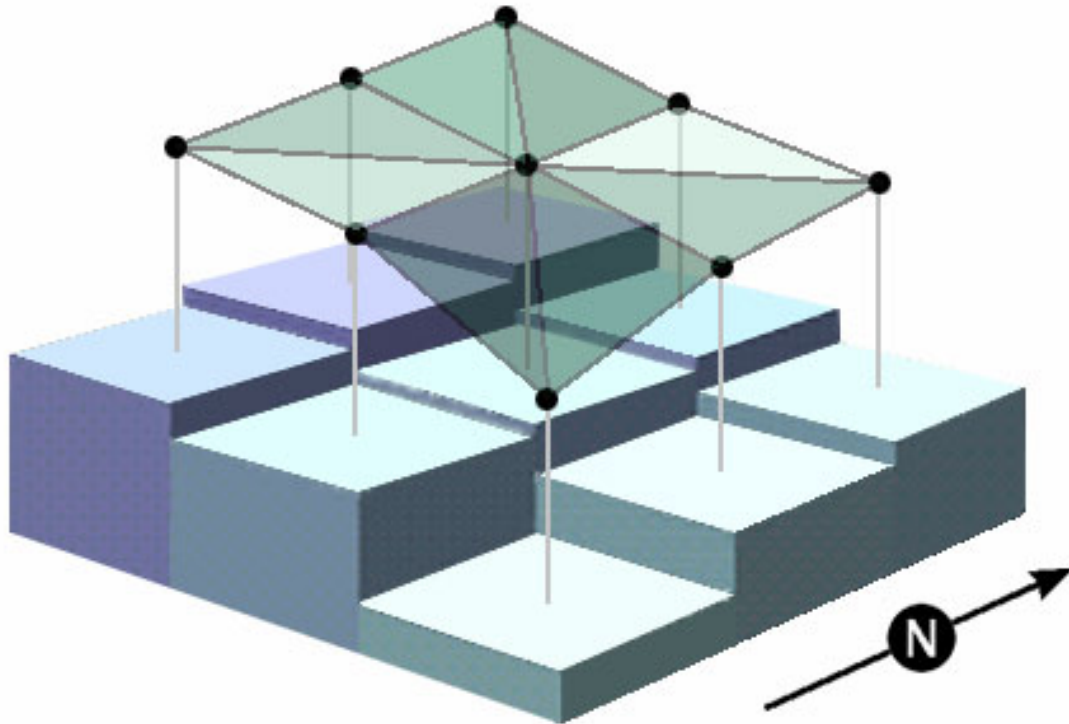
and the surface area for the "165" cell based on the eight elevation value surrounding it:

210	190	170	155	140	135
204	183	165	145	125	120
200	175	160	122	110	100
208	187	165	150	126	120

The following example of a Surface Area calculation will be based on the illustration above, where we will calculate the surface area for a cell with an elevation value of 165, surrounded by cells at 190, 170, 155, 183, 145, 175, 160 and 122. This grid of elevation values can be pictured in 3-dimensional space as a set of adjacent columns, each rising as high as its specified elevation value.



We take the 3-dimensional centerpoints of each of these 9 cells, and calculate the lengths of the 8 lines that connect the central cell's centerpoint with the center points of the 8 surrounding cells. We then calculate the lengths of the lines that connect each of the 8 surrounding cells with the one right next to it, so that we end up with the lengths of the sides of the 8 triangles that all meet at the center point of the central cell. Using these lengths, we can calculate the areas of each of the triangles.



These lengths are simple to calculate using the Pythagorean Theorem, which states that, in a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the other two sides. Thus, for any two points **A** and **B**:

Pythagorean Theorem:

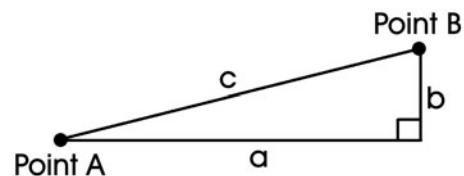
$$a^2 + b^2 = c^2 \quad \text{or} \quad c = \sqrt{a^2 + b^2}$$

where:

a = planimetric distance from Point A to Point B

b = difference in elevation between Point A and Point B

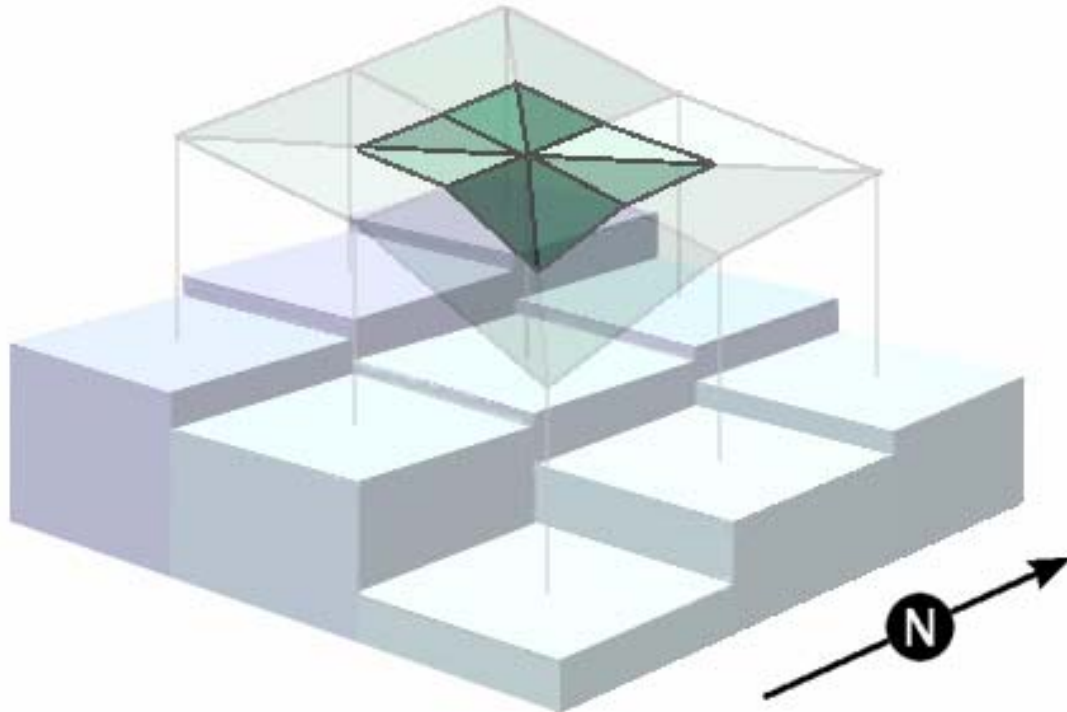
c = True surface distance from Point A to Point B



"**b**" is easy to calculate, since it is simply the difference between the two cell values. "**a**" is easy to get for the cells directly to the North, East, South and West, since it is simply the value of the Cell size. For the diagonals, we use the Pythagorean theorem again and calculate

$$a = \sqrt{2(\text{cell size})^2}.$$

However, since we're only concerned with the surface area within each cell, we cut all the triangle lengths in half. Now when we calculate the area of a triangle based on the lengths of three sides, the three sides will represent the portion of the triangle that lies within the cell boundaries.



This action is justified based on the Side-Angle-Side similarity criterion for similar triangles (Euclid 1956:204), which states that “If two triangles have one angle equal to one angle and the sides about the equal angles proportional, the triangles will be equiangular and will have those angles equal which the corresponding sides subtend.”. Each original triangle is “similar” to its corresponding clipped triangle because the 2 sides extending from the center cell in the original triangle are exactly twice as long as the respective sides in the clipped triangle, and the angles defined by these 2 sides are the same in each triangle. Therefore, the third side of the clipped triangle must be exactly half as long as the corresponding side of the original triangle.

Technical Note: The above description only describes the logic used to derive the surface area. The extension is not actually connecting any lines or using any vector data at all. Rather, the extension creates 16 new grids in which the cell values represent the lengths of one of the 16 triangle edges pictured above. Using 3 of these 16 "Triangle Edge Length" grids at a time, it then calculates 8 new grids representing the area of the triangle that would be bounded by those three edge lengths. Finally, the extension adds those 8 "Triangle Area" grids to produce the final Surface Area Grid.

Calculating the Area of a Triangle from the Lengths of the Three Sides: The author is aware of two formulae that will produce the area of a triangle given the lengths of the three sides, and will describe both of them here. The first one was derived by the author from a couple of basic mathematical theorems and the second was discovered by the author after he figured out the first one. This extension uses the first one because it was all worked out by the time the author found out about the second formula. The author has checked to confirm that these two formulae produce identical results.

Formula 1: Derived by the author –

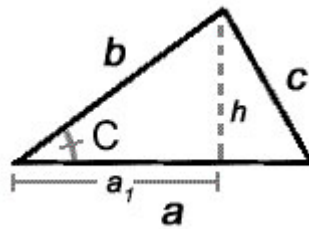
$$\text{Area} = \frac{a * \sqrt{b^2 - \left(\frac{a^2 + b^2 - c^2}{2a}\right)^2}}{2}$$

Formula 2: Adapted from the "[Triangle Area Calculator](#)" (click to visit web site), and attributed to Heron of Alexandria and Archimedes (see also Abramowitz and Stegun [1972, p. 79]):

$$\begin{aligned} \text{Triangle half-perimeter} &= S = \frac{a+b+c}{2} \\ \text{Area} &= \sqrt{S(S-a)(S-b)(S-c)} \end{aligned}$$

Derivation of Formula 1:

- 1) Given a triangle with sides **a**, **b**, and **c**:



and the standard formula for the area of a triangle:

$$\text{Area} = \frac{ah}{2}$$

The main problem was to figure out what **h** was equal to, given that we knew what **a**, **b**, and **c** were.

- 2) The Law of Cosines is useful for identifying the length of a third side if you know the lengths of two sides and the angle between them:

$$\text{Law of Cosines: } c^2 = a^2 + b^2 - 2ab\cos C$$

- 3) Since we know the three sides, we can isolate "**cos C**" on one side of the equation:

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

- 4) In a right triangle (see triangle illustrated above, but look at sides **a1**, **h**, and **b**), **cos C** is also equal to the adjacent side divided by the hypotenuse:

$$\cos C = \frac{a_1}{b}$$

- 5) Therefore,

$$\frac{a_1}{b} = \frac{a^2 + b^2 - c^2}{2ab}$$

6) Multiplying both sides by **b**, we now have isolated **a1**:

$$a_1 = \frac{a^2 + b^2 - c^2}{2a}$$

7) We can use the Pythagorean Theorem to isolate **h**:

$$\text{Pythagorean Theorem: } a_1^2 + h^2 = b^2 \quad \text{or} \quad h = \sqrt{b^2 - a_1^2}$$

8) Substituting for **a1**:

$$h = \sqrt{b^2 - \left(\frac{a^2 + b^2 - c^2}{2a}\right)^2}$$

9) Finally, plugging **h** back into the Area equation, we get:

$$\text{Area} = \frac{a * \sqrt{b^2 - \left(\frac{a^2 + b^2 - c^2}{2a}\right)^2}}{2}$$

Appendix B: TIN vs. GRID Surface Area Comparisons

SUMMARY

I compared the output of the Surface Area and Surface Ratio grids derived by this extension with surface areas and ratios derived from a TIN-based analysis using the ESRI's [3D Analyst](#) extension (ESRI 2000) and the author's [Surface Tools extension](#) (Jenness 2001). I computed the surface areas and ratios within 650 circular polygons ranging in size from 100m radius (approximately 3 grid cells) to 20,000m radius (approximately 146,454 grid cells). I used two methods of determining grid statistics within polygons (the "Standard" method and the "Modified" method) because I felt the standard method was giving me inaccurate results (see ["Appendix C: Problems with 'ZonalStatsTable' Request"](#) for details).

Surface Ratios: I found that Surface Ratios correlated very well at all scales and regardless of the method of determining grid statistics. The lowest correlation between Surface Ratio values was 0.981 (circle radius = 100, Standard Method). GRID-based values tended to be very close to TIN-based values, never differing more than 4% from their corresponding TIN-based value and rarely differing more than 1%. In almost all cases, both accuracy and precision increased as the size of the circle increased, so that the circles with the greatest numbers of grid cells tended to be the most accurate and precise.

Surface Areas; Standard Method: The Standard method of calculating statistics for surface areas tended to be variable and unreliable at cell counts ≤ 234 (circle radius = 800m). Correlations, accuracy and precision increased at higher cell counts except for the case of the 2,000m circles (mean cell count = 1,472), in which the Standard Method appeared to be considering too many cells in some of the circles. I suspect that the problem with the 2,000m circles had more to do with an apparent bug in the Standard Method than it did with the surface area grids, because correlations, precision and accuracy for the 2,000m circles greatly increased when I disregarded the 19 problematic circles.

Surface Areas; Modified Method: The Modified method of calculating statistics tended to produce much more precise results than the Standard method, with correlations ranging from 0.993 to 1.000. GRID-based values tended to be higher than their TIN-based counterparts at low cell counts (< 264), but accuracy greatly increased at cell counts higher than 234 (circle radius = 800m). For polygons with ≥ 264 cells, GRID-based values were almost identical to TIN-based values and both accuracy and precision were nearly perfect.

The overall accuracy of this method, based on several additional analyses, can be also reviewed in Jenness (In Press), "Calculating Landscape Surface Area from Digital Elevation Models".

METHODS

I tested the output accuracy of this extension by generating surface areas and ratios for 650 circular polygons in 7 different sizes and comparing results between a TIN-based method and the GRID-based method used by this extension. These 650 polygons were randomly distributed across an elevation grid and all the polygons were completely within the elevation grid boundaries (i.e. none of the polygons overlaid any "No Data" areas). For a control group, I used the [Surface Tools extension](#) (Jenness 2001) to calculate what I assume to be the most accurate Surface Area and Surface Ratio values possible from the elevation data. I used a Z-factor = 0 (meaning that the new TIN will go through all the elevation points and therefore be as complex as possible. See Surface Tools manual for an explanation).

I then generated surface areas and ratios using the Surface Area and Surface Ratio grids produced by this extension. I compared the GRID-based surface areas with the TIN-based surface areas for each of the 650 circular polygons using scatterplots, correlation statistics, and descriptive statistics. I used SPSS 9.0 to generate the statistics.

I used both a standard method and a modified method to generate grid cell statistics for the 650 circular polygons because I was uncertain that the standard method was producing accurate results (see [Appendix C](#) for details). Results from both methods are presented here

Standard Method: I used Spatial Analyst's "Summarize Zones..." menu item to generate total surface areas and mean surface ratios for the 650 sample circular polygons. For purposes of this comparison, this "Summarize Zones" operation treated each polygon as a separate "zone", then generated grid statistics for all those grid cells that lay within each zone.

Modified Method: Conceptually, this modified process can be pictured as taking each polygon separately, using it like a cookie cutter to clip out only those grid cells that lie within the polygon, then running 'ZonalStatsTable' on only that polygon and only those clipped grid cells. Technically, the modified version takes each polygon in the theme and converts it into a grid with the same cell size as the Input Grid, producing a grid of "1" values in the shape of the polygon. It then multiplies that grid by the Input Grid, thereby converting all cells outside of the polygon into "No Data" cells while retaining the original cell values inside the polygon. Finally, this extension uses the 'ZonalStatsTable' request on the modified input grid to generate statistics for that single polygon. This process eliminates the possibility that the 'ZonalStatsTable' request will select cells outside of the polygon boundary by converting all cells outside of the polygon boundary into "No Data" values.

Polygon radii are in meters and grids had a cell size of approximately 92.6 meters. The modified process causes the circular polygons to have identical numbers of cells for each radius size while the standard process can have variable cell counts in each radius size. The means, standard deviations and ranges reflect the ratios of (GRID Surface Area) divided by (TIN Surface Area), or the ratio of (GRID Surface Ratio) divided by (TIN Surface Ratio).

RESULTS

Surface Area/Ratio Comparison using Standard Method: In all cases, Surface Ratio values using the Standard method were very close between the TIN-based and GRID-based calculations. Accuracy and precision increased with higher cell counts, but even in the worst case the GRID-based calculation was only 4% different than the TIN-based calculation.

At low cell counts, this Standard method produced highly variable results with low correlations between TIN-based and GRID-based surface areas. Circular polygons can have variable cell counts depending on their placement on the grid and these different cell counts can have significant effects on the internal areas. The scatterplot for the 100m circles comparing Surface Areas clearly shows 4 groupings of points, and the groups are explained by the cell counts. There are very distinct separations between those circles with 2, 3, 4 or 5 cells. These distinct groupings are still visible in the 200m and 400m circles, but they start to blend together at higher cell counts.

At high cell counts this Standard method generally produced very accurate and precise surface areas, with the exception of the 2,000m circles. Something strange happened there, and surface areas for 19 of these circles apparently included grid cells that were outside the circle. The scatterplot for the 2,000m surface areas shows two distinct groupings of points and I suspect that one of these groups was somehow calculated incorrectly. This problem with the 2,000m points led me to develop the Modified method of generating statistics.

Table 1: Surface Area Comparisons - TIN vs. GRID-based Output using Standard ZonalStatsTable Request

Circle Radius	n	Avg. # Grid Cells in Polygon	Surface Area Comparison			
			Pearson Correlation	(GRID Surface Area) / (TIN Surface Area)		
				mean	sd	range
100	100	3.58	0.323	0.9793	0.1469	0.54 - 1.37
200	100	14.65	0.400	1.0013	0.0498	0.94 - 1.10
400	100	58.42	0.675	.09978	0.0261	0.95 - 1.06
800	100	234.32	0.969	1.0005	0.0065	0.99 - 1.03
1,000	100	366.12	0.914	1.0006	0.0069	0.99 - 1.02
2,000	100	1,472.39	0.733	1.0059	0.0133	1.00 - 1.04
2,000*	81	1,463.12	0.996	0.9995	0.0014	1.00 - 1.00
20,000	50	146,454.22	0.999	1.0002	0.0006	1.00 - 1.00

* Disregarding 19 apparently incorrect surface area values

Table 2: Surface Ratio Comparisons - TIN vs. GRID-based Output using Standard ZonalStatsTable Request

Circle Radius	n	Avg. # Grid Cells in Polygon	Surface Ratio Comparison			
			Pearson Correlation	(GRID Surface Ratio) / (TIN Surface Ratio)		
				mean	sd	range
100	100	3.58	0.981	1.0007	0.0060	0.98 - 1.04
200	100	14.65	0.994	1.0002	0.0027	0.99 - 1.01
400	100	58.42	0.998	0.9999	0.0015	0.99 - 1.01
800	100	234.32	1.000	0.9997	0.0008	1.00 - 1.00
1,000	100	366.12	1.000	0.9999	0.0005	1.00 - 1.00
2,000	100	1,472.39	1.000	0.9998	0.0004	1.00 - 1.00
2,000*	81	1,463.12	1.000	0.9997	0.0004	1.00 - 1.00
20,000	50	146,454.22	1.000	0.9995	0.0004	1.00 - 1.00

* Disregarding 19 apparently incorrect surface ratio values

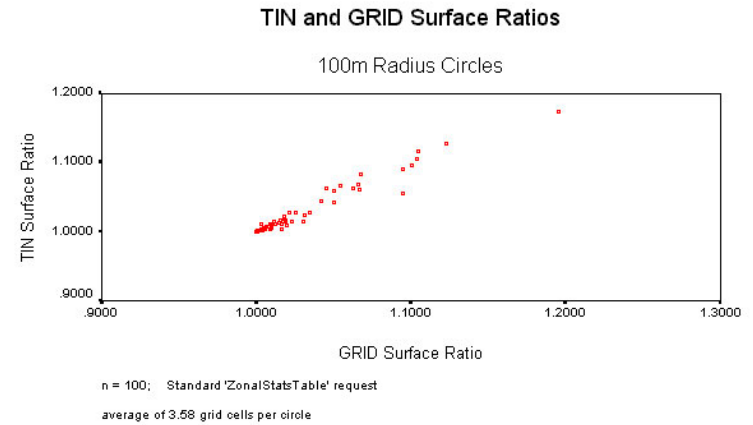
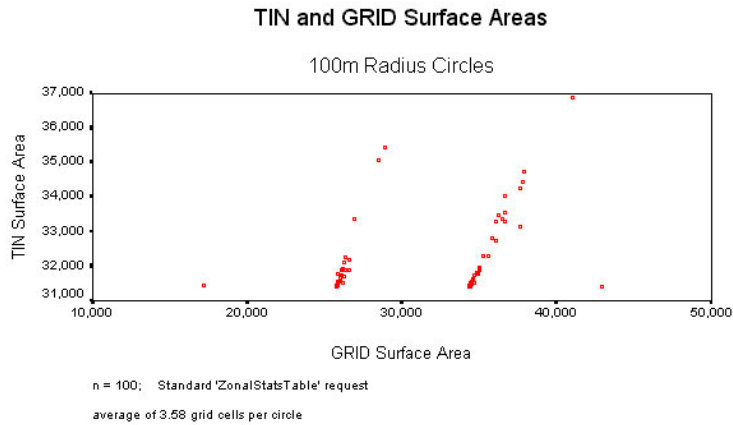
Table 3: TIN vs. GRID Comparison Scatterplots, using Standard ZonalStatsTable Request

Circle Data

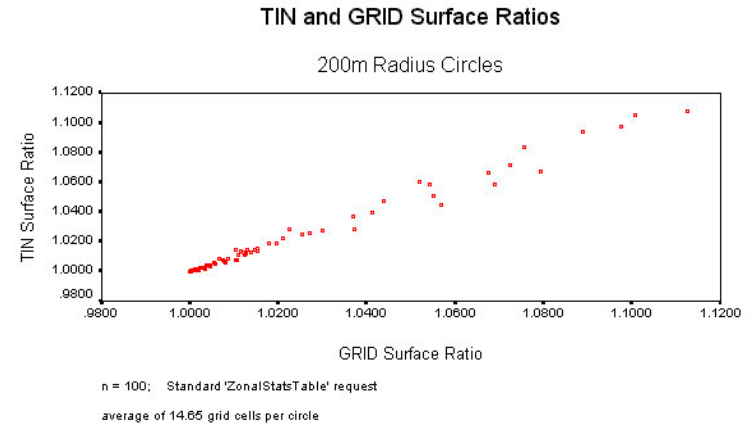
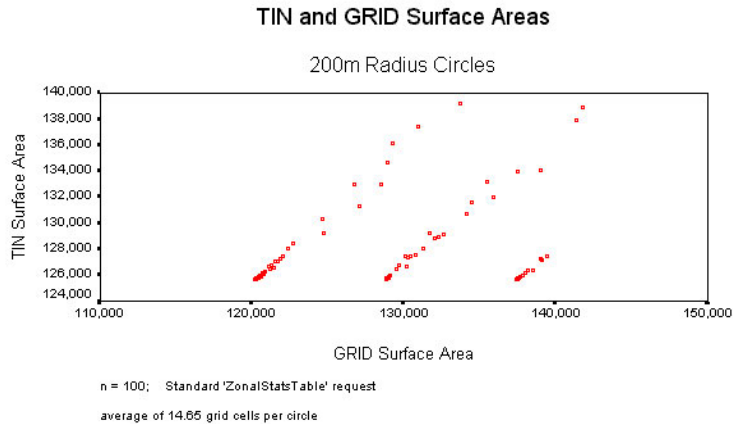
Surface Area Scatterplots

Surface Ratio Scatterplots

radius = 100
3.58 cells
n = 100

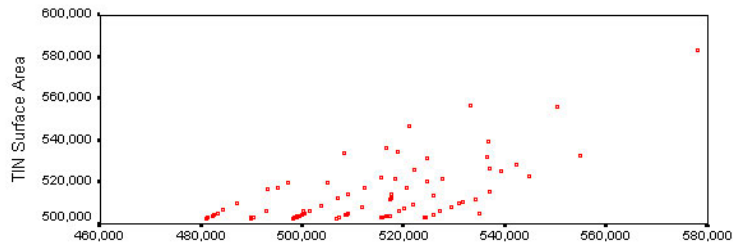


radius = 200
14.65 cells
n = 100



TIN and GRID Surface Areas

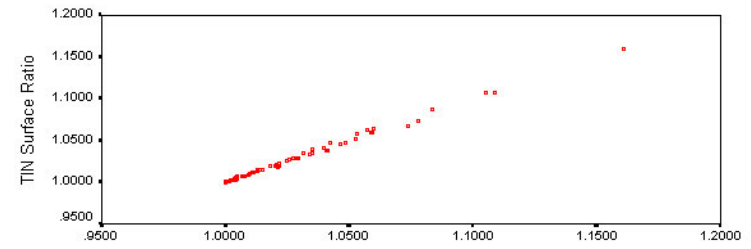
400m Radius Circles



n = 100; Standard 'ZonalStatsTable' request
average of 58.42 grid cells per circle

TIN and GRID Surface Ratios

400m Radius Circles

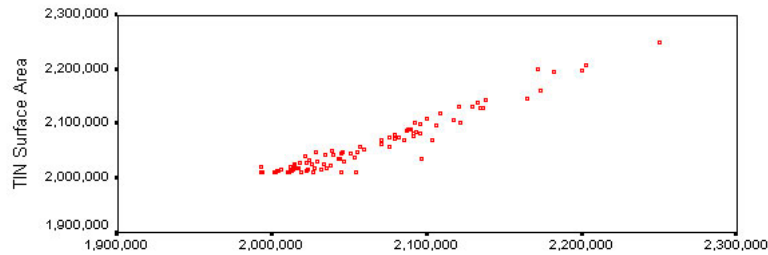


n = 100; Standard 'ZonalStatsTable' request
average of 58.42 grid cells per circle

radius = 400
58.42 cells
n = 100

TIN and GRID Surface Areas

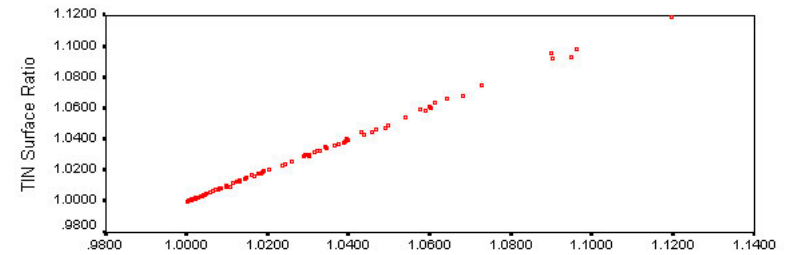
800m Radius Circles



n = 100; Standard 'ZonalStatsTable' request
average of 234.32 grid cells per circle

TIN and GRID Surface Ratios

800m Radius Circles

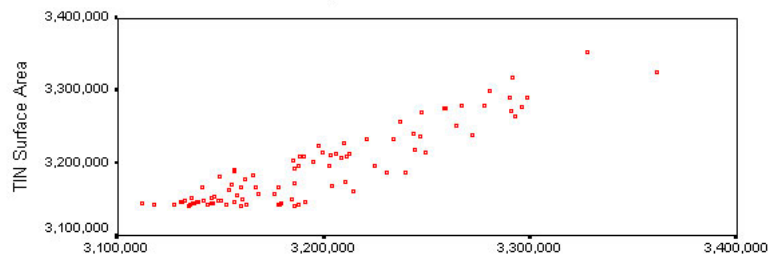


n = 100; Standard 'ZonalStatsTable' request
average of 234.32 grid cells per circle

radius = 800
234.32 cells
n = 100

TIN and GRID Surface Areas

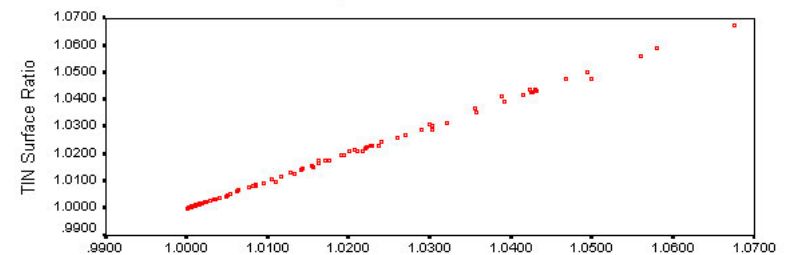
1,000m Radius Circles



n = 100; Standard 'ZonalStatsTable' request
average of 366.12 grid cells per circle

TIN and GRID Surface Ratios

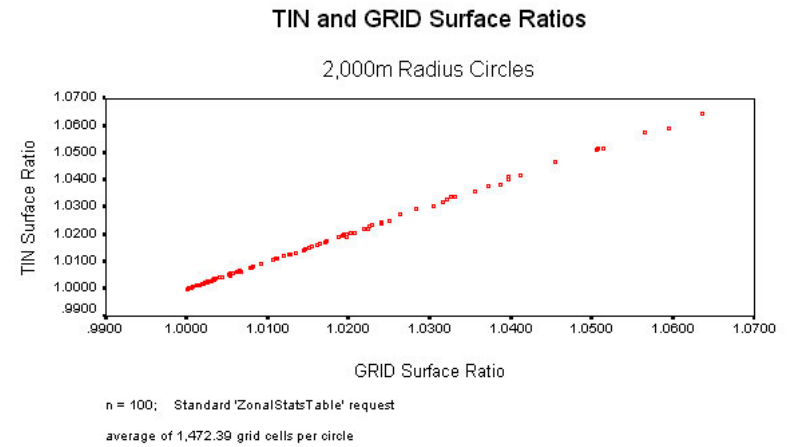
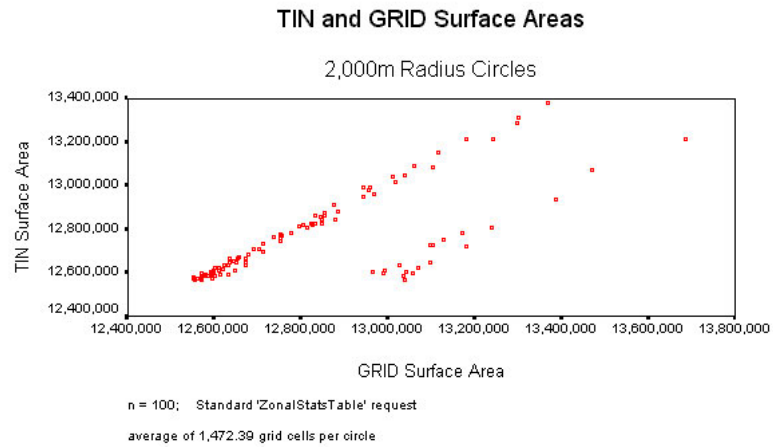
1,000m Radius Circles



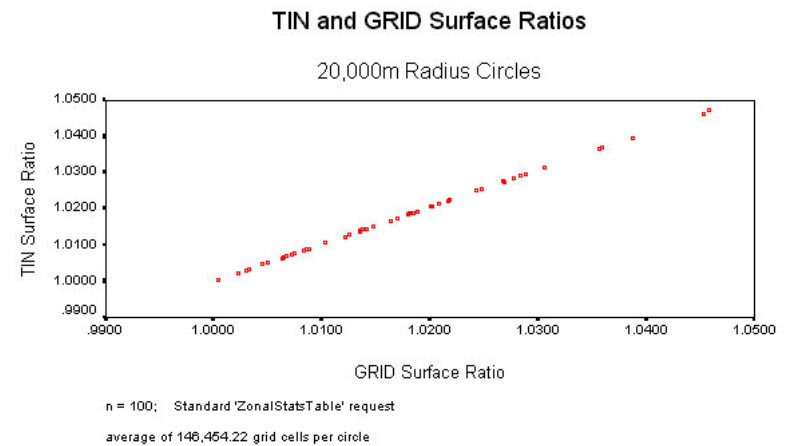
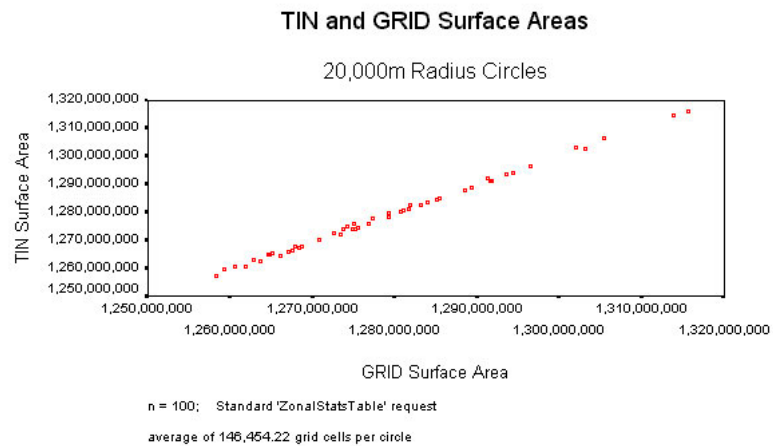
n = 100; Standard 'ZonalStatsTable' request
average of 366.12 grid cells per circle

radius = 1,000
366.12 cells
n = 100

radius = 2,000
1,472.39 cells
n = 100



radius = 20,000
146,454.22 cells
n = 50



Surface Area/Ratio Comparison using Modified Method: In all cases, Surface Ratio values using the Modified method were very close between the TIN-based and GRID-based calculations. Accuracy and precision increased with higher cell counts, but even in the worst case the GRID-based calculation was only 1% different than the TIN-based calculation.

Surface Area values were always highly correlated, but GRID-based calculations tended to be higher than TIN-based calculations at low cell counts. Surface area values for the 100m circles, with 4 grid cells each, tended to be over 9% higher when calculated from the Grids. GRID-based surface areas for circles with up to 60 cells tended to be about 2.5% higher than their corresponding TIN-based surface areas.

Surface areas for circles with cell counts ≥ 234 tended to be almost identical between the GRID- and TIN-based calculations, and correlations were almost perfect.

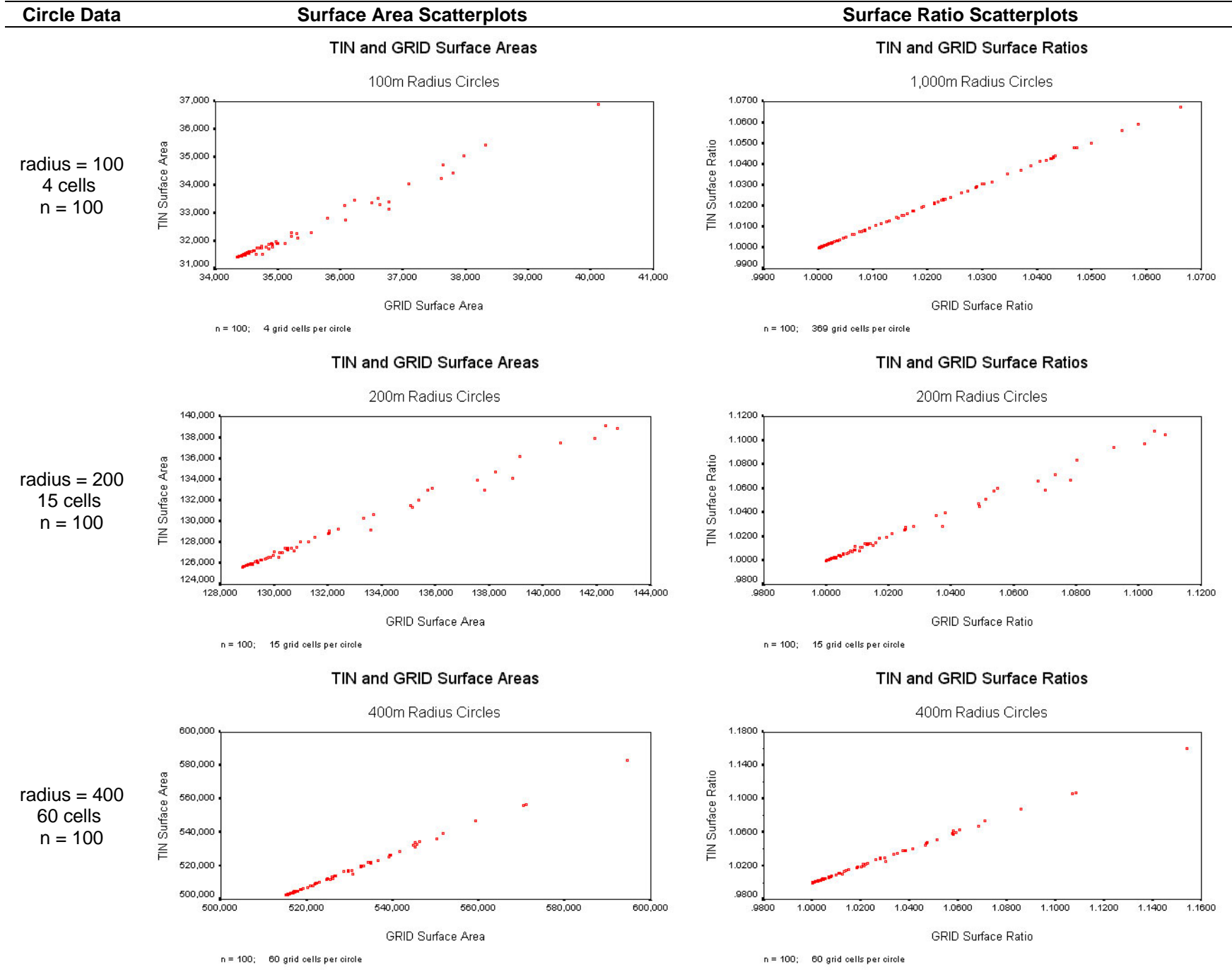
Table 4: Surface Area Comparisons - TIN vs. GRID-based Output using Modified ZonalStatsTable Request

Circle Radius	n	Avg. # Grid Cells in Polygon	Surface Area Comparison			
			Pearson Correlation	(GRID Surface Area) / (TIN Surface Area)		
				mean	sd	range
100	100	4	0.993	1.0937	0.0039	1.08 - 1.11
200	100	15	0.996	1.0252	0.0022	1.02 - 1.04
400	100	60	0.999	1.0249	0.0011	1.02 - 1.03
800	100	234	1.000	0.9991	0.0004	1.00 - 1.00
1,000	100	369	1.000	1.0084	0.0003	1.01 - 1.01
2,000	100	1,465	1.000	1.0008	0.0003	1.00 - 1.00
20,000	50	146,361	1.000	0.9996	0.0003	1.00 - 1.00

Table 5: Surface Ratio Comparisons - TIN vs. GRID-based Output using Modified ZonalStatsTable Request

Circle Radius	n	Avg. # Grid Cells in Polygon	Surface Ratio Comparison			
			Pearson Correlation	(GRID Surface Ratio) / (TIN Surface Ratio)		
				mean	sd	range
100	100	4	0.993	1.0004	0.0036	0.99 - 1.01
200	100	15	0.996	1.0002	0.0021	1.00 - 1.01
400	100	60	0.999	0.9999	0.0011	0.99 - 1.00
800	100	234	1.000	0.9998	0.0004	1.00 - 1.00
1,000	100	369	1.000	0.9998	0.0003	1.00 - 1.00
2,000	100	1,465	1.000	0.9998	0.0003	1.00 - 1.00
20,000	50	146,361	1.000	0.9995	0.0002	1.00 - 1.00

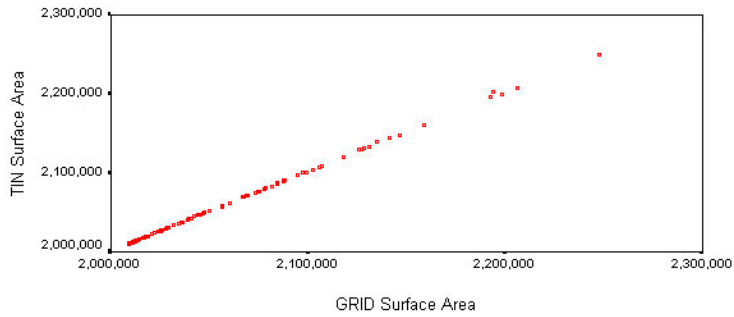
Table 6: TIN vs. GRID Comparison Scatterplots, using Modified ZonalStatsTable Request



TIN and GRID Surface Areas

800m Radius Circles

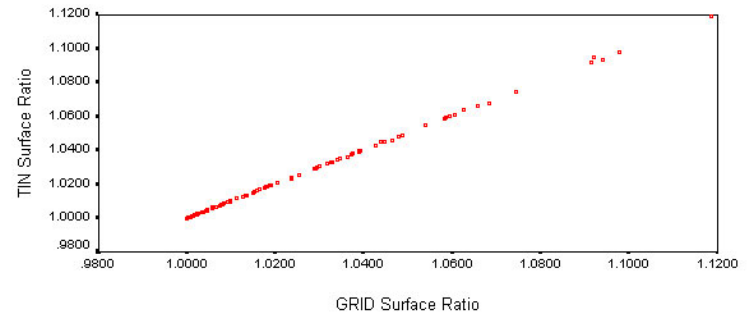
radius = 800
234 cells
n = 100



n = 100; 234 grid cells per circle

TIN and GRID Surface Ratios

800m Radius Circles

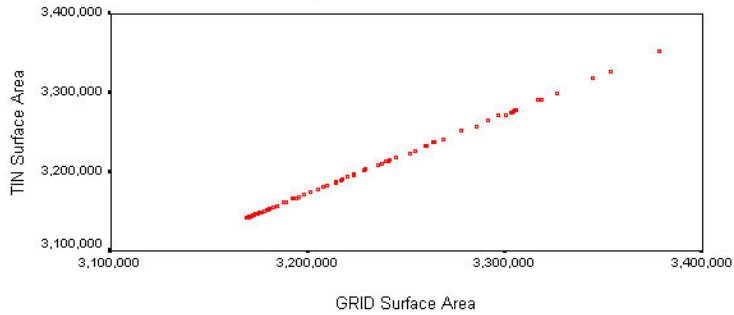


n = 100; 234 grid cells per circle

TIN and GRID Surface Areas

1,000m Radius Circles

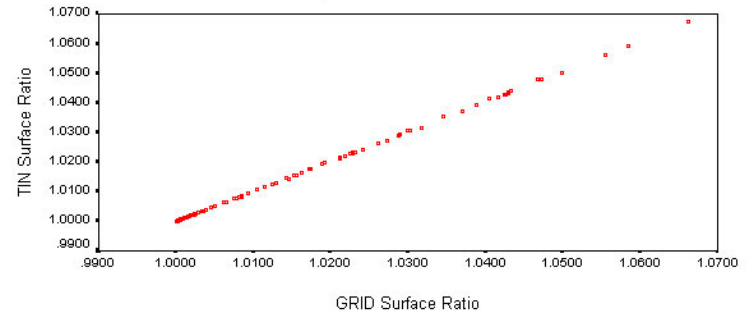
radius = 1,000
369 cells
n = 100



n = 100; 369 grid cells per circle

TIN and GRID Surface Ratios

1,000m Radius Circles

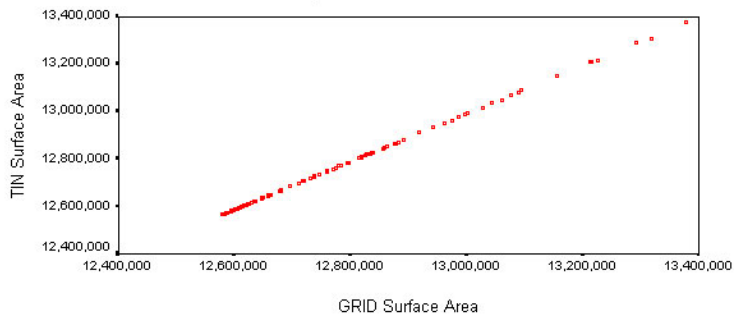


n = 100; 369 grid cells per circle

TIN and GRID Surface Areas

2,000m Radius Circles

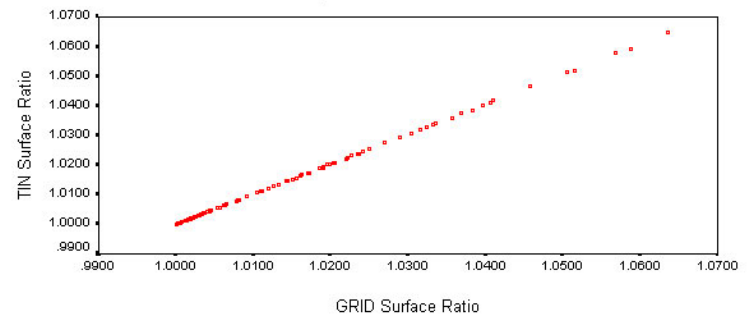
radius = 2,000
1,465 cells
n = 100



n = 100; 1,465 grid cells per circle

TIN and GRID Surface Ratios

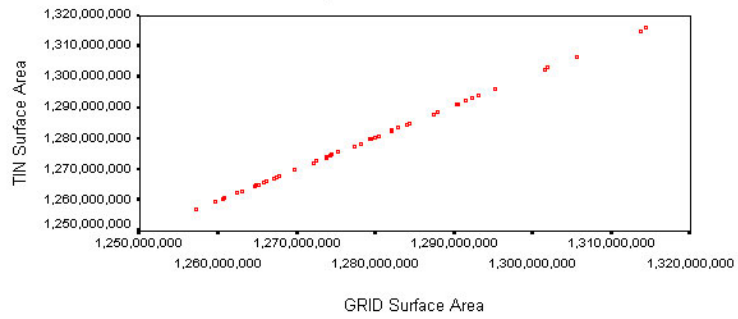
2,000m Radius Circles



n = 100; 1,465 grid cells per circle

TIN and GRID Surface Areas

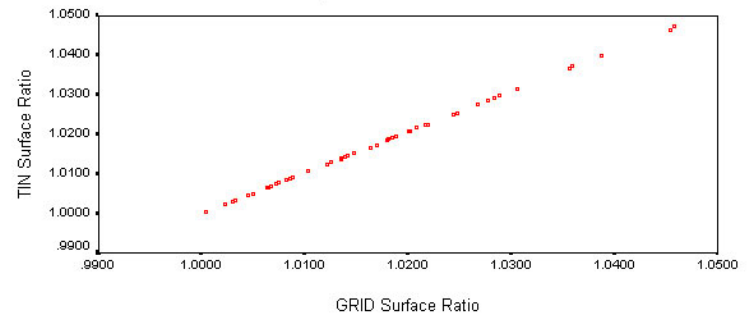
20,000m Radius Circles



n = 50; 146,361 grid cells per circle

TIN and GRID Surface Ratios

20,000m Radius Circles



n = 50; 146,361 grid cells per circle

radius = 20,000
146,361 cells
n = 50

DISCUSSION

This extension appears to generate accurate surface area and surface ratio grids. A cell value in a surface area grid appears to accurately reflect the surface area within that cell's boundaries.

Surface areas within polygons can be accurately measured with this method provided the polygon contains a sufficient number of cells. With the circular polygons used in this analysis, accuracy and precision tended to be very high when cell counts equaled or exceeded 264. However, accuracy decreased significantly when using the Standard method on 2,000m circles (mean cell count = 1,472.39), and I suspect this lack of accuracy was due to a bug in the calculation method. There was clearly something wrong with the calculation method in the case of the 2,000m circles, and therefore I cannot be certain that the problem does not also exist in other cases.

Surface areas using the Modified method appeared to be consistently precise, accurate and reliable for cell counts ≥ 264 .

Accuracy in cases of low cell counts is highly variable when using the Standard method, although most of this variation is likely due to the fact that identically-sized circles randomly placed on a grid will not generally have the same number of cells in each circle. A cell is considered to be included in a circle if that cell's center is located within the circle boundary, and slight deviations in position can cause large numbers of cells to be included or excluded. Accuracy is consistently low in cases of low cell counts when using the Modified method, with GRID-based values tending to be significantly higher than TIN-based values.

RECOMMENDATIONS

I feel that this extension does produce reliable surface area and surface ratio values when the area of interest includes a large number of cells (≥ 264). I leave it to the user to decide whether the Standard or Modified method of generating statistics is preferable. This extension offers the choice to generate statistics using either method. The Standard method is much faster, but did produce some mysterious results in this data set. The modified method appears to consistently overestimate the surface area at low cell counts, but appears highly accurate at high cell counts.

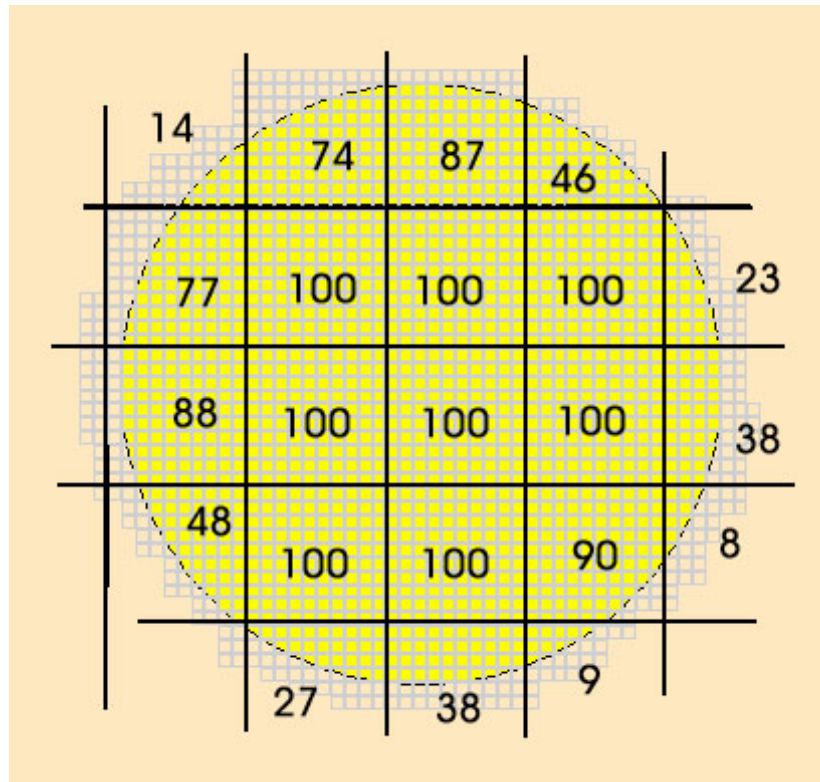
Appendix C: Problems with 'ZonalStatsTable' Request

I created 100 identical circular polygons, each with a radius of 2,000 meters. I randomly distributed them across the landscape, then I did ran a "ZonalStatsTable" request to get polygon statistics based on an underlying grid. All the circles were made exactly the same way, and appear identical except for location. They all have the same number of vertices and the same polygonal area. The grid cell size was 92.662394 meters and the polygon areas were 12,565,732.6848 sq. meters.

$$(12,565,732.6848) / (92.662394\text{-squared}) = 1.463.46$$

Therefore I expected each polygon to have close to 1,463 cells. Naturally this number should vary some because the polygon locations were random. For the most part, this expectation was accurate. 81 of the 100 circular polygons contained between 1460-1468 cells.

The remaining 19 polygons, however, ranged between 1506-1518 cells, with no polygons containing cell counts between 1468 and 1506. I suspected that the cell counts of those polygons containing 1506-1518 cells were not accurate, so I took the circular polygon with the highest number of cells (1518) and actually counted all the cells whose cell centers lay within the polygon boundary. This polygon is pictured below. I used the "CellTools" sample extension to draw a grid representing the cell boundaries, then broke that grid up into 100-cell blocks to make it easier to count.



A cell is considered "inside" the polygon if the cell center is contained within the polygon boundary. According to my count, this polygon only contains about 1,467 cells. According to the ZonalStatsTable request, this polygon contains 1,518 cells.

Therefore, it appears that in some cases the "ZonalStatsTable" request is using more cells than it should in the analysis, and thus the statistics may not reflect the true set of cells enclosed by the polygon.

Alternative Modified Version of ZonalStatsTable:

This extension offers a modified version of the "ZonalStatsTable" request that appears to produce more reliable results. Conceptually, this modified process can be pictured as taking each polygon separately, using it like a cookie cutter to clip out only those grid cells that lie within the polygon, then running 'ZonalStatsTable' on only that polygon and only those clipped grid cells. Technically, the modified version takes each polygon in the theme and converts it into a grid with the same cell size as the Input Grid, producing a grid of "1" values in the shape of the polygon. It then multiplies that grid by the Input Grid, thereby converting all cells outside of the polygon into "No Data" cells while retaining the original cell values inside the polygon. Finally, this extension uses the 'ZonalStatsTable' request on the modified input grid to generate statistics for that single polygon. This process eliminates the possibility that the 'ZonalStatsTable' request will select cells outside of the polygon boundary by converting all cells outside of the polygon boundary into "No Data" values.

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