Lecture 16 Cloud-topped mixed layers II

An idealized marine CTML

We now step back and try to understand what controls the depth, cloud thickness and evolution timescales of a stratocumulus-capped boundary layer. Following Lilly (1968) and Schubert et al. (1979, *J. Atmos. Sci.*, **36**, 1286-1307) we consider a simplified model of a cloud-topped mixed layer (CTML), making the following assumptions:

(i) Surface transfer given by bulk aerodynamic formulas with an exchange velocity $C_T V$, where V is a BL wind speed and $C_T \approx 1.5 \times 10^{-3}$ is a neutral drag coefficient. For a typical wind speed of 7 m s^{-1}, $C_T V \approx$ 1 cm s^{-1}.

(ii) Radiation idealized as a fixed cloud-top jump ΔR_N.

(iii) No drizzle

(iv) $\overline{w} = -Dz$, where D is a specified horizontal divergence (typically 0-6 $\times$ 10^{-6} s^{-1})

(v) Geometric flux-partitioning entrainment closure with $\overline{w'b'}|_{\min} = -c\overline{w'b'}|_{\mathrm{av}}$. Schubert et al. take $c = 0.2$.

(vi) We are following a PBL air column, so there is no horizontal advection term.

The mixed layer equations are then

$$dh/dt = w_e - Dh \tag{1}$$

$$dq_{tM}/dt = \{w_e(q_t^+(h) - q_{tM}) + C_T V(q_{t0} - q_{tM})\}/h \tag{2}$$

$$d\theta_{eM}/dt = \{w_e(\theta_e^+(h) - \theta_{eM}) - \Delta R_N/\rho c_p + C_T V(\theta_{e0} - \theta_{eM})\}/h \tag{3}$$

It is convenient to replace (3) by an equivalent equation for the liquid water virtual potential temperature $\theta_{vl} = \theta_e - (\mu L/c_P)q_t$. Noting that $\theta_{vl} = \theta_v$ in the free troposphere and at the surface (and everywhere else outside the cloud), can subtract $\mu L/c_P \times$(2) from (3) to obtain

$$d\theta_{vlM}/dt = \{w_e(\theta_v^+(h) - \theta_{vlM}) - \Delta R_N/\rho c_p + C_T V(\theta_{v0} - \theta_{vlM})\}/h \tag{4}$$

Four possibly time-dependent external parameters force the mixed layer. These are the radiative flux divergence ΔR_N, the divergence D, the SST, and the surface transfer velocity $C_T V$. In addition we must specify the free tropospheric profiles $q_t^+(z)$ and $\theta_v^+(z)$.

Steady-state structure

If the external parameters and profiles are time-independent, we may seek a steady-state solution and investigate its dependence on these parameters. It is important to understand that in reality the geographic distribution of the BL structure feeds back on the entire circulation of the troposphere, so that what we treat as ‘external’ depends on our perspective!

For simplicity of analysis we will for the moment take $c = 0$ (rather than 0.2) in the entrainment closure; this has little impact on the steady state solutions. Given our assumptions, the flux of θ_{vl} must be linear with height, and because there is no flux divergence in a steady state, the steady-state θ_{vl} flux must be height independent. The buoyancy flux is proportional to the θ_{vl} flux below cloud base, and will be larger above cloud base. Thus the condition that the minimum buoyancy flux in the mixed layer be zero implies that the θ_{vl} flux is zero everywhere below cloud base, including down to the sea-surface. Thus, by the bulk aerodynamic formula, the steady state or ‘equilibrium’ solution must satisfy

$\theta_{vl,eq} = \theta_{v0}$ (to ensure negligible surface buoyancy flux)

From the steady-state versions of (1) and (2),

$w_e = Dh_{eq}$ (subsidence balances entrainment rate)

$q_{t,eq} = \chi q_t^+ + (1 - \chi)q_{t0}$ (surface moistening balances entrainment drying)

where $\chi = w_e/(w_e + C_T V)$ can be interpreted as the mixing fraction of above inversion (vs. sea-surface) air in the mixed layer The steady-state PBL energy balance implied by (4) is

$$w_e(\theta_v^+ - \theta_{v0}) = \Delta R_N/\rho c_p$$

Setting $w_e = Dh_{eq}$, we must have

$$Dh_{eq}(\theta_v^+(h_{eq}) - \theta_{v0}) = \Delta R_N/\rho c_p \tag{5}$$

This is a quadratic for h_{eq} with one positive root. We can work backward to get the rest of the mixed layer parameters.

We will first consider a specific example using idealized free-tropospheric conditions conditions off the California coast, $\Delta R_N = 50$ W m^{-2}, $D = 5 \times 10^{-6}$ s^{-1}, SST = 290 K, and $C_T V = 1$ cm s^{-1}, a surface pressure $p_0 = 1020$ mb, and

$q_t^+(z) = 4$ g kg^{-1} and $\theta_v^+(z) = 303 + 0.004z$ K (z in meters)

At the saturated sea-surface

$q_{t0} = q^*(p_0, \text{SST}) = 12$ g kg^{-1},

$\theta_{v0} = (1000/1020)^{.285}\text{SST}(1 + .61q_{t0}) = 290.5$ K

Thus $\theta_{vl,eq} = 290.5$ K. Solving the quadratic (5) for the given parameters, we get

$h_{eq} = 564$ m, $w_e = 0.3$ cm s^{-1}, $\theta_v^+ - \theta_{v0} = 14.8$ K

The mixing fraction of above inversion air $\chi = w_e/(w_e + C_T V) = 0.22$, so

$q_{tM} = \chi q_t^+ + (1 - \chi)q_{t0} = 10.3$ g kg^{-1}.

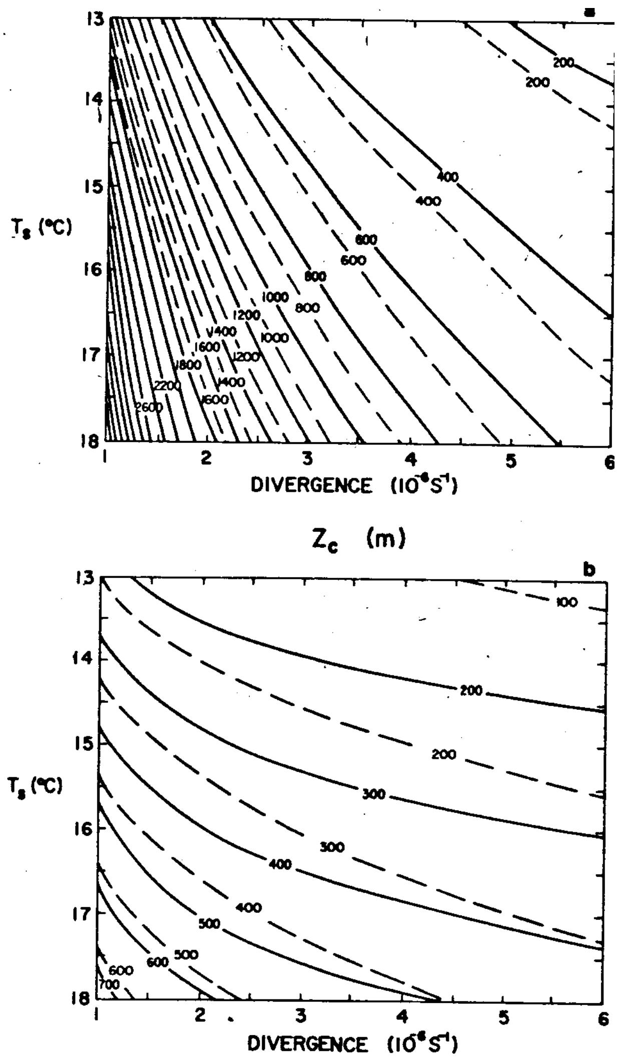
Together with the requirement that $\theta_{vlM} = 290.5$ K, this allows us to deduce the equilibrium mixed layer temperature 290.3 K just above the sea-surface, and the saturation mixing ratio 12.3 g kg^{-1} at that temperature . Using a formula for how fast saturation mixing ratio decreases with height in a well-mixed (dry adiabatic) stratification (4.9 g kg^{-1} km^{-1} at the given temperature and pressure), we can deduce the cloud base height at which BL air is exactly saturated:

$z_b = (12.3$ g kg^{-1} - 10.3 g kg^{-1})/(4.9 g kg^{-1} km^{-1}) = 414 m.

The cloud base is 150 m beneath the inversion, consistent with the assumption that the mixed layer is cloud-topped. However, this in not guaranteed. If the wind is too weak or the divergence much stronger than assumed above, the predicted equilibrium cloud base will be above the BL top, so the BL cannot be cloud-topped. However, for weak to moderate subsidence and typical BL wind speed and above-BL profiles, the equilibrium BL is cloud-topped.

The plots below show the variation of equilibrium cloud base and top as SST and *D* are varied for a model similar to that above, taken from Schubert *et al.* Disregard the dashed lines. For weak divergence and high SST, typical of conditions further downwind in the mean trade wind circulation, deep BLs are obtained but their cloud base is still only 500-600 m, implying very thick stratocumulus layers. An obvious question is how the cloud-top mixed layer structure breaks up into the observed shallow trade cumulus boundary layers that are seen.

Schubert et al. 1979



Contours of equilibrium well mixed BL top (top) and cloud base (bottom) as functions of SST and mean horizontal divergence.

Timescales

Over land, the strong diurnal cycle guarantees that the daytime convective BL never achieves a steady state. However, marine CTBLs are closer to a steady state structure. Following a very nice paper by Schubert et al. (1979, *J. Atmos. Sci.*, **36**, 1308-1324), we consider the timescales for the BL to relax toward a steady state, by rephrasing the mixed layer equations as follows:

$$dh/dt = w_e - Dh = (h_{eq} - h)/\tau_h \tag{6}$$

$$\partial q_{tM}/\partial t \;=\; \{w_e(q_t^+ - q_{tM}) + C_T V(q_{t0} - q_{tM})\}/h \;=\; (q_{t,eq} - q_{tM})/\tau_M \tag{7}$$

$$\partial \theta_{eM}/\partial t \;=\; \{w_e(\theta_e^+ - \theta_{eM}) - \Delta R_N/\rho c_p + C_T V(\theta_{e0} - \theta_{eM})\}/h \;=\; (\theta_{e,eq} - \theta_{eM})/\tau_M \tag{8}$$

where the relaxation timescales are

$\tau_h = D^{-1}$ for BL depth

$\tau_M = h/(w_e + C_T V)$ for internal thermodynamic adjustment

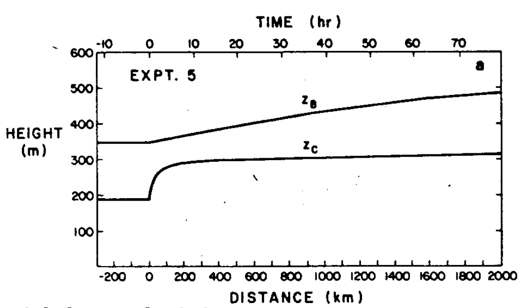
For typical values for subtropical Sc-topped mixed layers, $D = 5 \times 10^{-6}$ s^{-1}, $w_e = 0.5$ cm s^{-1}, $C_T V = 1$ cm s^{-1}, we find that:

τ_M = (500 m)/(0.5 + 1 cm s^{-1}) = 30,000 s $\approx$ 0.4 days,

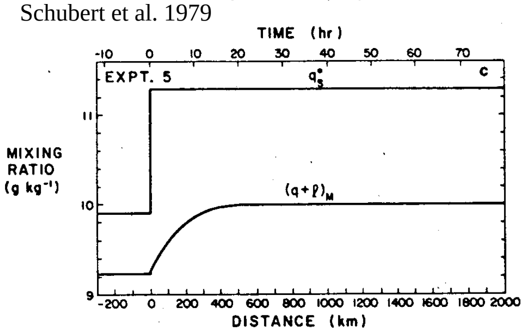
τ_h = 200,000 s = 2.3 days.

The internal thermodynamic state of the BL rapidly adjusts on the time τ_M to changes in SST and free-atmospheric properties. The BL depth relaxes to an equilibrium value on a much longer timescale τ_h = 200, 000 s = 2.3 days . During this time, slow thermodynamic changes also continue as the entrainment rate and the temperature and humidity of the entrained air adjust to the changing depth of the boundary layer.

The figure below shows the response of a cloud-topped mixed layer to a 2 K step change in SST. The rapid adjustment of cloud base (i. e. the internal thermodynamic state) to the changed SST contrasts with the much slower adjustment of cloud top. In this figure, BL changes are envisioned as occuring as the BL air column advects over a changing surface with a fixed wind speed V = 7 m s^{-1}, so 1 day's evolution corresponds to a distance of 600 km In reality, the conditions following a BL air column are rarely nearly constant over periods of many days, so the BL height is usually not in equilibrium.



Schubert et al. 1979



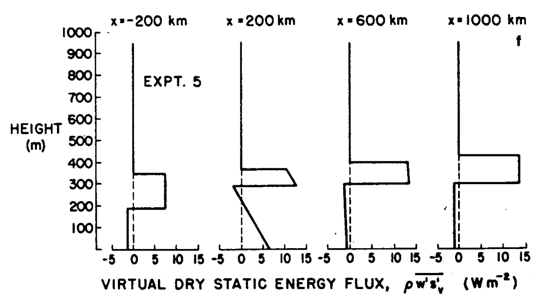


Fig. 11. Results for Experiment 5 (constant divergence, instantaneous 2°C increase in sea surface temperature).

Diurnal decoupling

Solar absorption in stratocumulus clouds has a large impact on the diurnal cycle of the marine CTBL. This was first discussed in detail by Nicholls (1984, QJRMS, **110**, 783-820). The 1987 FIRE-MSC experiment documented the diurnal cycle in Sc 100 km off the California coast. Considerable cloud thinning (a factor of four decrease in liquid water path) and a reduction in cloud albedo from 50% to 30% is observed during the morning (see figure on next page). This is not simply cloud 'burning off' due to warming of cloudy air by the absorbed sunlight, resulting in evaporation of the cloud. Instead it is mainly driven by a change in the BL turbulent structure called **diurnal decoupling**. To understand this process it is helpful to consider the effect of absorption-induced heating on the buoyancy flux profile of a mixed layer. The heating is trying to stabilize the region below it. Buoyancy fluxes must be more negative (helping keep the subcloud layer as warm as the cloud) to maintain a mixed layer.

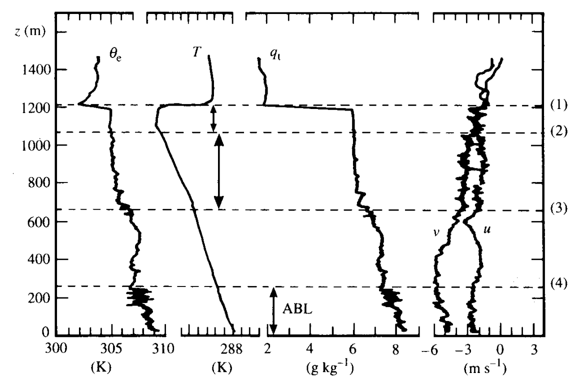
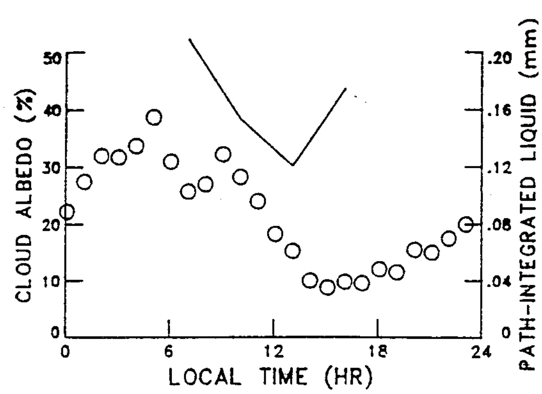
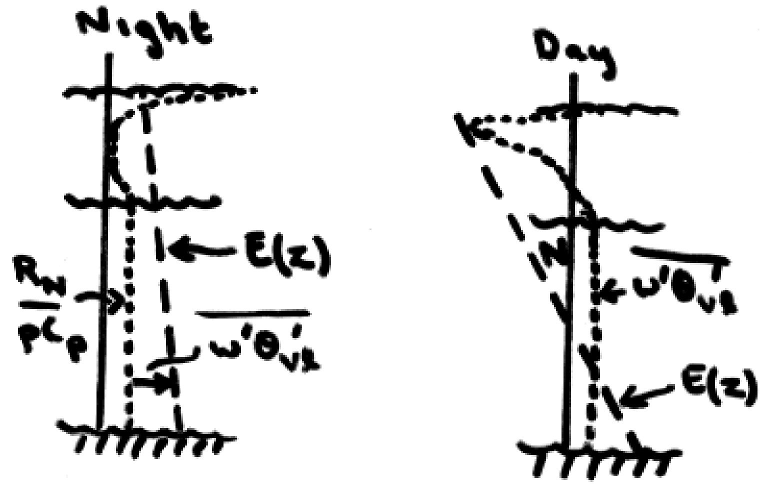


Fig. 7.5 Observed mean profiles of thermodynamic variables and wind components made in the CTBL over the ocean during JASIN, for a decoupled stratocumulus layer. The pecked horizontal lines delineate layer boundaries as follows: (1) cloud top; (2) cloud base; (3) bottom of subcloud layer; (4) top of the surface-related Ekman layer. After Nicholls and Leighton (1986), *Quarterly Journal of the Royal Meteorological Society*.

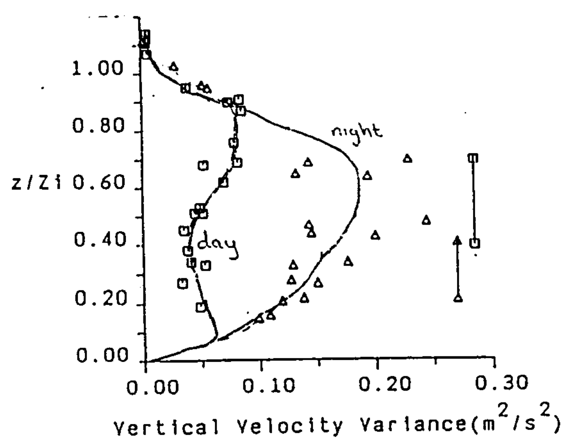
A decoupled boundary layer near local noon.



The diurnal cycle of cloud albedo (solid) and liquid water path (circles) averaged over 23 Jun - 15 Jul 1987 at San Nicholas Island off the California coast .



Nighttime and daytime profiles of radiative, θ_{vl}-and total enegy fluxes in a mixed layer.



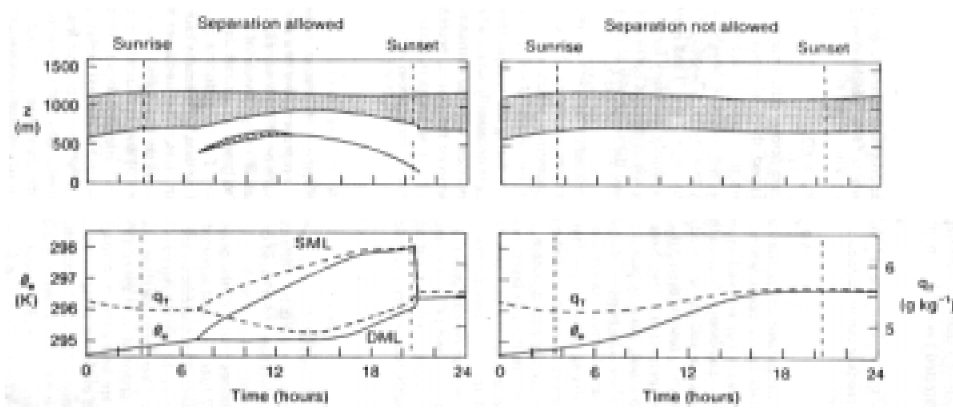
Tethered balloon measurements during three days of FIRE-MSC, 1987, showing impact of decoupling on vertical velocity variance (Hignett 1991).

The top figure above shows nighttime and daytime mixed layer profiles of θ_{vl} flux (which is proportional to buoyancy flux below cloud base) visualized as the difference between the total flux $E(z) = \ + R_N/\rho c_p$ forcing θ_{vl} and the radiative flux. During the day, cloudtop R_N is very small so $E(h)$ is dominated by the negative contribution of entrainment. This forces a large area, N, of negative buoyancy fluxes beneath cloud base, that suggests that the mixed layer must break down due to decoupling.

As the zone of negative buoyancy fluxes below cloudbase expands, transport of TKE becomes insufficient to sustain convection within this region, and it becomes stably stratified, separating **decoupled** convective layers near the surface (driven by surface fluxes) and within the cloud (driven by the cloud-top cooling that overlies the absorption heating). An example of a decoupled CTBL over the summertime N Atlantic around local noon is shown on the previous page. Within the two convective layers, well-mixed profiles of θ_e, q_t, u, and v are seen, separated by a stably stratified 'transition' layer characterized by intermediate values of these quantities. Mean daytime and nighttime vertical velocity variance profiles measured by a tethered balloon during FIRE-MSC are shown above, and illustrate that during the night, the vertical velocity variance is maximum in the middle of the layer where large-eddy up and downdrafts are maximum, while during the day separate surface layer and cloud layer maxima exist, indicative of two convective layers separated by a stable layer with gravity wave activity and perhaps intermittent turbulence.

After decoupling occurs, the cloud mixed layer dries due to entrainment, so the cloud base steadily rises and the cloud may partly or fully dissipate. The surface mixed layer moistens due to surface fluxes. ‘Scud’ cloud may start forming at its top, well beneath the main Sc cloud base. The transition layer is usually conditionally unstable, so more vigorous scud clouds may begin to rise as cumuli into the upper Sc layer. In shallow coastal Sc, this process is not thought to contribute significantly to the overall fluxes of heat or moisture, but in deeper CTBLs it becomes of paramount importance.

Late in the afternoon, shortwave heating becomes less potent, and the upper mixed layer begins to cool more rapidly due to longwave cooling. As it cools, it penetrates further back down into the transition layer and during the evening usually ‘reconnects’ with the surface mixed layer. When the two layers reconnect, the cloud rapidly deepens again and a single mixed layer is reestablished. Turton and Nicholls (1987) presented an elegant simulation of this process in which two mixed layers are separated by a nonturbulent stable layer. While a mixed layer model (figure below, right) shows almost no daytime thinning of the cloud (shaded region in upper plot), their model (left) predicts that the upper mixed layer dries by 0.5 g kg^{-1} while the lower mixed layer moistens almost 1 g kg^{-1}, resulting in a 70% thinning of the upper Sc layer while thin scud develops atop the lower mixed layer. The lower panels show the corresponding diurnal evolution of the conserved variables in the two models.



Turton and Nicholls (1987) multiple mixed-layer simulation of diurnal decoupling