IMPULSIVE SOURCE SEPARATION WITH APPLICATION TO SPERM WHALE CLICKS

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ABSTRACT

This paper proposes a new method for impulsive source separation in multi-input single-output (MISO) systems based on impulse spacing. The primary application of the method is the classification of click trains from multiple marine mammals. The impulse spacing can be modeled as a renewal process, so to classify sources, an algorithm is developed to maximize the likelihood of click assignment based on the impulse spacing distributions. A theoretical lower bound on the performance is derived and the algorithm shown to get reasonably close to the bound. Additionally, the algorithm is further extended for use when the impulse spacing distribution parameters are unknown. The algorithm is tested with recordings of sperm whales, and the separation is shown to be accurate.

***Index Terms*—Source separation, renewal process, marine mammals, impulsive sources, multi-input single-output**

I. INTRODUCTION

In this paper, the problem considered is the following. There are K sources that generate impulses at random intervals, and a single sensor receives a superposition of these impulsive signals. The problem is to classify which source generated each impulse from the received signal. See Fig. 1 for an illustration.

The motivation for the problem comes from passive acoustic monitoring of marine mammals. Marine mammals vocalize for various reasons, including communication, navigation and foraging. Passive acoustic monitoring refers to the recording and processing marine mammal vocalizations for the purpose of gaining information about the animals [1]. A common problem in passive acoustic monitoring is separation of source signals. That is, given a recording with vocalizations from multiple animals, how many animals are recorded and which animal made which vocalization? Although this problem can be greatly simplified by using a compact array of hydrophones, passive acoustic systems are often limited to single or widely-spaced sensors due to financial and practical constraints.

In the single sensor case, various clues can help with association, such as the similarity of signal characteristics between associated vocalizations and the presence of multipath arrivals. This paper is concerned with another clue: vocalization timing. This approach is based on a common vocalization called the "click." Clicks are quasi-impulsive signals that are produced by odontocetes [2]. Although the specific characteristics of clicks (e.g. spectral characteristics and range, duration, source level) vary widely between species, they often occur in sequences called "click trains" in which clicks are (somewhat) regularly spaced in time. That is, the time between successive clicks from a single animal (called the "inter-click-interval" among biologists, or "impulse spacing" here) is randomly distributed about some mean. Animals can and do vary their mean impulse spacing over time, but a constant-mean can be a good approximation over short time periods.

In this paper we consider classification based *solely* on timing. That is, given K sources generating events, is it possible to correctly classify events based solely on timing? This problem could have other applications. For example, in a mixture of heartbeats (e.g. in fetal ECG), is it possible to say which heartbeat belongs to the mother and which to the fetus based solely on timing? Or if a radioactive decay process is observed and only the timing of particle detection is recorded, not the kind of particle, is it possible to determine which type of particle generated each event? Another example is traffic counting: is it be possible to say what the origins of vehicles are (i.e. where they came from) based only on time of arrival at an intersection? In spite of

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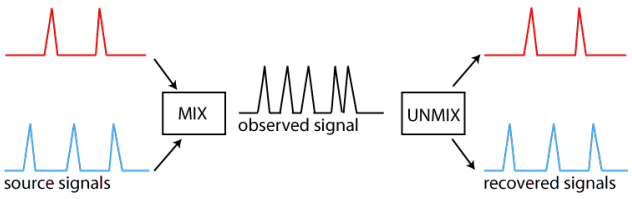


Fig. 1. This is an illustration of the problem considered in this paper for two arbitrary sources. The original signals are mixed when recorded by the sensor, and the goal is to recover the original input signals from the mixture using timing.

a wide range of possible applications, and a well-defined mathematical setup, the authors have not been able to find any other published paper considering this problem. Previous work on separating impulse sources include [3], [4], [5], [6], but none of these consider separation based on timing only. One reason could be that the most common point process is the Poisson process, and Poisson processes cannot be exactly separated based on timing (see Sec. IV).

II. SYSTEM MODEL

Each source k transmits a sequence of impulses at times $\tau_{k,i}$. These are transmitted through a linear dispersive system that is due to the source, the environment, and the receiver. Therefore, the total received signal from K sources is

$$s(t) = \sum_{k=1}^{K} \sum_{i=0}^{N_k-1} h_k(t - \tau_{k,i}) + w(t), \qquad (1)$$

where N_k is the number of impulses source k transmits, h_k is the impulse response for source k, and $w(t)$ is receiver noise. We assume that the support of the impulse response is much shorter than the pulse spacing, so there is no overlap.

The problem we consider is: given the received signal $s(t)$, decide which impulses belong to which source. To make this possible we assume the following

- The interimpulse spacing $\tau_{k,i} - \tau_{k,i-1} > 0$ is distributed according to some prior known distribution $\mathcal{T}_k(t)$.
- The impulses constitute a homogenous Markov process, that is $\tau_{k,i} - \tau_{k,i-1}$ is iid.
- The number of sources K is known.

Thus, the set of event times for a single source constitutes a renewal process, but the total set of event times is generally not a renewal process [7].

The performance criterion for the algorithms developed here is the error probability, that is the probability that an impulse is wrongly classified. Let the impulse times in the mixed signal (1) be τ_i (i.e., with unknown source). Let $D_i \in \{1, \dots K\}$ denote the source that originated the impulse at time τ_i, and $\hat{D}_i$ an estimate. We then define the error probability for source k as $P_{e,k} = \lim_{j\to\infty} P(\hat{D}_j \neq D_j | D_j = k)$ and the total error probability as $P_e = \lim_{j\to\infty} P(\hat{D}_j \neq D_j)$. The algorithms developed here are aimed at minimizing P_e.

For the prior distributions $\mathcal{T}_k(t)$, we use a truncated Gaussian which can be argued for using the central limit theorem or maximum entropy [8]. The distribution is truncated at 0 to the enforce the constraint of non-negative pulse spacings. However, in the application we consider, $\mu \gg \sigma$. Therefore the correction due to truncation is very small and can be ignored in many calculations.

III. TIMING-BASED SEPARATION

We assume some other algorithm has detected the clicks (impulses) from (1) and estimated their timing. The problem is therefore to classify the impulses purely using the prior distributions $\mathcal{T}_k(t)$. It is easily seen that the optimum classification is done by maximizing the log-likelihood function [9]

$$L = \sum_{k=1}^{K} \sum_{i} \log \mathcal{T}_k(\tau_{S_k(i)} - \tau_{S_k(i-1)}) \qquad (2)$$

with respect to $S_k(i)$. Here $S_k(i)$ denotes the i-th impulse *assigned* to the k-th source, and is a function of $\hat{\mathbf{D}} = (D_1, D_2, \dots)$. For example, if for two sources $\hat{\mathbf{D}} = (1, 1, 2, 1, 2, 2)$ then $\mathbf{S}_1 = (1, 2, 4)$ and $\mathbf{S}_2 = (3, 5, 6)$. Brute force maximization has complexity $O(K^M)$, but similar to the Viterbi algorithm [10], [11], an algorithm with complexity $O(M)$ can be found.

Define an optimum path $\mathcal{P}_{i,k}$ as a set of decisions $\{\hat{D}_j; j = 1 \dots i-1\}$ leading to $\hat{D}_i = k$ that maximizes the log-likelihood function up to time τ_i. Define the continuation $\bar{\mathcal{P}}_{i,k}(m)$ as a set of decisions $\hat{D}_{i+1}, \dots \hat{D}_{i+m} \neq k$ that maximizes the log-likelihood function subject to the constraint $\hat{D}_{i+1}, \dots \hat{D}_{i+m} \neq k$; let $L(\bar{\mathcal{P}}_{i,k}(m))$ be the corresponding value of the log-likelihood function.

Consider a given pulse time τ_j and the problem of finding the optimum path $\mathcal{P}_{j,k}$. For any $d \geq 1$ consider paths with the following property: the path has $\hat{D}_j, \hat{D}_{j-d} = k$, but $\hat{D}_i \neq k, i = j-d+1, \dots, j-1$. The maximum value of the log-likelihood function under that constraint can then be written as

$$L_d \quad = \quad \log \mathcal{T}_k(\tau_j - \tau_{j-d}) + L(\bar{\mathcal{P}}_{j-d,k}(d-1)). \qquad (3)$$

The key observation is that similar to the Viterbi algorithm, if we have an optimum path ending in $\hat{D}_i = k$ then future extension of this path will have the same optimum decisions prior to time τ_i. In the Viterbi algorithm decisions at stage j only depends on decisions at stage $j-1$, so paths $\mathcal{P}_{i,k}$ can then be discarded as soon as the paths $\mathcal{P}_{i+1,k}$ have been found. However, in the timing problem the decision at stage j can depend on decisions at stage $j-d$ for $d > 1$, so it is more difficult to determine when paths can be discarded. A strict criterion for when to discard paths can be developed for two sources, but it is very difficult to generalize to more than two sources. We therefore use a heuristic approach instead: a fixed number, a, of paths with the largest log-likelihood is memorized (see Alg. 1). It is not clear how many paths need to be kept. Based on trial and error with K and K^3, K^3 seems to suffice.

IV. ERROR ANALYSIS

We develop a lower bound on the error probability for two sources; the bound can be generalized to more sources. Consider the i-th impulse of source 1, and let $j(i)$ be the nearest impulse of source 2. Let $\mathcal{A}_i$ be the event that impulses i and $j(i)$ are nearest neighbors and that the classifications of the i-th impulse and the $j(i)$-th impulse

Algorithm 1 Separation based solely on timing

Given K sources and $\mathcal{T}_k$ with observations $\tau_i, i = 1, \ldots, M$, and $a = K^b, b \in \mathbb{Z}^+$ paths

1) Initialize the assignment of τ_1 such that in the first K paths τ_1 is labeled $1, \ldots, K$. The assignment of the remaining $a - K$ paths is arbitrary. Initialize each path's likelihood to 1. Set $i = 1$.
2) For each path consider the extensions where τ_{i+1} is assigned to source $k = 1, \ldots, K$
3) Compute the likelihood of all extensions according to (3).
4) Examine all extensions for all paths and choose the a most likely paths as the paths to keep.
5) If $i = M$, done. Choose the most likely path as the assignment. Otherwise, increment i and go to step 2.

are swapped from their correct values (i.e. i is assigned to 2 and j is assigned to 1). If $\mathcal{A}_i$ happens, one error for source 1 happens. Since i and $j(i)$ are nearest neighbors, the pair is disjoint from any other nearest neighbor pair. We discount all other possible errors because we are developing a lower bound. A lower bound on the error probability of source 1 therefore is

$$\begin{aligned} P_{e,1} &\geq \lim_{n\to\infty} E\left[\frac{1}{n}\sum_{i=1}^{n} I(\mathcal{A}_i)\right] \\ &= \lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} P(\mathcal{A}_i) \\ &= P(\mathcal{A}_i), \end{aligned}$$

where n is the number of impulses from source 1 and $I(\cdot)$ is the indicator function for event $\mathcal{A}_i$. We arrive at the final result since for large n, the $\mathcal{A}_i$ have the same probability (though they are not independent).

We can find a lower bound for$P(\mathcal{A}_i)$ by letting a genie provide the correct decision for all impulses except i and $j(i)$. The genie also tells us that impulses i and $j(i)$ are from different sources. Denote $\delta = \tau_{j(i)} - \tau_i$, t_{kp} as the time-difference from pulse i to the nearest prior impulse from source k, and t_{ks} as the time-difference to the nearest following impulse. Let $\mathbf{t} = (t_{1p}, t_{2p}, t_{1s}, t_{2s})$ and let $d \in \{(12), (21)\}$, where $d = (12)$ means that the first impulse is from source 1, the second from source 2. The optimum decision is now

$$\begin{aligned} \hat{d} &= \text{argmax} P(d|\mathbf{t}) = \text{argmax}\frac{f(\mathbf{t}|d)P(d)}{f(\mathbf{t})} \\ &= \text{argmax} f(\mathbf{t}|d), \end{aligned}$$

where $f(\mathbf{t})$ is the joint pdf of $\mathbf{t}$ obtained from $\mathcal{T}_k$. Here we have used that, due to symmetry, $P(d) = \frac{1}{2}$. We normalize time so that the midpoint between pulses i and $j(i)$ is zero. We have

$$\begin{aligned} f(\mathbf{t}|d=(12)) =& f_1(t_{1p} - \tfrac{1}{2}\delta | t_{1p} > \tfrac{3}{2}\delta) \\ & \times f_1(t_{1s} + \tfrac{1}{2}\delta | t_{1s} > \tfrac{3}{2}\delta) \\ & \times f_2(t_{2p} + \tfrac{1}{2}\delta | t_{2p} > \tfrac{3}{2}\delta) \\ & \times f_2(t_{2s} - \tfrac{1}{2}\delta | t_{2s} > \tfrac{3}{2}\delta) \\ =& \frac{f_1(t_{1p} - \frac{1}{2}\delta) f_1(t_{1s} + \frac{1}{2}\delta) f_2(t_{2p} + \frac{1}{2}\delta)}{P_1(t > \frac{3}{2}\delta)^2 P_2(t > \frac{3}{2}\delta)^2} \\ & \times f_2(t_{2s} - \tfrac{1}{2}\delta) \\ f(\mathbf{t}|d=(21)) =& \frac{f_1(t_{1p} + \frac{1}{2}\delta) f_1(t_{1s} - \frac{1}{2}\delta) f_2(t_{2p} - \frac{1}{2}\delta)}{P_1(t > \frac{3}{2}\delta)^2 P_2(t > \frac{3}{2}\delta)^2} \\ & \times f_2(t_{2s} + \tfrac{1}{2}\delta), \end{aligned}$$

where f_i is the pdf for source i. If the timing process is a Poisson process, these two are equal due to the memoryless property of the exponential distribution, which shows that Poisson processes cannot be separated. Let's assume $d = (12)$. An error happens if

$$\begin{aligned} & f_1(t_{1p} + \tfrac{1}{2}\delta) f_1(t_{1s} - \tfrac{1}{2}\delta) f_2(t_{2p} - \tfrac{1}{2}\delta) f_2(t_{2s} + \tfrac{1}{2}\delta) \\ & > f_1(t_{1p} - \tfrac{1}{2}\delta) f_1(t_{1s} + \tfrac{1}{2}\delta) f_2(t_{2p} - \tfrac{1}{2}\delta) f_2(t_{2s} + \tfrac{1}{2}\delta). \end{aligned}$$

In particular if we let $f_i \sim \mathcal{N}\left(m_i, \sigma_i^2 \middle| 0\right)$ (where $\mathcal{N}\left(m, \sigma^2 \middle| t\right)$ denotes a Gaussian with mean m and variance σ^2 truncated to t), an error happens if

$$\begin{aligned} & \frac{1}{\sigma_1^2}(t_{1p} + \frac{1}{2}\delta - m_1)^2 + \frac{1}{\sigma_1^2}(t_{1s} - \frac{1}{2}\delta - m_1)^2 \\ & \quad + \frac{1}{\sigma_2^2}(t_{2p} - \frac{1}{2}\delta - m_2)^2 + \frac{1}{\sigma_2^2}(t_{2s} + \frac{1}{2}\delta - m_2)^2 \\ & < \frac{1}{\sigma_1^2}(t_{1p} - \frac{1}{2}\delta - m_1)^2 + \frac{1}{\sigma_1^2}(t_{1s} + \frac{1}{2}\delta - m_1)^2 \\ & \quad + \frac{1}{\sigma_2^2}(t_{2p} + \frac{1}{2}\delta - m_2)^2 + \frac{1}{\sigma_2^2}(t_{2s} - \frac{1}{2}\delta - m_2)^2. \end{aligned}$$

Equivalently,

$$\begin{aligned} 0 < & -\frac{1}{\sigma_1^2}(t_{1p} - m_1) + \frac{1}{\sigma_1^2}(t_{1s} - m_1) - \frac{1}{\sigma_2^2}(t_{2s} - m_2) \\ & + \frac{1}{\sigma_2^2}(t_{2p} - m_2). \end{aligned}$$

Let us rewrite this in terms of the inter-impulse spacing, $r_{1p} = t_{1p} - \frac{1}{2}\delta$, $r_{1s} = t_{1s} + \frac{1}{2}\delta$ etc.,

$$\begin{aligned} \left(\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}\right)\delta < & -\frac{1}{\sigma_1^2}(r_{1p} - m_1) + \frac{1}{\sigma_1^2}(r_{1s} - m_1) \\ & - \frac{1}{\sigma_2^2}(r_{2s} - m_2) + \frac{1}{\sigma_2^2}(r_{2p} - m_2). \end{aligned}$$

We can further lower bound the error probability by

$$\begin{aligned} P_{e,1} \geq & P\left(-\frac{1}{\sigma_1}(r_{1p} - m_1) + \frac{1}{\sigma_1}(r_{1s} - m_1) > \frac{1}{\sigma_1}\delta\right) \\ & \times P\left(-\frac{1}{\sigma_2}(r_{2p} - m_2) + \frac{1}{\sigma_2}(r_{2s} - m_2) > \frac{1}{\sigma_2}\delta\right). \end{aligned}$$

Here $\frac{1}{\sigma_1}(r_{1p}-m_1) \sim \mathcal{N}\left(0,1\left|\frac{-m_1+\delta}{\sigma_1}\right.\right)$ and $\frac{1}{\sigma_1}(r_{1s}-m_1) \sim \mathcal{N}\left(0,1\left|\frac{-m_1+2\delta}{\sigma_1}\right.\right)$ and if $X_1 = -\frac{1}{\sigma_1}(r_{1p}-m_1)+\frac{1}{\sigma_1}(r_{1s}-m_1)$ then

$$f_{X_1}(t) = -\frac{e^{-\frac{t^2}{4}}}{4\sqrt{\pi}Q\left(\frac{-m_1+\delta}{\sigma_1}\right)Q\left(\frac{-m_1+2\delta}{\sigma_1}\right)} \times\left(\text{Erf}\left[\frac{-m_1+\delta}{\sigma_1}-\frac{t}{2}\right]+\text{Erf}\left[\frac{-m_1+2\delta}{\sigma_1}-\frac{t}{2}\right]\right)$$

where $-2m_1+3\delta \le t$. Let

$$\begin{aligned} g_{X_1}(t) =& -\frac{1}{4\sqrt{\pi}}e^{-\frac{t^2}{4}}\left(\text{Erf}\left[\frac{-m_1+\delta}{\sigma_1}-\frac{t}{2}\right]\right. \\ &\left.+\text{Erf}\left[\frac{-m_1+2\delta}{\sigma_1}-\frac{t}{2}\right]\right) \\ =&\frac{2}{4\sqrt{\pi}}e^{-\frac{t^2}{4}}\left(Q\left[\sqrt{2}\left(\frac{-m_1+\delta}{\sigma_1}-\frac{t}{2}\right)\right]\right. \\ &\left.+Q\left[\sqrt{2}\left(\frac{-m_1+2\delta}{\sigma_1}-\frac{t}{2}\right)\right]-1\right). \end{aligned}$$

where $Q(x) = 1-\Phi(x)$, $\Phi(x)$ being the cdf of the normal Gaussian [10]. According to renewal theory [7, Theorem 10.3.5], $D = \frac{\delta}{\sigma_2}$ has distribution

$$f_D(d) = \frac{\sigma_2}{m_2}Q\left(d-\frac{m_2}{\sigma_2}\right).$$

We can then low bound the error probability by

$$P_{e,1} \ge \int_0^\infty \left(\int_d^\infty g_{X_2}(t)dt\right)\left(\int_{\sigma_2/\sigma_1 d}^\infty g_{X_1}(t)dt\right) \frac{\sigma_2}{m_2}Q\left(d-\frac{m_2}{\sigma_2}\right)dd. \quad (4)$$

V. UNKNOWN PARAMETERS

So far we assumed that the distributions $\mathcal{T}_k$ are known. We now extend this to the case where $\mathcal{T}_k$ depends on an unknown parameter vector $\boldsymbol{\theta}_k$ (e.g. $\boldsymbol{\theta}_k = [m_k, \sigma_k^2]$ where m_k and σ_k^2 are the mean and variance of a Gaussian $\mathcal{T}_k$). (2) is extended to

$$L = \sum_{k=1}^{K}\sum_i \log \mathcal{T}_{k,\hat{\boldsymbol{\theta}}_k}(\tau_{S_k(i)} - \tau_{S_k(i-1)}) \quad (5)$$

(recall that $S_k(i)$ is a function of $\hat{\mathbf{D}}$ as explained after (2)). This is a mixed optimization problem: $\mathbf{D}$ is discrete and $\boldsymbol{\theta}_k$ is continuous. Separately, the problems are not difficult to solve; the challenge is to solve them jointly. In order to understand the nature of the problem we have tried brute-force optimization calculating L for all possible M^K assignments, and for variable values of $\boldsymbol{\Theta} = [\boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_K]$. Results confirmed that L is maximized at the correct $\boldsymbol{\Theta}$. However, the function has many local maxima, which makes optimization challenging.

Algorithm 2 KM timing separation algorithm

Given the observed impulse times $\tau_i, i = 1, \ldots, M$ that we want to assign to K sources

1) Choose some initial parameters $\boldsymbol{\Theta}^{[0]}$
2) Assignment: For iteration t, assign each each point maximize overall likelihood by using Alg. 1 with $\boldsymbol{\Theta}^{[t]}$ (i.e. find $\hat{\mathbf{D}}^{[t]}$)
3) Update: Given the new assignment, compute $\boldsymbol{\Theta}^{[t+1]}$ using maximum likelihood estimation on $\hat{\mathbf{D}}^{[t]}$
4) If $\|\boldsymbol{\Theta}^{[t+1]} - \boldsymbol{\Theta}^{[t]}\| \le \varepsilon$ where ε is some acceptable error, then done. Otherwise, go to step 2.

Another way to cast the problem is to think of it in terms of clustering. The goal of clustering is to group similar data points together without specified targets. For example, given a set of animal pictures, group the pictures by animal type. Or in our case, given a set of impulse times, group the impulses by mean impulse spacing. In general, we have a set of observations x and an unobservable/missing data set z (e.g. source assignment) which make up the complete data set $y = [x, z]$. We need to find z. One method to do this is the k-means (KM) algorithm. KM seeks to find the centroids/mean of K groups such that we minimize the the error

$$J = \frac{1}{M}\sum_{i=1}^{M}\|x(i) - z(i)\|, \quad (6)$$

where $\|\cdot\|$ is an appropriate distance measure, $x(i)$ denotes the i-th observation out of M and $z(i)$ denotes the centroid assigned to the i-th observation. It is a simple algorithm where we assign points at each step to the closest centroid and then recompute the centroids. Eventually we will converge because at each iteration we can only reduce J or remain the same. [12], [13]

We can apply this method to the timing problem by maximizing similarity (i.e. likelihood) instead of minimizing error. That is, instead of minimizing (6), maximize (5) by updating the unknown means in $\boldsymbol{\Theta}$ at each step based on the assignment from Alg. 1 (see Alg. 2). To make the algorithm equivalent to KM, we assume that the variance of all $\mathcal{T}_k$ are fixed. Experimentally, this assumption has proved beneficial and reasonable.

With this algorithm there is some concern with initialization (i.e. how do we choose $\boldsymbol{\Theta}^{[0]}$?). Since the underlying structure is complicated, the simplest way to investigate sensitivity to initialization is to try many points. For these experiments, we consider only two sources so our results are tractable. Furthermore, we only evaluate the success of the algorithm on an appropriate range for sperm whales. That is, interimpulse times ranging from 0.5s up to 2s with average standard deviation 0.1276s. [6] Based on these experiments, we determine that we should choose initial means that are just outside the range interimpulse times with each mean

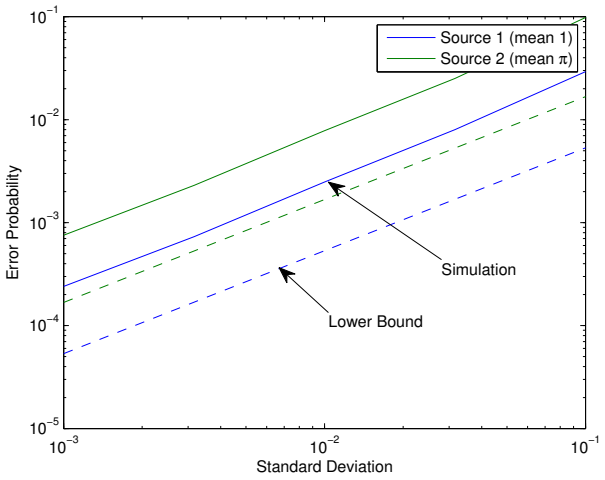


Fig. 2. For two synthetic sources with fixed mean impulse spacings and identical standard deviations, the error probability of each source from Alg. 1 with $a = K^3$ was averaged over 500 trials. The theoretical lower bound (4) was also calculated.

separated by about 2σ. For two sperm whale sources we choose $\mathbf{\Theta}^{[0]} = [(2.2, 0.13^2), (2.5, 0.13^2)]$.

VI. SIMULATED RESULTS

VI-A. Algorithm with known parameters

To validate Alg. 1 and the derived lower bound (4), we consider two synthetic sources with mean impulse spacings of 1s and πs with identical standard deviations ranging from 10^{-3} to 10^{-1}. Each source is generated to have 400 clicks. The algorithm with $a = K^3$ is applied to this data and the error probability for each source is averaged over 500 trials. Fig. 2 compares the lower bound with the result. There is a constant factor 5 in difference between the lower bound and the algorithm performance, but we cannot say if this due to the suboptimality of the algorithm or if it is due to the bound being loose.

VI-B. Algorithm with unknown parameters

To validate the KM algorithm for unknown parameters, we consider two synthetic sources with mean impulse spacing varying from 0.5s to 2s and standard deviations of 0.1s. Each source is generated to have 100 clicks. As mentioned earlier, we initialize the KM algorithm with $\mathbf{\Theta}^{[0]} = [(2.2, 0.13^2), (2.5, 0.13^2)]$. Averaging over 100 trials we find that the performance, total error probability of predicted assignment, of this algorithm approaches that of the algorithm when we know $\mathbf{\Theta}$ (see Fig. 3). Additionally we see that the KM modification to the algorithm is an improvement over using a static guess for $\mathbf{\Theta}$ especially when the difference in the means is large. Here we also notice

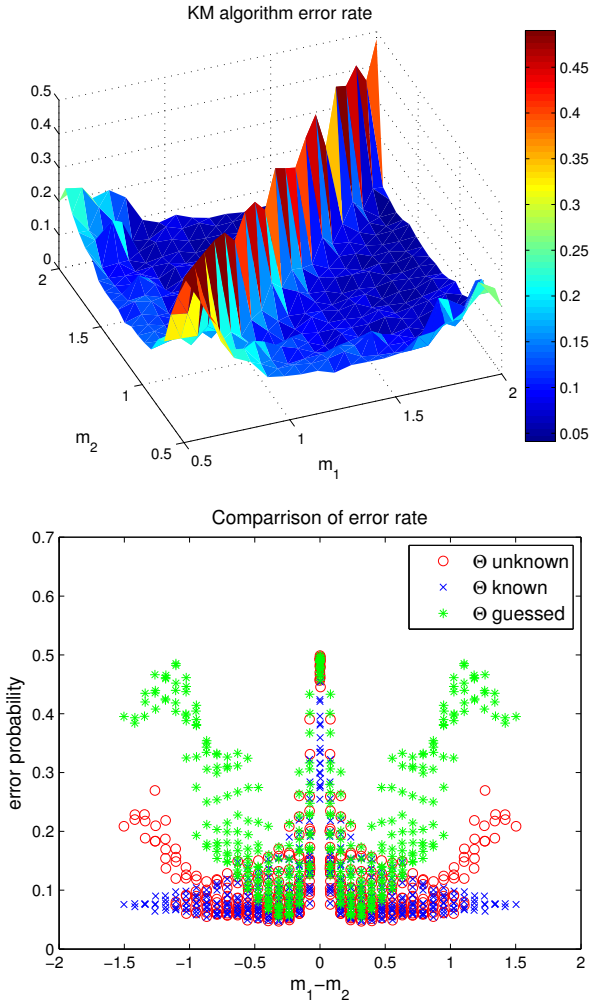


Fig. 3. (top) The total error probability was computed and averaged over 100 trials for two synthetic sources with fixed variance ($\sigma = 0.1$s) and variable means using the KM algorithm initialized with means 2.2s and 2.5s and $\sigma = 0.13$s. (bottom) Also as a function of the difference between source means, this result ($\mathbf{\Theta}$ unknown) was compared with the performance of Alg. 1 using correct means and standard deviations ($\mathbf{\Theta}$ known), and Alg. 1 guessing the means using the same initialization as the KM algorithm ($\mathbf{\Theta}$ guessed).

that the algorithm performs poorly when the input means are equal. This is because when we have equal means we can assign clicks to any source with the same likelihood.

Based on these simulations, we conclude that our KM algorithm can be used in practice when $\mathbf{\Theta}$ is unknown as we will get approximately the same performance as if we had known $\mathbf{\Theta}$.

VII. RESULTS ON SPERM WHALE CLICKS

A general difficulty with marine mammal bioacoustics is that it is usually not possible to make controlled experiments.

We therefore tested the algorithm on recordings of a single sperm whale where we mixed the timings from two different segments. The data set used in this paper comes from [14]. To extract click times we use the techniques outlined in [15].

Five pairs of 40s segments were chosen to test the algorithm. In Table 1, we list the sample means of the segments chosen; the sample standard deviations are not shown but were all around 0.1s. The algorithm for $a = K$ and $a = K^3$ maintained paths with known distribution parameters and the proposed KM algorithm (initialized with means 0.75 and 1.5s and standard deviation of 0.13s) were applied and resulting percent error is recorded in Table 1. From this sample set, it is clear that K^3 has better performance than K. We also see that the KM algorithm performance does not deviate too far from the cases when the parameters are known.

Table 1. Algorithm performance on sperm whale data

m_1	m_2	%err K	%errK^3	%err KM
1.39	0.93	0	0	0
0.80	1.27	10	0	0
0.88	0.92	4.3	0	0
0.87	0.89	8.2	5.9	5.9
0.82	1.39	1.3	0	5.2

VIII. CONCLUSIONS

In this paper we have shown that separation of impulse sources is possible based solely on timing information. The performance of both known and unknown distributions was shown to be fairly good. To improve the performance, we plan to incorporate pulse shape in classification. In the future, we also plan to combine this algorithm with the detection step in hopes that it will further enhance the results for both detection and classification.

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