Lecture 5. The logarithmic sublayer and surface roughness

In this lecture…

- Similarity theory for the logarithmic sublayer.
- Characterization of different land and water surfaces for surface flux parameterization

Monin-Obukhov scaling in the surface layer

Near a solid boundary, in the `surface layer', vertical fluxes are transported primarily by eddies with a lengthscale much smaller than in the center of the BL. A very successful similarity theory is based on dimensional reasoning (Monin and Obukhov, 1954). It postulates that near any given flat surface, the vertical gradients of the mean wind and thermodynamic profiles should be determined purely by the height z above the surface (which limits the vertical size of the eddies that carry the turbulent fluxes) and the surface fluxes which drive turbulence:

1. Surface mom. flux. often expressed as **friction velocity** $u_* = \left(\overline{u'w'}_0^{\,2} + \overline{v'w'}_0^{\,2}\right)^{1/4}$.
2. Surface buoyancy flux $B_0 = \overline{w'b'}_0$.

In the ABL, a typical u_* might be 0.3 m s^{-1}. A typical range of B_0 would be -3×10^{-4} m^{2}s^3 (nighttime) to 1.5×10^{-2} m^{2}s^{-3} (midday), corresponding to a virtual heat flux of -10 W m^{-2} at night and 500 W m^{-2} at midday. One can construct from these fluxes the

$$\textbf{Obukhov length } L = -u_*^3/kB_0 \qquad (5.1)$$

Here $k = 0.4$ is the **von Karman constant**, whose physical significance we'll discuss shortly. L is positive for stable BLs and negative for unstable BLs. The example values above give $L = 200$ m (nighttime) and -5 m (midday).

The logarithmic sublayer (Garratt, p. 41)

At height z, the characteristic eddy size, velocity, and buoyancy scale with z, u_* and B_0/u_*. If the buoyant acceleration acts over the eddy height, it would contribute a vertical velocity $(z\delta b)^{1/2} = (z B_0/u_*)^{1/2}$. If $z < |L|$, this buoyancy driven contribution to the vertical velocity is much smaller than the shear-driven inertial velocity scale u_*, so buoyancy will not significantly affect the eddies. In this case, the mean wind shear will depend only on u_* and z. From now on, we will use an unprimed variable to refer an ensemble mean (dropping the overline) unless otherwise stated. We will also rotate our coordinates so $+x$ is the near-surface mean wind direction and the near-surface momentum flux is $-\overline{u'w'}_0\,\mathbf{i}$. By dimensional reasoning,

$$du/dz = u_*/kz \qquad (z << |L|) \qquad (5.2)$$

This can also be viewed in terms of an eddy viscosity consistent with mixing length theory:

$$\overline{u'w'}_0 = -u_*^2 = -u_*kz\, du/dz = -K_m du/dz$$

$$K_m \propto (u_*)(kz) \quad = \text{(eddy velocity)(eddy length)} \qquad (5.3)$$

The von Karman constant k is the empirically determined constant of proportionality in (5.2). Integrating (5.2), we get the **logarithmic velocity profile** law:

$$u(z)/u_* = k^{-1} \ln(z/z_0) \qquad (z << |L|) \qquad (5.4)$$

The constant of integration z_0 depends on the surface and is called the **roughness length**. The figure below shows measured near-surface velocity profiles from the Wangara experiment that compare very well with the predicted log-layer structure and are consistent with z_0 = 5 mm.

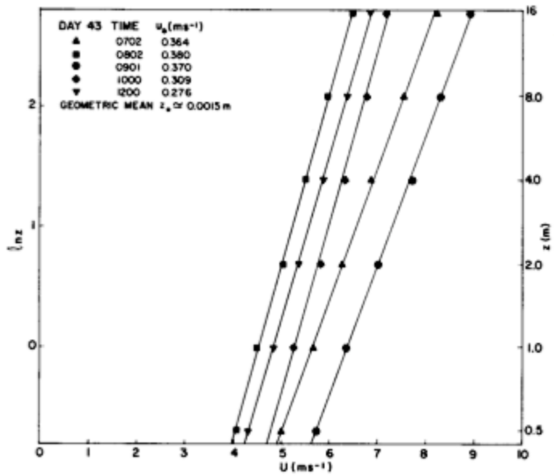

Fig. 10.4 Comparison of the observed wind profiles in the neutral surface layer of day 43 of the Wangara Experiment with the log law [Eq. (10.6)] (solid lines). [Data from Clarke et al. (1971).]

Roughness length of various surfaces (Garratt, Ch. 4)

The roughness length is loosely related to the typical height of closely spaced surface obstacles, often called roughness elements (e. g. water waves, trees, buildings, blades of grass). It depends on the distribution as well as the height h_c of roughness elements (see figure below), but as a rule of thumb,

$$z_0 \sim 0.1 h_c \,. \tag{5.5}$$

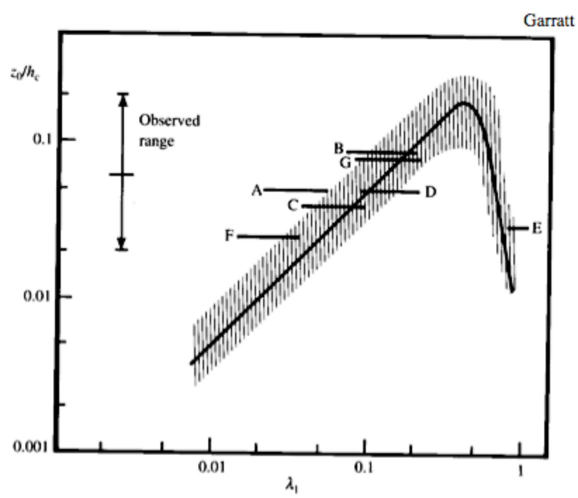

Fig. 4.1 Variation of z_0/h_c with element density, based on the results of Kutzbach (1961), Lettau (1969) and Wooding *et al.* (1973), represented by the shaded area and solid curve. Some specific atmospheric data are also shown as follows: A and B, trees; C and D, wheat; E, pine forest; F, parallel flow in a vineyard; G, normal flow in a vineyard. Analogous wind-tunnel data are described in Seginer (1974). From Garratt (1977b).

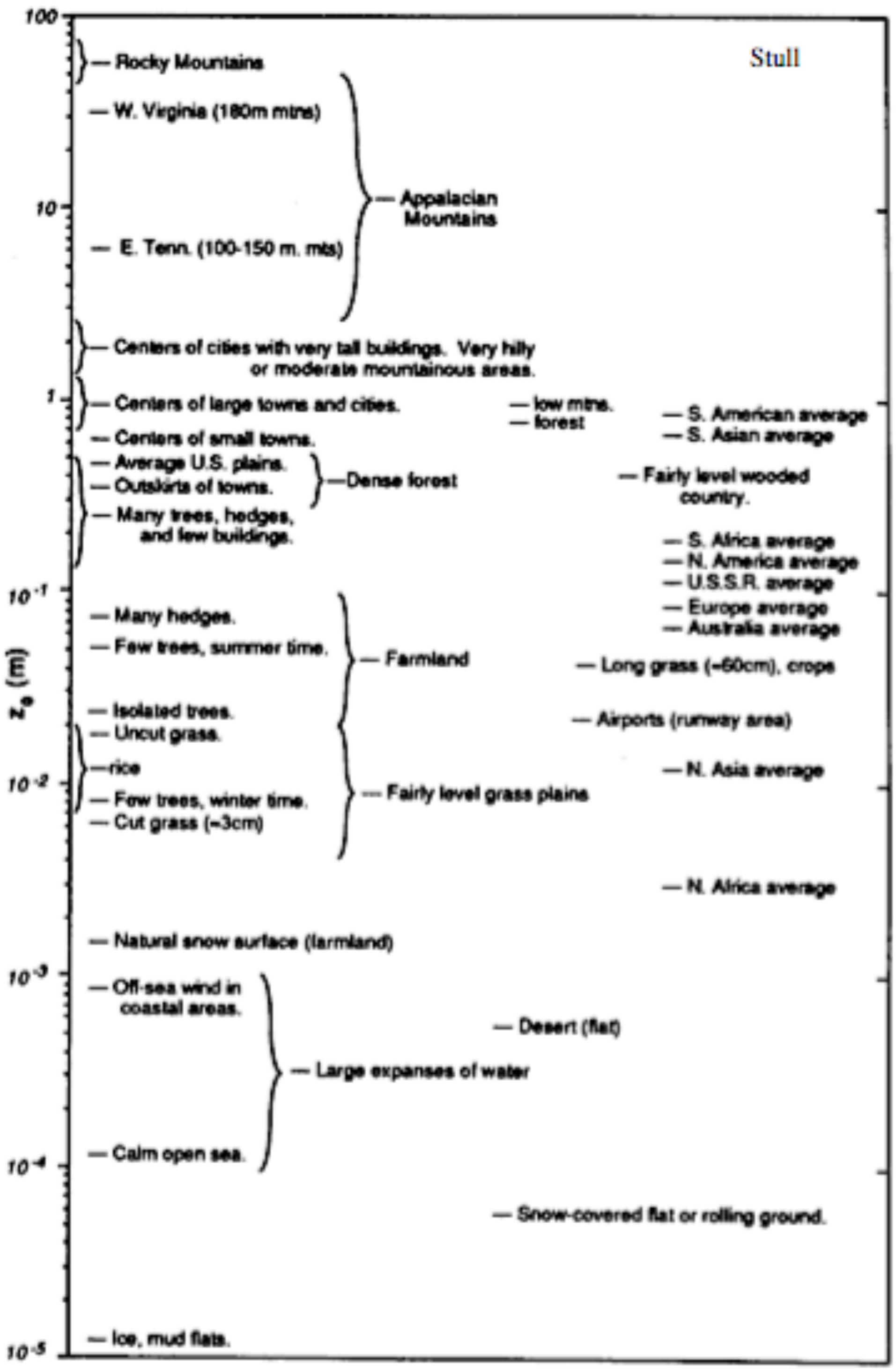

Fig. 9.6 Aerodynamic roughness lengths for typical terrain types. (After Garratt 1977, Smedman-Högström & Högström 1978, Kondo & Yamazawa 1986, Thompson 1978, Napo 1977, and Hicks 1975).

z_0 varies greatly depending on the surface, but a typical overall value for land surfaces is $z_0 = 0.1$ m (see table on previous page). In rare circumstances, the surface is so smooth that the viscous sublayer is deeper than roughness elements, whence

$$z_0 \sim 0.1\nu/u_* \sim 0.015 \text{ mm for } u_* = 0.1 \text{ m s}^{-1} . \qquad (5.6)$$

Near the surface, the log profile fits best if z is offset by a **zero-plane displacement** d_0 which lies between 0 and h_c, and is typically around 0.7 h_c:

$$u(z)/u_* = k^{-1} \ln([z-d_0]/ z_0) \qquad (z << |L|) \qquad (5.7)$$

Roughness of Water Surfaces (Garratt p. 97-100)

The roughness of a water surface depends on wind speed and the spectrum of waves. A strong wind blowing from S to N across the SR 520 bridge shows the importance of fetch on wave spectrum. On the south side, large waves will be crashing onto the bridge deck. On the N side, the water surface will be nearly smooth except for short wavelength ripples (`cats paws') associated with wind gusts. As one looks further N from the bridge, one sees chop, then further downwind, longer waves begin to build. It can take a fetch of 100 km for the wave spectrum to reach the steady state or fully developed sea assumed by most formulas for surface roughness. It is thought that much of the wind stress is associated with boundary layer separation at sharp wave crests of breaking waves or whitecaps, which start forming at wind speeds of 5 m s^{-1} and cover most of the ocean surface at wind speeds of 15 m s^{-1} or more.

For wind speeds below 2.5 m s^{-1}, the water surface is approximately aerodynamically smooth, and the viscous formula for z_0 applies. For intermediate wind speeds, the flow is aerodynamically smooth over some parts of the water surface but rough around and in the lee of the breaking whitecaps, and for wind speeds above 10 m s^{-1} it is fully rough. For rough flow, Charnock (1955) suggested that z_0 should depend only on the surface stress on the ocean and the gravitational restoring force, i. e., u_* and g, leading to **Charnock's formula**:

$$z_0 = a_c\, u_*^2/g, \quad (a_c = 0.016 \pm 20\% \text{ from empirical measurements}). \qquad (5.8)$$

This formula appears reasonably accurate for 10 m wind speeds of 4-50 m s^{-1}. For 10 m wind speeds of 5-10 m s^{-1}, this gives roughness lengths of 0.1 - 1 mm, much less than almost any land surface. Even the heavy seas under in a tropical storm have a roughness length less than mown grass! This is because (a) the large waves move along with the wind, and (b) drag seems to mainly be due to the vertical displacements involved directly in breaking, rather than by the much larger amplitude long swell. The result is that near-surface wind speeds tend to be much higher over the ocean, while surface drag tends to be smaller over the ocean than over land surfaces.

Snow and Sand Surfaces (Garratt, p. 87-88)

The roughness of sand or snow surfaces also increases of wind speed, apparently due to suspension of increasing numbers of particles. Charnock's dimensional argument again applies, and remarkably, the same a_c appears to work well, though now the minimum z_0 is larger (typically at least 0.05 mm), associated with the roughness of the underlying solid surface.

Bulk Aerodynamic Drag Formula (Garratt, p. 100-101)

Suppose that a wind measurement is taken at a standard reference level z_R within the log layer (A typical shipboard height of z_R = 10 m is often used for ocean measurements). Then (ignoring zero-plane displacement for simplicity), $u(z_R) = u_* k^{-1} \ln(z_R/z_0)$. The bulk aerodynamic formula relates the surface stress $\rho_0 \overline{u'w'}_0$ to the reference wind speed in terms of a **neutral drag coefficient** C_{DN} which depends on surface roughness:

$$-\rho_0 \overline{u'w'}_0 = \rho_0 u_*^2 = \rho_0 C_{DN} u^2(z_R), \qquad (5.9)$$

$$\text{where } C_{DN} = k^2/\{\ln(z_R/z_0)\}^2 \qquad (5.10)$$

The N, for 'neutral', in the suffix is to remind us that this formula only applies if when $z_R << |L|$. At typical reference heights (2 m or 10 m), this requires fairly neutrally stratified conditions, as usually observed over the oceans but less reliably over land. For z_R = 10 m and z_0 = 0.1 m, $C_{DN} = 8\times10^{-3}$.

Over the water, C_{DN} is a function of surface roughness u_* and hence implicitly of wind speed. While Charnock's formula gives an awkward transcendental equation to solve for C_{DN} in terms of $u(z_R)$, a good approximation using mean 10 m wind speed u_{10} is:

$$C_{DN} = (0.75 + 0.067u_{10}) \times10^{-3} \quad \text{(water, neutrally stratified BL)} \qquad (5.11)$$

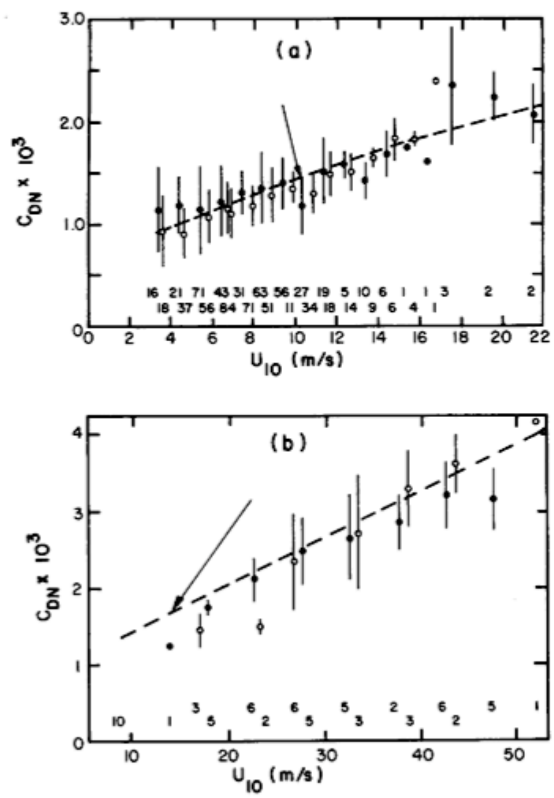

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Fig. 13.4 Neutral drag coefficient as a function of wind speed at a 10-m height compared with Charnock's formula [Eq. (13.5), indicated by the arrows in (a) and (b)] with a = 0.0144. Block-averaged values are shown for (a) 1-m sec^{-1} intervals, based on eddy correlation and profile methods, and (b) 5-m sec^{-1} intervals, based on geostrophic departure method and wind flume simulation experiments. [After Garratt (1977).]

Heat and Moisture Transfer in Neutral Conditions

Let a be a scalar (θ, q, etc.) transported by the turbulence. In the log-layer, we again might hope for a flux-gradient relation of the form

$$\overline{w'a'} = -K_a da/dz, \quad K_a = k_a z u_* \tag{5.12}$$

The nondimensional constant k_a need not equal the von Karman constant k, since momentum perturbations of fluid parcels are affected by eddy-induced pressure gradients, while scalars are not. However, empirical measurements do suggest that $k_a = k$ in a neutral BL. A scale for turbulent perturbations a¥ in the log layer is:

$$a_* = \overline{w'a'}_0 / u_* \tag{5.13}$$

Since the flux is approximately equal to its surface value throughout the surface layer,

$$da/dz = -\overline{w'a'}_0/(kzu_*) = -a_*/kz \tag{5.14}$$

$$a(z) - a_0 = -a_*/k \ln(z/z_a) \tag{5.15}$$

This has the same logarithmic form as the velocity profile, but the **scalar roughness length** z_a need not be (and usually isn't) the same as z_0. In fact, it is often much smaller, because pressure (form) drag on roughness elements helps transfer momentum between the interfacial (viscous) sublayer around roughness elements to the inertial sublayer. No corresponding nonadvective transfer mechanism exists for scalars, so they will be transferred less efficiently out of the interfacial layer ($z_a < z_0$) unless their molecular diffusivity is much larger than that of heat.

Note that temperature T is *not* an adiabatically conserved scalar; to use (5.15) you must adiabatically correct the temperature measured at height z to the surface ($z = 0$) by adding gz/c_p (this is equivalent to using dry static energy $s = c_pT + gz$ as your conserved scalar).

The surface humidity over a water surface depends on whether it is fresh or salty, since salt (which make up 2% of the molecules in sea water) does not evaporate, corresponding reducing the vapor pressure over salt water. Also, one must use the skin temperature, which may differ slightly from the bulk temperature measured below the water surface:

$$\text{Over water:} \quad q_0 = q_{sat}(p, T_{skin}) \cdot \begin{cases} 1, & \text{freshwater lakes} \\ 0.981, & \text{ocean} \end{cases} \tag{5.15a}$$

(5.15) can be converted into a bulk aerodynamic formula like (5.10), but the transfer coefficient is different:

$$\rho_0 \overline{w'a'}_0 = \rho_0 C_{aN} u(z_R)\{a_0 - a(z_R)\}, \tag{5.16}$$

$$C_{aN} = k^2/\{\ln(z_R/z_0)\ln(z_R/z_a)\} \tag{5.17}$$

For most land surfaces, the heat and moisture scaling lengths z_H and z_q are 10-30% as large as z_0, resulting in a typical C_{HN} of 0.7-0.95C_{DN} . For water surfaces, the heat and moisture coefficients are comparable to C_{DN} for 10 m winds of 7 m s^{-1} or less, but remain around 1.1-1.3$\times10^{-3}$ (see figures below) rather than increasing as wind speed increases. This corresponds to heat and moisture scaling lengths appropriate for laminar flow even at high wind speeds. For instance, ECMWF uses z_H, z_q = (0.4, 0.62)ν/u_* following Brutsaert (1982).

Bulk aerodynamic formulas are quite accurate as long as (i) an appropriate transfer coefficient is used for the advected quantity, the reference height, and the BL stability, and (ii) Temporal variability of the mean wind speed or air-sea differences are adequately sampled. The figure below shows comparisons between direct (eddy-correlation) measurements of moisture flux in nearly neutrally stratified BLs over ocean surfaces compared with a bulk formula with constant $C_{qN} = 1.32\times10^{-3}$. In individual cases, discrepancies of up to 50% are seen (which are as likely due to sampling scatter in the measured fluxes as to actual problems with the bulk formula), but the overall trend is well captured. Due to this type of scatter, no two sources exactly agree on the appropriate formulas to use, though all usually agree within about 10-20%.

Garratt

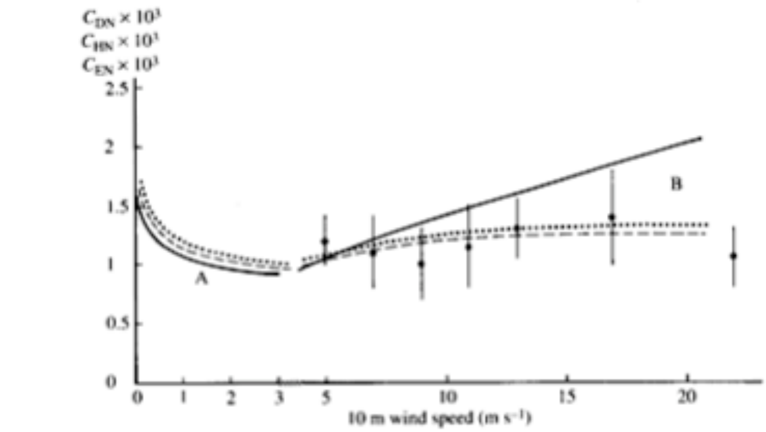

Fig. 4.9 Drag coefficient C_{DN}, heat transfer coefficient C_{HN} and water vapour transfer coefficient C_{EN} as functions of the 10 m wind speed. Curves A are for smooth flow: solid curve C_{DN} (Eq. 4.22); pecked curve, C_{HN} (Eqs. 4.10 and 4.26a); dotted curve, C_{EN} (Eqs. 4.11 and 4.26b). Curves B are for rough flow: solid curve, C_{DN} (Eq. 4.23); pecked curve, C_{HN} (Eqs. 4.10 and 4.27); dotted curve, C_{EN} (Eqs. 4.11 and 4.28). Observational data are from Large and Pond (1982).

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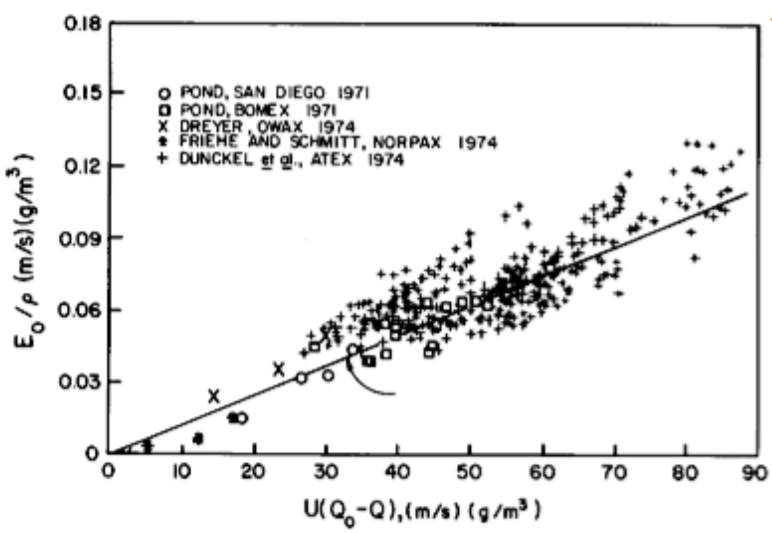

Fig. 13.6 Observed moisture flux at the sea surface as a function of $U(Q_0 - Q)$ compared with Eq. (13.8) with $C_W = 1.32 \times 10^{-3}$, indicated by the arrow. [After Friehe and Schmitt (1976).]