Vector operators in Cartesian coordinates

Notation: velocity components (u,v,w) and unit vectors ($\underline{\hat{i}}$, $\underline{\hat{j}}$, $\underline{\hat{k}}$) are in the (x,y,z) directions.

Gradient of a scalar p

$$\nabla p = \frac{\partial p}{\partial x}\underline{\hat{i}} + \frac{\partial p}{\partial y}\underline{\hat{j}} + \frac{\partial p}{\partial z}\underline{\hat{k}}$$

Divergence of a vector $\underline{u}$

$$\nabla \cdot \underline{u} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

Curl of a vector $\underline{u}$

$$\nabla \times \underline{u} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\underline{\hat{i}} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\underline{\hat{j}} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\underline{\hat{k}}$$

Advective operator on a vector $\underline{u}$

$$(\underline{u} \cdot \nabla)\underline{u} = \left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)\underline{u}$$

Laplacian operator on a vector $\underline{u}$

$$\nabla^2\underline{u} = \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)\underline{\hat{i}} + \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)\underline{\hat{j}} + \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)\underline{\hat{k}}$$

Reference: Vector Analyis by M R Spiegel, Schaum Pub. Co.