IMPULSIVE SOURCE SEPARATION WITH APPLICATION TO SPERM WHALE CLICKS

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ABSTRACT

We propose a new method for impulsive source separation in MISO systems based on impulse spacings to classify multiple sperm whale click trans. It is a maximum likelihood algorithm using impulse spacing distributions. Additionally, we develop and test a lower bound, and extend the algorithm to work when the distribution parameters are unknown.

INTRODUCTION

We consider the following problem: Given that there are K sources that generate impulses at random intervals, and a single sensor receives a superposition of these impulsive signals. How do we determine which source generated each impulse from the received signal. (See Fig. 1)

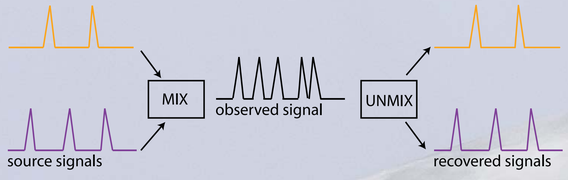


Figure 1. Illustration of source separation problem with only one sensor for impulsive sources. The original signals are mixed when recorded by the sensor, and the goal is to recover the original input signals from the mixture using timing.

The motivation for this problem comes from passive acoustic monitoring of marine mammals. Marine mammals vocalize for various reasons (e.g. communication, navigation and foraging). Passive acoustic monitoring refers to the recording and processing marine mammal vocalizations for the purpose of gaining information about the animals. A common problem in passive acoustic monitoring is separation of source signals. Although this problem can be greatly simplified by using a compact array of hydrophones, passive acoustic systems are often limited to single sensor situation due to financial and practical constraints.

Our approach is based on a common vocalization called the "click." Clicks are quasi-impulsive signals that are produced by odontocetes that often occur in sequences called "click trains" in which clicks are (somewhat) regularly spaced in time. (See Fig. 2)

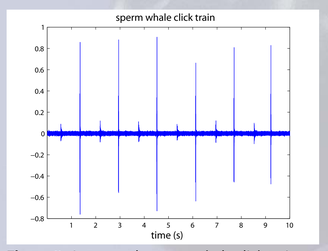


Figure 2. An example sperm whale click train.

Here we will consider classification based solely on timing. That is, given K sources generating events, is it possible to correctly classify events based solely on timing? This problem could have other applications:

- Fetal ECG: Which heartbeat belongs to the baby and which belongs to the mother?
- Particle classification in a radioactive decay process
- Traffic counting: Which car came from where?

SYSTEM MODEL

Each source k transmits a sequence of impulses at times $\tau_{k,i}$. These are transmitted through a linear dispersive system that is due to the source, the environment, and the receiver. Therefore, the total received signal from K sources is

$$s(t) = \sum_{k=1}^{K} \sum_{i=0}^{N_k-1} h_k(t-\tau_{k,i}) + w(t)$$

where N_k is the number of impulses source k transmits, h_k is the impulse response for source k, and $w(t)$ is receiver noise. We assume that the support of the impulse response is much shorter than the pulse spacing, so there is no overlap.

The problem we consider is: given the received signal s(t) , decide which impulses belong to which source. To make this possible we assume:

(continued in next column)

(system model continued)

- The interimpulse spacing $\tau_{k,i}$-$\tau_{k,i-1}$ >0 is distributed according to some prior known distribution $T_k(t)$.
  - We choose it to be a truncated Gaussian.
- The impulses constitute a homogenous Markov process, that is $\tau_{k,i}$-$\tau_{k,i-1}$ is iid.
- The number of sources K is known.

The performance criterion for the algorithms developed here is the error probability, that is the probability that an impulse is wrongly classified. Let the impulse times in the mixed signal $s(t)$ be τ_i (i.e. with unknown source). Let $D_i \in \{1,...,K\}$ denote the source that originated the impulse at time τ_i, and $\hat{D}_i$ an estimate. We then define the error probability for source k as $P_{e,k} = \lim_{j\to\infty} P(\hat{D}_j \neq D_j \mid D_j = k)$ and the total error probability as $P_e = \lim_{j\to\infty} P(\hat{D}_j \neq D_j)$. The algorithms developed here are aimed at minimizing P_e.

TIMING-BASED SEPARATION

We assume some other algorithm has detected the impulses from $s(t)$ and estimated their timing. The problem is therefore to classify the impulses purely using the prior distributions $T_k(t)$ which is done by maximizing the log-likelihood function

$$L = \sum_{k=1}^{K} \sum_{i} \log \mathcal{T}_k(\tau_{S_k(i)} - \tau_{S_k(i-1)})$$

with respect to $S_k(i)$ where $S_k(i)$ denotes the i-th impulse assigned to the k-th source. Brute force maximization has complexity $O(K^M)$, but similar to the Viterbi algorithm, an algorithm with complexity $O(M)$ can be found.

Define an optimum path $\boldsymbol{P}_{i,k}$ as a set of decisions $\{\hat{D}_j; j=1,...,i\text{-}1\}$ leading to $\hat{D}_i = k$ that maximizes the log-likelihood function up to time τ_i. Define the continuation $\boldsymbol{P}_{i,k}(m)$ as a set of decisions that maximizes the log-likelihood function subject to the constraint $\hat{D}_{i+1},...,\hat{D}_{i+m} \neq k$, and let $L(\boldsymbol{P}_{i,k}(m))$ be the corresponding value of the log-likelihood function.

Consider a given pulse time τ_j and the problem of finding the optimum path $\boldsymbol{P}_{j,k}$. For any $d \geq 1$, consider paths with the following property: the path has $D_j = \hat{D}_{j-d} = k$, but $\hat{D}_i \neq k$, $i = j\text{-}d+1,...,j\text{-}1$. The maximum value of the log-likelihood function under that constraint can then be written as

$$L_d = \log \mathcal{T}_k(\tau_j - \tau_{j-d}) + L(\bar{\mathcal{P}}_{j-d,k}(d-1))$$

The key observation is that similar to the Viterbi algorithm, if we have an optimum path ending in $D_j = k$, then future extension of this path will have the same optimum decisions prior to time τ_i. In the Viterbi algorithm, decisions at stage j only depends on decisions at stage j-1 , so paths $\boldsymbol{P}_{i,k}$ can then be discarded as soon as the paths $\boldsymbol{P}_{i,k+1}$ have been found. However, in the timing problem the decision at stage j can depend on decisions at stage j-d for $d > 1$, so it is more difficult to determine when paths can be discarded. We therefore use a heuristic approach instead: a fixed number of paths with a the largest log-likelihood is memorized.

THE ALGORITHM

1. Initialize the assignment of τ_1 such that in the first K paths τ_1 is labeled 1,..., K . The assignment of the remaining a - K paths is arbitrary. Initialize each path's likelihood to 1. Set i=1.
2. For each path consider the extensions where τ_{i+1} is assigned to source k=1,..., K
3. Compute the likelihood of all extensions.
4. From all paths and choose the a most likely paths as the paths to keep.
5. If all τ_i checled, done-- choose the most likely path as the assignment. Otherwise i++ and go to step 2.

ERROR ANALYSIS

We develop a lower bound on the error probability for two sources; the bound can be generalized to more sources. Consider the i-th impulse of source 1, and let $j(i)$ be the nearest impulse of source 2. Let A_i be the event that impulses i and $j(i)$ are nearest neighbors and that the classifications of the i-th impulse and the $j(i)$-th impulse are swapped from their correct values. Since i and $j(i)$ are nearest neighbors, the pair is disjoint from any other nearest neighbor pair. We discount all other possible errors because we are developing a lower bound. A lower bound on the error probability of source 1 therefore is

$$P_{e,1} \geq \lim_{n\to\infty} E\left[\frac{1}{n}\sum_{i=1}^{n} I(\mathcal{A}_i)\right] = \lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} P(\mathcal{A}_i) = P(\mathcal{A}_i)$$

where n is the number of impulses from source 1 and $I(\cdot)$ is the indicator function for event A_i. We arrive at the final result since for large n, the A_i have the same probability (though they are not independent).

We can find a lower bound for $P(A_i)$ by letting a genie provide the correct decision for all impulses except i and $j(i)$. The genie also tells us that impulses i and $j(i)$ are from different sources. Denote $\delta = \tau_{j(i)} - \tau_i$, t_{kp} as the time difference from pulse i to the nearest prior impulse from source k, and t_{ks} as the time difference to the nearest following impulse. Let $\mathbf{t} = (t_{1p}, t_{2p}, t_{1s}, t_{2s})$ and let $d \in \{(12),(21)\}$, where $d = (12)$ means that the first impulse is from source 1, the second from source 2. The optimum decision is now

$$\hat{d} = \arg\max P(d|\mathbf{t}) = \arg\max \frac{f(\mathbf{t}|d)P(d)}{f(\mathbf{t})} = \arg\max f(\mathbf{t}|d)$$

(continued in next column)

(error analysis continued)

where $f(\mathbf{t})$ is the joint pdf of $\mathbf{t}$ obtained from T_k. Here we have used that, due to symmetry, $P(d) = ½$. We normalize time so that the midpoint between pulses i and $j(i)$ is zero. We have

$$f(\mathbf{t}|d=(12)) = \frac{f_1(t_{1p}-\frac{1}{2}\delta)f_1(t_{1s}+\frac{1}{2}\delta)f_2(t_{2p}+\frac{1}{2}\delta)f_2(t_{2s}-\frac{1}{2}\delta)}{P_1(t>\frac{3}{2}\delta)^2 P_2(t>\frac{3}{2}\delta)^2}$$

$$f(\mathbf{t}|d=(21)) = \frac{f_1(t_{1p}+\frac{1}{2}\delta)f_1(t_{1s}-\frac{1}{2}\delta)f_2(t_{2p}-\frac{1}{2}\delta)f_2(t_{2s}+\frac{1}{2}\delta)}{P_1(t>\frac{3}{2}\delta)^2 P_2(t>\frac{3}{2}\delta)^2}$$

where f_i is the pdf for source i. If the timing process is a Poisson process, these two are equal due to the memoryless property of the exponential distribution, which shows that Poisson processes cannot be separated. Let's assume d=(12). An error happens if

$$f_1(t_{1p}+\tfrac{1}{2}\delta)f_1(t_{1s}-\tfrac{1}{2}\delta)f_2(t_{2p}-\tfrac{1}{2}\delta)f_2(t_{2s}+\tfrac{1}{2}\delta) > f_1(t_{1p}-\tfrac{1}{2}\delta)f_1(t_{1s}+\tfrac{1}{2}\delta)f_2(t_{2p}-\tfrac{1}{2}\delta)f_2(t_{2s}+\tfrac{1}{2}\delta)$$

Using renewal theory (Grimmett and Stirzaker, *Probability and Random Processes*, Theorem 10.3.5], for the Gaussian case this can be show to happen with probability (see paper for detail)

$$P_{e,1} \geq \int_0^\infty \left(\int_d^\infty g_{X_2}(t)dt\right)\left(\int_{\sigma_2/\sigma_1 d}^\infty g_{X_1}(t)dt\right)\frac{\sigma_2}{m_2}Q\left(d-\frac{m_2}{\sigma_2}\right)dd$$

UNKNOWN PARAMETERS

Consider the case where T_k depends on an unknown parameter vector. Now we must optimize the discrete assignments and the continuous parameter vector jointly. We will consider the method of clustering: grouping similar data points together. One method to do this is the k-means (KM) algorithm. KM seeks to find the centroids of K groups such that we minimize the the grouping error. To do this at each step we assign points to the closest centroid and then recompute the centroids.

We apply this tehcnique to the timing problem by maximizing similarity instead of minimizing error. That is, iteratively choose the assignment based on our algorithm then we update the unknown means based on the assignment.

RESULTS

Lower Bound Test

- 2 synthetic sources
- fixed means: 1 s and π s
- variable standard deviations: 10^{-3} - 10^{-1}
- 400 clicks
- 500 trials

There is a constant factor 5 in difference between the lower bound and the algorithm performance, but we cannot say if this due to the suboptimality of the algorithm or if it is due to the bound being loose.

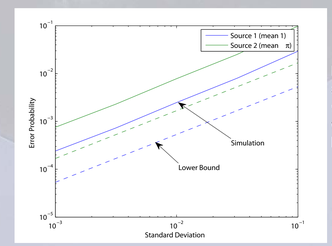
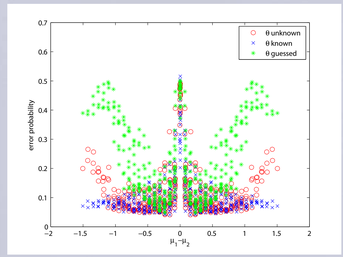


Figure 3. Error probability two synthetic sources is compared with its theoretical lower bound.



Unknown Parmeter Test

- 2 synthetic sources
- variable means: 0.5 - 2 s
- fixed standard deviation: 10^{-1}
- 100 clicks
- 100 trials
- Initialize: m_1=2.2, m_2=2.5, & σ_1=σ_2=0.13

The unknown parameter algorithm performance approaches that of the original algorithm if we know the means. Also we see an improvement over a static guess.

Figure 4. Error probability is compared between the unknown parameter alg., the original alg. with known parameters and with static guess.

Tests on Sperm Whale Data

- Artificial mix of sperm whale recordings (Adam et al., *Overview of the 205 workshop...*)
- 5 pairs of 40 s segments

From this sample set, it is clear that $a = K^3$ has better performance than $a = K$. We also see that the unknown parameter algorithm performance does not deviate too far from the cases when the parameters are known.

Table 1. Algorithm performance on sperm whale clicks

m_1	m_2	%err K	%errK^3	%err KM
1.39	0.93	0	0	0
0.80	1.27	10	0	0
0.88	0.92	4.3	0	0
0.87	0.89	8.2	5.9	5.9
0.82	1.39	1.3	0	5.2

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