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Air flow over a two-dimensional hill: studies of velocity speed-up, roughness effects and turbulence

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SUMMARY

Wind tunnel measurements have been made of the streamwise mean and turbulent velocities over a rough, bell-shaped, two-dimensional hill, with height h and maximum slope 0·26, placed in a neutrally stable boundary layer of thickness $10\,h$ and roughness length $z_0 = 0{\cdot}02\,h$. Close agreement is found between the mean velocity and predictions obtained from Taylor's (1977) computational model and Jackson and Hunt's (1975) analytical linearized model, for locations at or upwind of the hill top but not in the wake. Examination of the models shows that the shear stresses are only important in an inner region close to the hill surface, so that, as suggested by Jackson and Hunt (1975), the perturbation to the mean flow outside this region is essentially inviscid. The theory shows that over very rough surfaces, such as wooded or urban terrain, the height of this inner region can be of the same order as the height of the roughness elements (so that in our experiments no measurements could be made in this region).

In a second experiment flow over a *smooth* hill on a rough surface was studied. The additional increase of wind speed over the hill top can be estimated by assuming a linear superposition of the velocity changes produced by the changes in elevation and in surface roughness (in this case rough to smooth). In the lee of a hill, however, the change in roughness significantly alters the flow with flow separation being suppressed and here a linear superposition is not appropriate.

Finally we consider why observed changes in turbulence structure close to the surface differ from those well above the surface. Calculations of these changes based on the simple theoretical arguments of equilibrium shear layers and rapidly distorted turbulent flows agree well with turbulence measurements in wind tunnels and in the field.

1. INTRODUCTION

Recent research on neutrally stable air flow over hills has increased our understanding of how the wind speed increases over hills of different shapes, of how roughness changes affect the wind over a hill, and of how the turbulence structure changes over hills. But in all these cases the research has also indicated some further problems that need solving, and it is these that we attempt to investigate here by theoretical and wind tunnel model studies.

The recent theoretical models of unstratified turbulent boundary layer flow over hills with low or moderate slopes have been shown by Jensen and Peterson (1978), Hunt (1979) and others to predict similar increases in surface wind speed over the hill top, despite their quite different assumptions about the relative size of the hill half length, L, to the boundary-layer thickness, δ. For example, Jackson and Hunt (1975) (hereafter referred to as JH) allowed $L \sim \delta$, while Taylor (1977) and Deaves (1976) assumed that $L \ll \delta$. So we thought it would be interesting to present some recent wind tunnel measurements and compare them with predictions of the JH and Taylor models, when the computations were performed for the *same* flow, i.e. with the *same assumptions about the ratio* L/δ. Most previous comparisons have been concentrated on the flow upwind of and on the hill top, and have excluded the wake. Even where there is no separation, because of the highly turbulent flow in the adverse pressure gradient on the lee side, the shear stresses are unlikely to be adequately described by the simple equilibrium or mixing-length models of JH and Taylor. Our wind tunnel results demonstrate the extent of this inadequacy.

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Over many hills or changes in surface elevation, the surface roughness changes. For example, Jensen and Peterson (1978) studied the air flow from the sea on to the land over a small coastal escarpment. They found that the changes in mean velocity produced by the roughness change were of the same order as those produced by the change in elevation. In their case they suggested, but did not quantitatively demonstrate, that the two effects could be *linearly superposed.* But over a rounded hill, where separation occurs, it is by no means obvious that such an assumption is justified. We examine this question by a wind tunnel study of flow over two hills one of which has a surface as rough as, and the other smoother than, the upwind surface.

Previous wind tunnel studies (e.g. Bouwmeester 1978) and field studies (e.g. Bradley 1980) have shown how different components of turbulence change over hills and how the changes vary with displacement from the surface. In section 3 we show that simple theoretical arguments based on the theory of equilibrium shear layers and rapidly distorted turbulent flow adequately predict the turbulence changes close to the surface and well above the surface respectively. We also show how different changes in turbulent structure can be expected inside and outside the inner layer.

In using a wind tunnel to test out some of the theoretical ideas we have had to consider carefully the limitations of wind tunnel modelling.

The theoretical arguments of JH and Sykes (1980) and the recent field measurements by Bradley (1980) show that it is only within a narrow inner region, of thickness l (see Fig. 1),

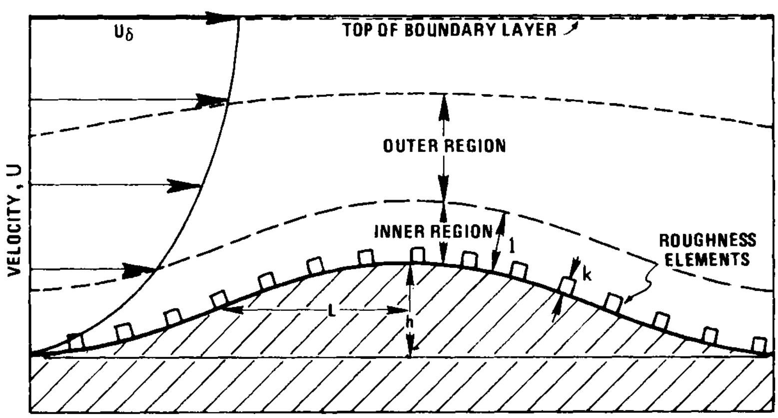

Figure 1. Definition sketch of flow regions.

where significant changes in shear stress occur and over the lower part of which the mean flow is affected by these shear stress changes. Bradley's measurements were over a hill whose height h was 170 m and half length L was 275 m and whose surface had a roughness length $z_0 = 1$ m on account of its covering by trees 25 m high. He found that l was about 28 m, in line with JH's theoretical estimate, which shows that over tree-covered or urban hillsides one can expect l to be of the same order as the height of the roughness elements k. Using the JH ratio for l/z_0 and the empirical result that $10 < k/z_0 < 30$, then l/k can be estimated as $l/k < 2\kappa^2(L/z_0)/\{10 \ln(L/z_0)\}$, where κ is von Kármán's constant, taken to be 0·4.

In some wind tunnel diffusion studies over hills (e.g. Cermak 1975), where low wind speeds are necessary for correct Froude-number scaling on buoyant plumes, if the same roughness scaling for z_0/δ or k/δ is applied on the model scale then the surface may not satisfy the condition for the surface to be aerodynamically rough, i.e. $u_*z_0/\nu < 5$, where u_* is the friction velocity and ν the kinematic viscosity. (E.g. over grassland $z_0/\delta \simeq 2\times10^{-5}$, and if in the wind tunnel the velocity outside the boundary layer $U_\delta = 3\,\mathrm{m\,s^{-1}}$, and $\delta = 1$ m, then undistorted scaling implies $k = 6\times10^{-4}$ m, $z_0 = 2\times10^{-5}$ m, so $u_*z_0/\nu < 0{\cdot}3$.) Consequently higher roughness elements are used, perhaps increased by a factor of 10, which results in l/k being much less on the model scale. (In our example k might be increased to 6 mm, so on a hill with length $L = 300$ m, l/k is decreased from 100 to 10.) This point is most important for modelling flow over smooth terrain (e.g. grassland).

In our experiments we model a very rough surface where k is large enough that $u_*z_0/\nu > 5$, so that the correct scaling of k/δ or z_0/δ can be used in the wind tunnel. However, this means that l/k is of the order of 1 and consequently details of the inner layer cannot be measured.

2. Wind tunnel measurements of mean velocity

2.1. *Undisturbed boundary-layer flow*

The wind tunnel studies were performed in the Warren Spring 4·3 m by 1·5 m by 22 m long wind tunnel. An artificially thickened boundary layer is generated to simulate the neutrally stable atmospheric boundary layer using the same method as Counihan (1969), except that the roughness elements are larger than his 4 mm Lego blocks being 20 mm 'Allen screws' at a spacing of 0·05 m. These large elements ensure fully rough conditions even at low wind speeds.

The measurements of mean and fluctuating longitudinal velocity were taken with a linearized constant temperature anemometer and a pulsed wire anemometer (Bradbury and Castro, 1971).

The mean velocity profiles were measured with the velocity at the outer edge of the boundary layer, U_δ, equal to $4{\cdot}0\,\mathrm{m\,s^{-1}}$. The boundary layer, whose mean velocity profile is shown in Fig. 2(a) is typical of a rough wall boundary layer, has a thickness, δ, of about 1 m at 20 m from the tunnel entrance.

Fitting the profile near the surface to the form

$$U/U_\delta = (u_*/U_\delta)\{2{\cdot}5\ln(z/z_0)\}$$

gives

$$z_0 = 2\times10^{-3}\,m \quad \text{and} \quad u_*/U_\delta = 0{\cdot}0685$$

where z_0 is the roughness length. Scaled up to the atmospheric boundary layer this value of z_0 corresponds to a value of about 1 m, i.e. typical of woodland or urban areas. Note that the roughness density was such that the zero plane displacement is of the order of or less than z_0.

The mean velocity data may be restated in the form

$$U/U_\delta = 1{\cdot}5(z/\delta)^n \quad \text{for } z/\delta < 0{\cdot}15$$

where $n = 0{\cdot}34$. The small discrepancy in the measurements between the pulsed wire anemometer and the linearized constant temperature anemometer (LCTA) is still to be explained.

Figure 2(b) is a plot of the r.m.s. longitudinal (σ_u) and vertical (σ_w) velocity fluctuations non-dimensionalized with the mean velocity at the edge of the boundary layer, U_δ.

The pulsed wire measurements near the top of the boundary layer were influenced by temperature fluctuations in the tunnel and may be ignored. (Note that $\sigma_u/u_* = 2{\cdot}19$ at

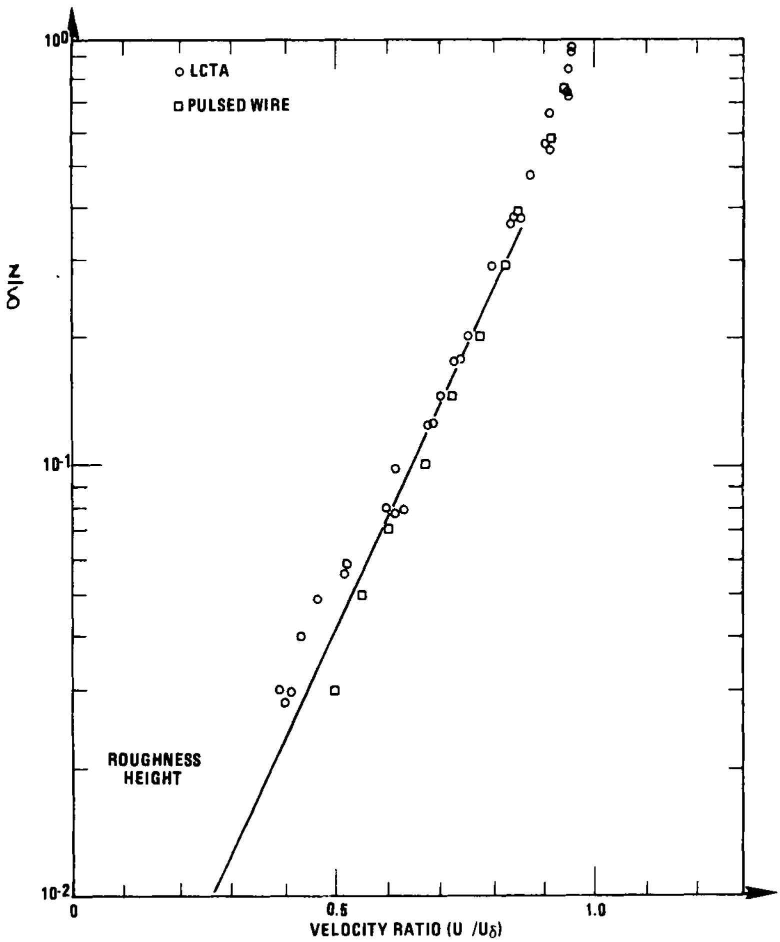

Figure 2(a). Measured mean velocity of the undisturbed boundary layer.

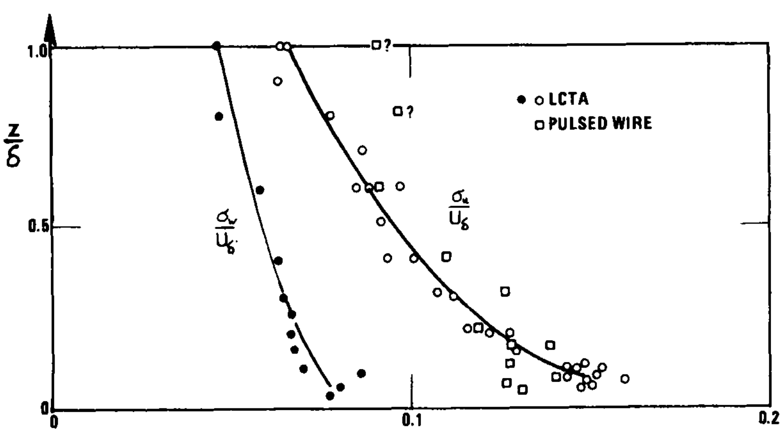

Figure 2(b). Ratio of the measured r.m.s. streamwise, σ_u, (○, □) and vertical, σ_w, (●) turbulent velocities of the undisturbed boundary layer to the free stream velocity U_δ.

$z/\delta = 0{\cdot}05$, which is a little low by comparison with atmospheric data in rural terrain, but within the range found in most wind tunnel simulations (Hunt and Fernholz 1975).)

Because we studied hills with and without surface roughness, a brief change of roughness experiment was also conducted in which the floor roughness was removed 17 m from the start of the boundary layer. The mean velocity profile was measured 1m from the roughness change, see section 2.4.

2.2 *Mean velocity over the hill*

The flows over two different hills were studied. The hill crests were located 17·5 m from the start of the simulated boundary layer and the hill shapes were

$$z = h\, f(x/L)$$

with

$$f(x/L) = \{1+(x/L)^2\}^{-1} \qquad (2.1)$$

where x is the downwind distance from the crest of the hill, $h = 0{\cdot}1$ m, $L = 0{\cdot}25$ m, and the maximum slopes were 0·26 (about $h/2L$). The model extended 1·2 m up and downwind of the centreline. Any smaller values of h or larger values of L were impracticable. Because the ceiling of the wind tunnel was not adjustable the height of the hill was restricted. A blockage ratio of 6·7 % obtained with $h = 0{\cdot}1$ m is estimated to produce a change in velocity over the hill of less than 1 %.

Measurements were made of the streamwise components of the mean (U_s) and r.m.s. turbulent (σ_{u_s}) velocities at 5 streamwise locations ($x = -0{\cdot}64$, $-0{\cdot}25$, 0, 0·25, 0·70 m). Vertical profiles were taken at values of z from 3 cm above the surface (which is only $1\frac{1}{2}$ roughness heights) to the top of the boundary layer ($z = 1$ m) (see Tables 1, 2).

TABLE 1. MEAN VELOCITY, U_s PROFILES OVER ROUGH HILL

All velocities are normalized with the velocity 1 m above the surface upstream of the hill

x(m) $\Delta z/\delta$	∞	$-0{\cdot}64$	$-0{\cdot}25$	0	$+0{\cdot}25$	$+0{\cdot}70$	$+1{\cdot}15$
1·00	1·00	1·00	1·01	0·99	1·00	1·03	1·04
0·98		1·00					
0·80	0·99	1·00 (I)	1·03	1·01	1·01	1·03	1·00
0·78		1·00					
0·60	0·96	0·96 (I)	0·96	0·99	0·99	1·00	0·97
0·58		0·95					
0·40	0·90	0·96 (I)	0·95	0·98	0·96	0·94	0·89
0·38		0·89					
0·20	0·79	0·77 (I)	0·86	0·91	0·87	0·78	0·73
0·18		0·76					
0·15	0·73	0·73 (I)	0·79	0·89	0·83	0·62	
0·13		0·71					
0·10	0·67	0·67 (I)	0·72	0·86	0·70	0·39	0·49
0·08	0·62	0·65	0·67	0·85	0·55	0·32	
0·06	0·55	0·54					0·43
0·05	0·50	0·51 (I)	0·62	0·80	0·23	0·23	
0·04	0·45	0·47 (I)		0·79		0·20	0·37
0·03	0·44	0·43		0·78			
0·02			0·45	0·69			0·37
0·01							0·31
0·005							

(i) Δz is measured from local hill surface.
(ii) measurements for $\Delta z/\delta < 0{\cdot}03$ are within roughness elements and unreliable.
(iii) (I) Interpolation.

TABLE 2. THE STREAMWISE r.m.s. TURBULENT VELOCITY u_s' FOR FLOW OVER A ROUGH HILL

The u_s' have been normalized with U_δ well upstream of the hill

x / $\Delta z/\delta$	∞	$-0{\cdot}64$ m	$-0{\cdot}25$ m	0	$+0{\cdot}25$ m	$+0{\cdot}70$ m
1·00	0·065	0·062(I)	0·060	0·060	0·060	0·061
0·98		0·063				
0·80	0·075	0·072(I)	0·072	0·069	0·067	0·070
0·78		0·073				
0·60	0·087	0·083(I)	0·082	0·079	0·079	0·078
0·58		0·084				
0·40	0·100	0·097(I)	0·11	0·089	0·091	0·098
0·38		0·098				
0·20	0·123	0·12(I)	0·11	0·11	0·12	0·14
0·18		0·12				
0·15	0·133	0·12(I)	0·13	0·12	0·12	0·19
0·13		0·13				
0·10	0·143	0·13(I)	0·14	0·13	0·16	0·18
0·08	0·147	0·14	0·14	0·13	0·18	0·19
0·06	0·151	0·14			0·12	
0·05	0·150	0·14(I)	0·13	0·13		
0·04	0·149	0·14(I)		0·13		
0·03	0·147	0·14	0·13	0·13		
0·02			0·13	0·15		
0·01						
0·005						

(i) (I) Interpolation.
(ii) Δz measured from hill surface.

We observe (Figs. 3(a)) a distinct sharpening of the velocity gradient near the hill upwind of the hill top and an elevated region of high shear downwind of the hill top. In Fig. 3(b) the mean velocity has been plotted as a function of x, with the displacement above the hill, Δz, as a parameter.

Observations of the velocity signal suggested that the flow separated in the lee of the hill. Flow visualization with smoke confirmed that the flow separated but also indicated that separation and reattachment were essentially intermittent, there being periods in which the flow did not separate at all. However, it was estimated that, on average, the separated region was approximately 3 cm thick 25 cm downstream from the top of the hill and that reattachment occurred about 70 cm (7 hill heights) downstream from the top of the hill.

From Fig. 4(a) we see an important distinction between the shallow region of intermittently separating flow and a deeper region whose thickness is of the order of h where the turbulent velocities are increased by 50% or more (see also Huber *et al.* (1976)).

Flow over a second model hill was studied which did not have any surface roughness elements. The measurements are described in section 2.4.

2.3 *Comparison with theory*

The measurements of mean flow are compared with the results of two recent theories of flow over hills, the analytical theory of JH and a modified version of the numerical model of Taylor (1977) for the hill shape (2.1). The calculations of the accelerations and the turbulence closures differ in the two models. By assuming that $h/L \ll 1$, JH linearized the inertial terms and used a mixing-length model. Taylor retains the non-linear inertial terms

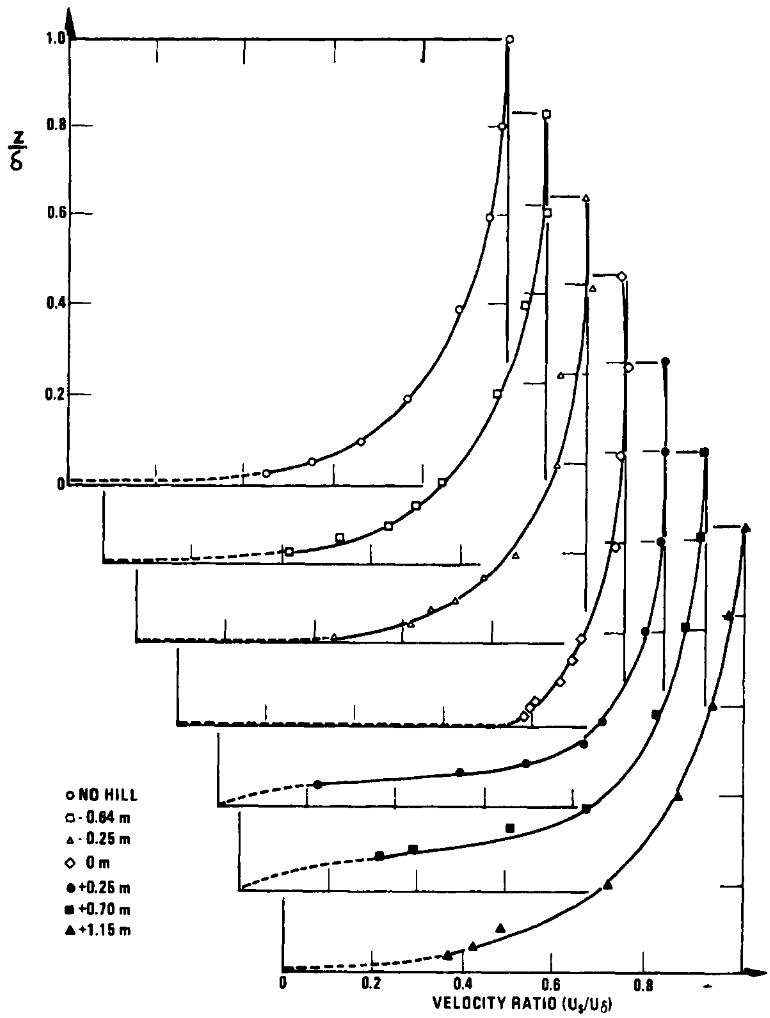

Figure 3(a). Measured velocity profiles for a number of positions over the rough hill as a function of height, z/δ.

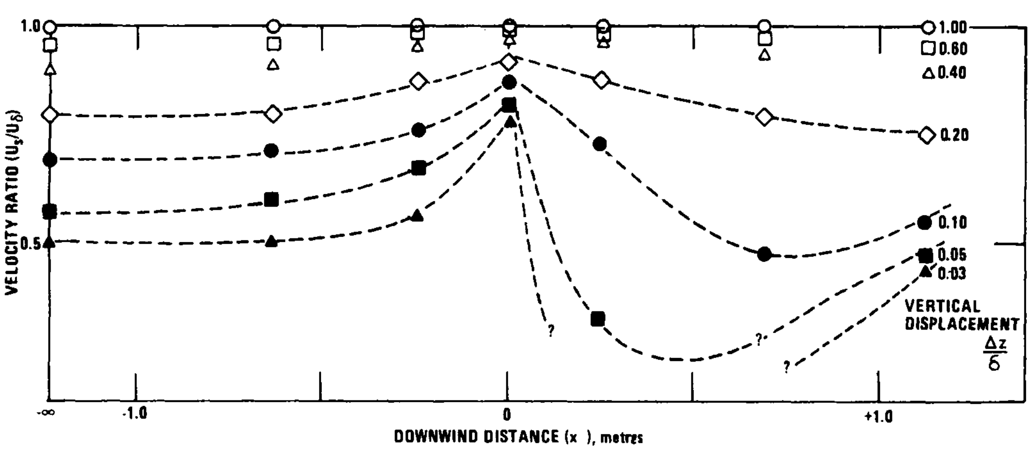

Figure 3(b). Measured mean velocity as a function of vertical displacement and x.

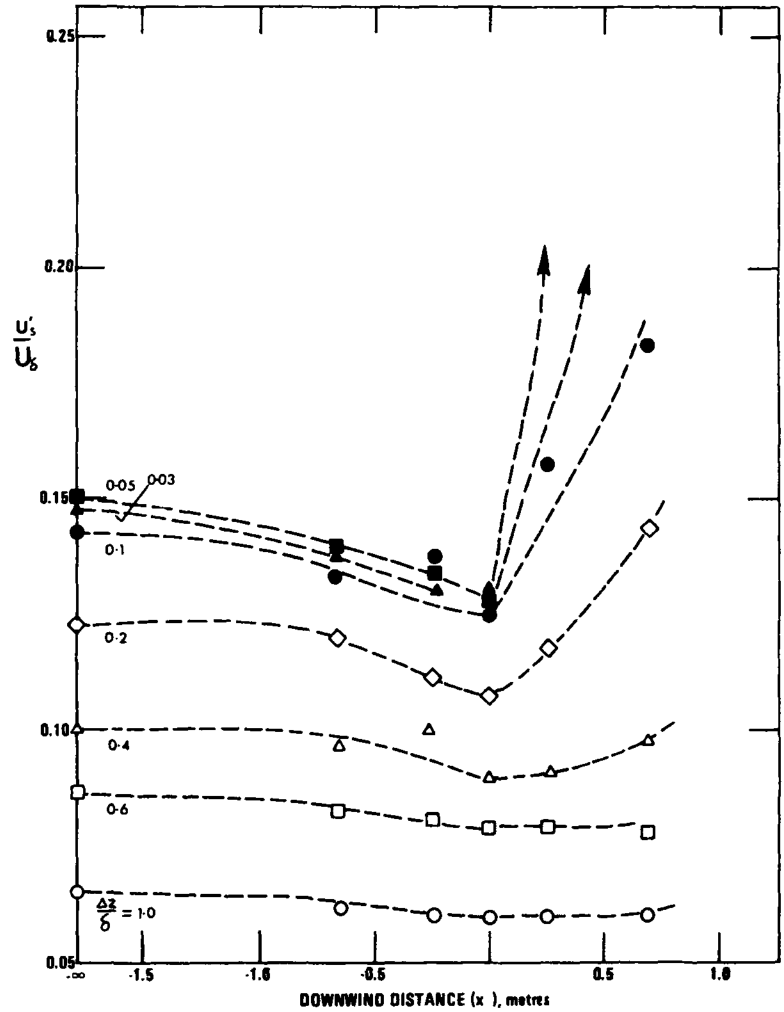

Figure 4(a) Ratio of the measured streamwise r.m.s. turbulence velocity, u_s', to U_δ over the rough hill as a function of x, with $\Delta z/\delta$ as a parameter.

and used an eddy viscosity closure based on the turbulent kinetic energy. Effectively Taylor only assumed that h/L is small enough that no separation occurs. Thus in this experiment, where $h/L = 0{\cdot}4$ and separation just occurs, Taylor's theory should be more applicable than JH's because of the inclusion of non-linear inertial terms in his model.

There are different ways of presenting the data, depending on which theory they are to be compared with and how they are to be applied. JH consider two regions shown in Fig. 1: (i) an inner region with thickness l in which significant changes in the Reynolds stresses occur and which affect the mean velocity (though, as pointed out by Sykes (1980), in a formal sense, as $u_*/U_\delta \to 0$, and $z_0/l \to 0$ the latter effect only occurs in the lowest part of the inner layer at a distance of order z_0 from the surface) and (ii) an outer region where the pressure gradient set up by the hill is balanced by the inertial forces. The height of the inner region is given by

$$(l/L)\ln(l/z_0) = 2\kappa^2 \quad . \quad . \quad . \qquad (2.2a)$$

Given the measured value of z_0 over the model hill, $l/z_0 = 15$, so that l is relatively rather small being only $1\frac{1}{2}$ times as high as the roughness elements.

For the inner region JH express the horizontal velocity as

$$U = U_0(\Delta z)+\Delta\hat{u}(x, \Delta z) \quad . \quad . \quad . \qquad (2.2b)$$

where $\Delta\hat{u}$ is a perturbation velocity on the upwind velocity U_0 at a displacement Δz above the surface. For practical purposes it is convenient to define $\Delta\hat{u}$ as a fraction ΔS of $U_0(\Delta z)$, so that

$$U(\Delta z) = (1+\Delta S)U_0(\Delta z). \quad . \quad . \quad . \qquad (2.2c)$$

$\Delta\hat{u}$ is found from the theory to have the form

$$\Delta\hat{u} = \frac{h}{L}\frac{\ln^2(L/z_0)}{\kappa\ln(l/z_0)}\hat{u}\left(\frac{x}{L},\frac{\Delta z}{Ll}\right) \quad . \quad . \quad . \qquad (2.2d)$$

where the function $\hat{u}(x/L, \Delta z/l)$ is 0(1).

In the outer region JH write $U(z)$ in terms of the upwind velocity at the *same height z* and a perturbation Δu, so

$$U = U_0(z)+\Delta u(x,z) \quad . \quad . \quad . \qquad (2.3)$$

Obviously there is a difficulty in matching (2.3) with (2.2b) near $\Delta z = l$. This problem is made easier by expressing U in the outer layer as

$$U = U_0(\Delta z)+hf\left(\frac{x}{L}\right)\frac{dU_0(\Delta z)}{dz}+\frac{h}{L}U_0(L)\tilde{u} \quad . \quad . \qquad (2.4)$$

Jackson's (1977) analysis shows that for the hill shape given by (2.1), on $x = 0$,

$$\tilde{u}(\Delta z) = \frac{1}{\{1+(\Delta z/L)\}^2}-\frac{1}{\ln(L/z_0)}\left[\frac{\ln(\Delta z/L)}{\{1+(\Delta z/L)\}^2}+\frac{1}{(\Delta z/L)\{1+(\Delta z/L)\}}\right] \qquad (2.5)$$

up to first order in $\ln^{-1}(L/z_0)$. Note that (2.4) tends to (2.3) when $z \gg l$ and then Δu is given by potential flow theory.

The changes in wind speed close to the surface of a hill are best indicated by plotting the ratio $U(\Delta z)/U_0(\Delta z)$ of the mean velocity at a height Δz above the hill to the mean velocity at the same displacement over level ground. This ratio is sometimes expressed as $1+\Delta S(\Delta z)$, where ΔS is the 'fractional speed-up ratio'. We have plotted the analytical prediction for the inner (2.2) and outer region (2.4), (2.5) and also the results of computations based on Taylor's model (Fig. 3(c)). These are compared with the measured velocities at the hill, top $x = 0$. (Direct comparison of the measured streamwise velocities and the theoretical horizontal velocities is valid here as the angle between them is very small.) The theories are within 15% of each other and of the measured values of $U(\Delta z)/U_0(\Delta z)$ outside the inner layer i.e. $\Delta z/l > 1$. For $\Delta z/l < 1$ the predictions of the two models diverge. The effect of the non-linear inertial terms is to reduce significantly the value of U/U_0 from that predicted by the linear theory, the maximum value of ΔS being reduced from 1·6 to 0·95. Unfortunately the height of the roughness elements, 0·67 l, prohibited any measurements being made below $\Delta z = l$ for comparison. The important practical point that emerges from Fig. 3(c) is that ΔS does not change by more than 20% over the inner region, so that ΔS is indeed a useful indicator of wind speed changes near the surface.

It is also of practical interest to compare $U(z)$ with the upwind velocity at the same height $U_0(z)$. This ratio is sometimes called $S(z)$ and is often assumed to be close to unity

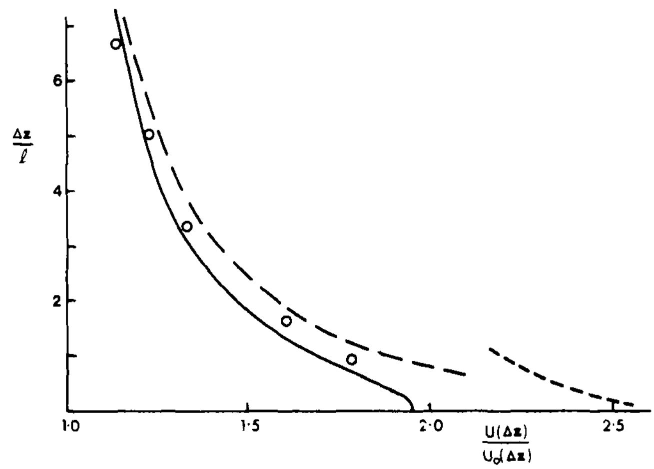

Figure 3(c). Ratio of the mean velocity at the hill top $U(\Delta z)$ to the undisturbed velocity $U_0(\Delta z)$ as a function of the displacement above the surface $\Delta z/l$ where l is the thickness of the inner region.

○ measured values
- - - - inner layer } JH theory
— — outer layer } JH theory
——— Taylor model

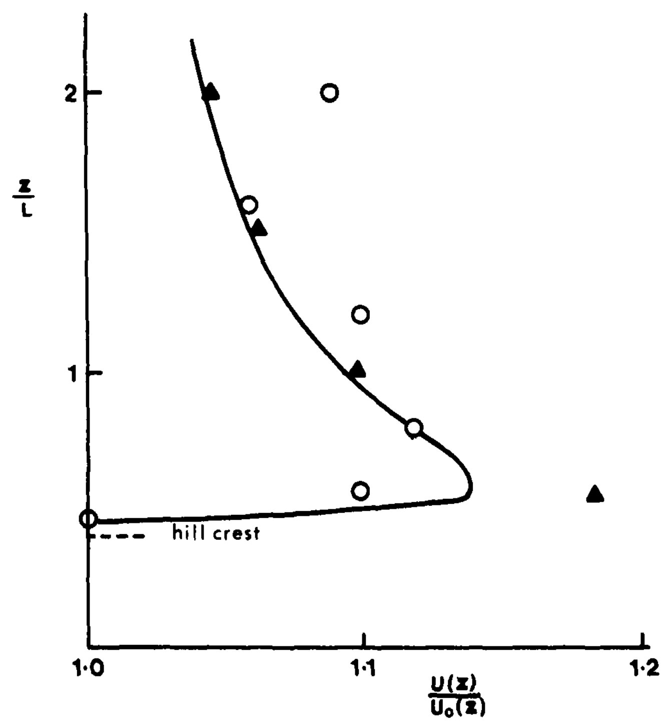

Figure 3(d). Ratio of the mean velocity at the hill top $U(z)$ to the undisturbed velocity $U_0(z)$, as a function of the height z/L. L is the half length of the hill.

○ measured values
▲ potential flow theory
——— Taylor model

above the inner region. The measured values of $S = U(z)/U_0(z)$ are plotted in Fig. 3(d) for the flow at the top of the hill and compared with the results of Taylor's model. Disregarding the uppermost data point, which is in the upper half of the boundary layer, the theory is within 20% of the measured values. Also plotted in Fig. 3(d) are the results using potential flow theory for a uniform upstream velocity. These are within 2% of those of Taylor's model down to a height $0{\cdot}3L$ above the top of the hill. Note how S changes by less than 10% above the inner layer.

Taylor's model assumes the length of the hill to be small compared with the depth of the constant stress region (i.e. $L \ll \delta/5$). To demonstrate the effect on this model of a finite depth boundary layer (where $L \sim \delta$) a series of computations were made using a modified form of the mixing length,

$$l_m = \kappa(z+z_0)\left\{1-\frac{z}{\frac{3}{2}(\delta+\frac{1}{3}z_0)}\right\}^{\frac{1}{2}} \qquad z \leqslant \delta$$

$$l_m = l_d, \text{ constant} \qquad z > \delta$$

Note that $(dl_m/dz)|_\delta = 0$.

The stress upstream was taken to vary linearly with height up to $z = \delta$ and the upstream velocity and turbulent energy profiles were computed from the undisturbed equations. Above $z = \delta$ the stress and turbulent energy were taken as zero and the velocity constant. The undisturbed boundary layer will develop with x and the changes in the velocity and turbulent energy over the integration region ($14L$) were approximately 5%.

The hill shape was Gaussian, i.e.

$$f(x/L) = \exp\{-(x/L)^2\}$$

We have taken $h = 2000\,z_0$ and $L = 5000\,z_0$, giving $l/z_0 = 300$. The speed-up ratios over different hill shapes are tabulated by Hunt (1979) and Taylor (1977). In Fig. 3(e) the predictions of ΔS at the hill top are plotted against $\Delta z/L$ for a number of values of δ/L and compared with those for a constant stress layer. With $\delta/L = 2$ the results are within 0·5% of those of the constant stress layer. Even with $\delta/L = \frac{1}{5}$, ($\delta/h = 0{\cdot}5$) the maximum difference is less than 15%. This supports the suggestion of JH that the perturbation shear stresses in the outer layer are small compared with the inertial and pressure stresses, and therefore the modelling of these stresses is unimportant.

In Fig. 3(f) the measured values of $\Delta u(\Delta z)/\{U_0(\Delta z)\}$ are compared with the results of Taylor's model for two positions, $x/L = 0{\cdot}85$ and $2{\cdot}8$, in the lee of the hill. Here the measurements and theory differ markedly. Taylor's model gives no mean flow separation, the surface shear stress being reduced by approximately $\frac{2}{3}$ of its upstream value. The wake is predicted to have effectively collapsed by $x/L = 2{\cdot}8$. The measurements show the wake to be still growing at this distance downwind of the top of the hill. Clearly the turbulence closure used in the model is inadequate in this region.

A possible explanation for the discrepancy is that the turbulent length scale in the model is based on the distance from the surface, whereas in the elevated region of high shear downwind of the hill top this should be based on the thickness of the shear layer itself. Thus the model over-predicts the values of the Reynolds stress in this region and mixes the velocity excess to the ground too quickly.

2.4. *Roughness change effects*

In the experiment where the roughness ended 1 m upstream of the hill crest there was a

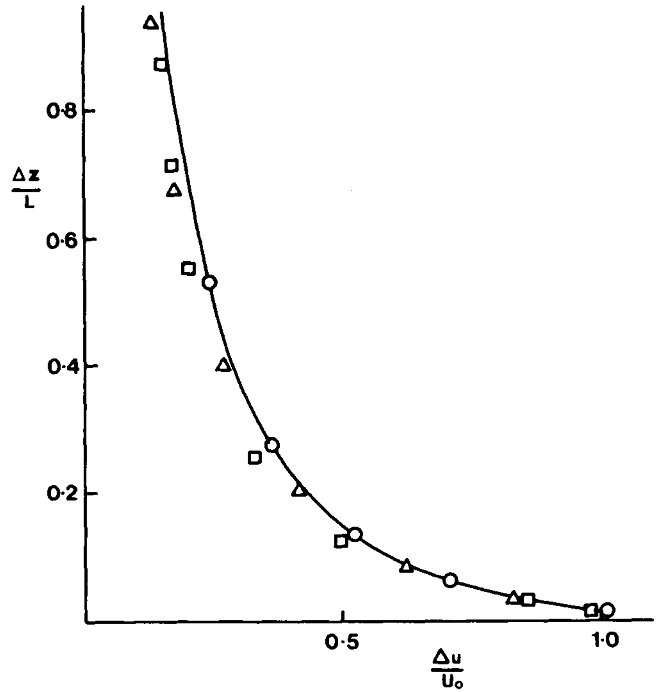

Figure 3(e). Calculations of perturbed velocity at top of a Gaussian hill, $h = 2000\,z_0$, $L = 5000\,z_0$, for varying boundary-layer thicknesses

——— constant stress layer
○ $\delta/L = 2$
△ $\delta/L = 3/5$
□ $\delta/L = 1/5$

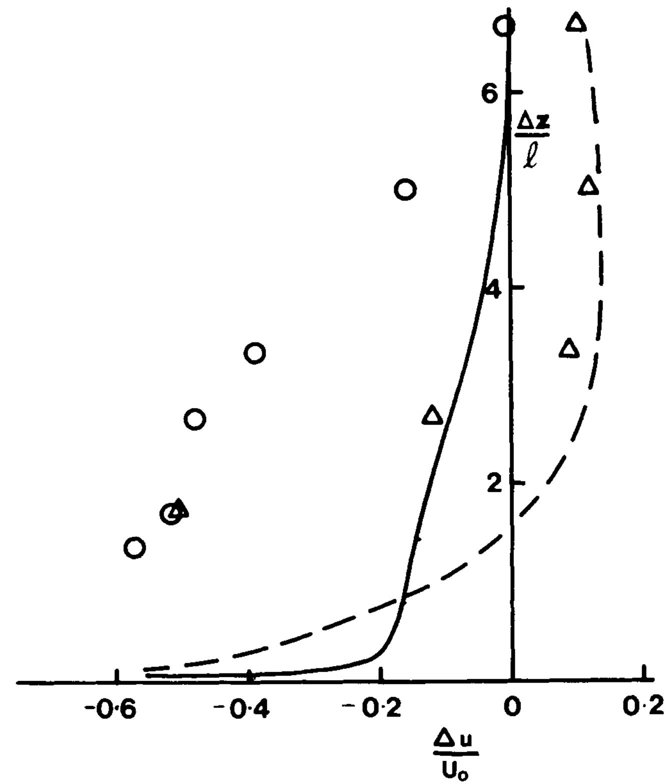

Figure 3(f). Perturbed velocity profile, $\Delta u/U_0$, in lee of the rough hill: at $x/L = 0{\cdot}85$ (Δ, measured value; — — — Taylor model) and at $x/L = 2{\cdot}8$, (○, measured value; ——— Taylor model) significant acceleration of the lower part of the boundary layer as a direct result of the change in roughness. Some comparisons with changes of roughness and change of elevation theories are interesting.

Assuming a self-similar development of the flow Townsend (1965) finds the fractional speed-up ratio (defined by (2.2c)) for a change of roughness from $(z_0)_1$ to $(z_0)_2$ at $x = x_0$ to be given approximately, when $\ln l/z_0 \gg 1$, by

$$\Delta S(x,\Delta z) = -[\ln\{(z_0)_2/(z_0)_1\}][\ln\{\delta_1/(z_0)_2\}]^{-1}[\ln\{\Delta z/(z_0)_1\}]^{-1}\int_\eta^\infty e^{-t}t^{-1}dt \quad (2.6a)$$

where $\eta = \Delta z/\delta_1$, and δ_1 is given by

$$\delta_1 \ln(\delta_1/z_0) = 2\kappa^2(x-x_0) \quad . \quad . \quad . \quad (2.6b)$$

The equivalent downstream roughness height for the smooth surface is taken as $\nu/9u_*$ where ν is the kinematic viscosity. This gives a value of $(z_0)_2 = 6\times10^{-6}$ m. The predicted values of ΔS are shown in Fig. 3(g) to agree within experimental error with measurements of

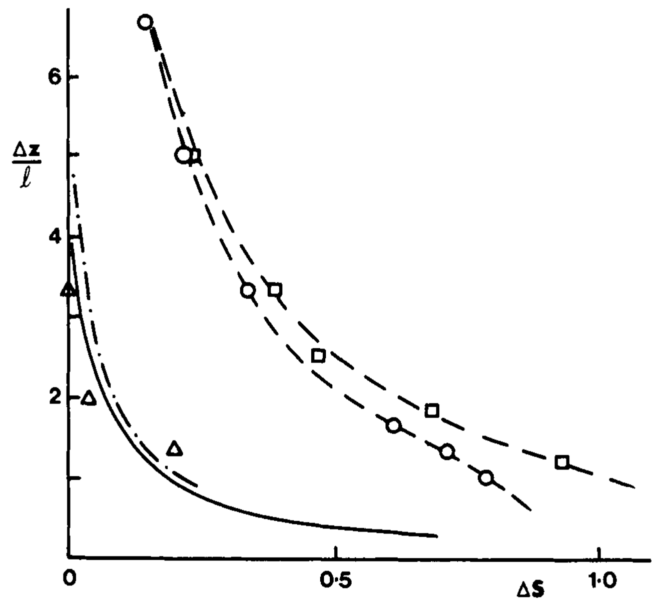

Figure 3(g). Measured fractional speed-up ΔS ($= \Delta u/U_0$) at top of the smooth and rough hills.
△ roughness change, no hill
—○— rough hill
—□— smooth hill
— · — (smooth) - (rough) hill
——— Townsend roughness change theory

ΔS taken with the roughness change but without the hill at a position equivalent to the top of the hill, a distance 1 m from the roughness change.

Following the suggestion of Jensen and Peterson (1978), if we assume a linear superposition of roughness and elevation effects on U, then over the smooth hill we can write

$$U(\Delta z) = U_0 + \Delta\hat{u}_{\text{hill}} + \Delta\hat{u}_{\text{roughness}} \quad . \quad . \quad (2.7)$$

where $\Delta\hat{u}_{\text{hill}}$ and $\Delta\hat{u}_{\text{roughness}}$ are the changes in velocity due to the hill and roughness changes alone, respectively. At the top of the hill, from Fig. 3(g), it is seen that the difference between the measured values of ΔS for the smooth and rough hills agree with the predicted value of ΔS for the roughness change alone to within 20%. At the height $\Delta z = l$, the expression (2.6) gives $\Delta S = 0{\cdot}18$ compared with a measured difference of ΔS for the two cases of 0·22.

In the lee of the smooth hill the flow is distinctly different from that over the rough hill (Fig. 3(h)). Velocity measurements and smoke releases indicated that the effects of the reduction of surface roughness were so great that the mean velocity on the lee side was actually *greater* than over *rough* level ground and that the flow did not separate, even intermittently, though the flow remained turbulent.

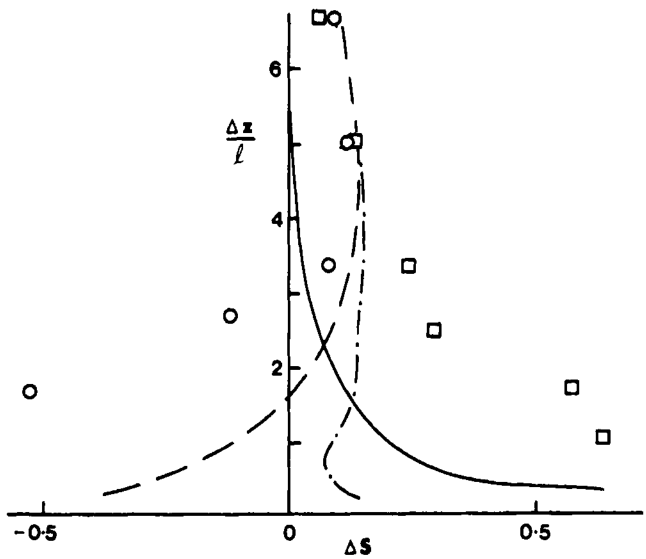

Figure 3(h). Measured fractional speed-up $\Delta S\,(=\Delta u/U_0)$ in the lee of the smooth and rough hills (at $x/L = 0{\cdot}8$)

○	rough hill – separated flow
□	smooth hill – unseparated
— —	Taylor model
———	Townsend roughness change theory
— · —	Taylor+Townsend

Paradoxically we can explain this result by adapting Prandtl's explanation (see Batchelor 1967, p. 362) for the delay of separation over a bluff obstacle when a *laminar* boundary layer becomes turbulent by the *introduction of roughness on the surface.* Although our case is different in that we are considering an initially turbulent boundary layer, these two types of separation delay are similar because in both the surface velocity near the separation point is increased so that the boundary-layer flow can penetrate further into the adverse pressure gradient on the lee side. (In the classical case the boundary layer is energized by transition, in our case by reducing the surface roughness and in other cases by vortex generators, blowing, etc.)

At a point $0{\cdot}8L$ downwind of the top of the hill the region affected by the roughness change extends up to a height $4l$ which corresponds well with the value 3·5 for δ_1 ($x = 4L$) from (2.6b). A linear superposition of Townsend's theory (Eq. (2.6a)) and the prediction of

Taylor's model severely underestimates the measured *increase* in velocity in this region.

3. Turbulence over the hill; theoretical considerations and comparison with experiment

The changes in the turbulence structure over a hill (or any obstacle) depend on whether the travel time, T, of an eddy starting upwind at height z_1 as it is advected along a streamline is large or small compared with the Lagrangian time scale or eddy 'turn-over' time scale at $z = z_1$, $\mathcal{T}_L(z)$. Along those streamlines where $T \ll \mathcal{T}_L$ then the turbulent kinetic energy is dissipated relatively slowly while the turbulent eddies are being distorted by the mean flow over the hill. Consequently the history of the straining of the eddies over the whole period that they traverse the hill has to be considered in estimating the changes in turbulence. Conversely along the streamlines where $T \gg \mathcal{T}_L$, the dissipation takes place rapidly and so the turbulence broadly depends on a balance between energy production by the *local* shear and the local dissipation, although the diffusion of turbulent energy and distortion of turbulence by curvature effects are also important.

These two regions where different changes of turbulence structure occur are roughly separated by a streamline at height l_T where $\mathcal{T}_L(z = l_T)$ is approximately equal to the travel time over the hill, $T \simeq L/U(z = l_T)$. Since $\mathcal{T}_L(z = l_T) \simeq 0{\cdot}3\, l_T/u_*$ (Hunt and Weber 1979),

$$0{\cdot}3\, l_T/u_* \simeq L/U(z = l_T),$$

so that $l_T \ln(l_T/z_0) \simeq L$.

Thus we conclude that l_T must be of the same order as the thickness of the inner shear stress layer. A more precise relation between l_T and l can only be provided by experiment, as we shall show.

Therefore in the inner region and close to the surface where the turbulence is in equilibrium, at or upwind of the hill top the variances of all three turbulent intensity components σ_u^2, σ_v^2, σ_w^2 are increased in proportion to $\Delta\tau/\rho u_*^2$, where $\Delta\tau$ is the change in surface shear stress due to the hill. Based on estimates for $\Delta\tau$ by JH over a hill with small slope, when $\Delta z \ll l$

$$\frac{\Delta\sigma^2}{\sigma^2} = \frac{\Delta\sigma_u^2}{(\sigma_u^2)_\infty} = \frac{\Delta\sigma_v^2}{(\sigma_v^2)_\infty} = \frac{\Delta\sigma_w^2}{(\sigma_w^2)_\infty} \simeq 4\left(\frac{h}{L}\right) \qquad (3.1)$$

where $\sigma^2 = (\sigma_u^2+\sigma_v^2+\sigma_w^2)$, and $\Delta\sigma_u$, etc. are the changes in the components of turbulence intensity from their upwind values.

In the outer region the turbulence changes can be estimated from rapid distortion theory (Hunt 1973; Townsend 1972). Assuming the distortion to be sufficiently rapid so that an eddy has insufficient time to adjust to the local strain rate and to interact with other eddies then the only effect on the turbulence of the distortion to the mean flow is to compress or lengthen and rotate individual vortex elements of the turbulence. By considering a distortion to the mean flow in the x,z plane (y being taken transverse to the mean flow) then the fluctuating vorticity at any point in the flow $\underline{\omega}$, can be related to the upstream value, $\underline{\omega}_\infty$, by

$$\omega_i(x, y, z) = \gamma_{ij}(x, z)\, \omega_{\infty j}(x, y, z_s) \qquad (3.2)$$

where

$$\gamma_{ij} = \begin{bmatrix} U/U_0(z_s) & 0 & -U_0\partial T/\partial z \\ 0 & 1 & 0 \\ W/U_0(z_s) & 0 & U_0\partial T/\partial x \end{bmatrix}$$

z_s is the upstream height of the streamline passing through the position (x, y, z) and

$$T = \lim_{X \to \infty} \left[\int_{-X}^{x} \frac{dx'}{U(x', z')} - \frac{(x+X)}{U_0(z_s)} \right]$$

the difference in travel time along the streamline, with coordinates (x', z'), from that of the undisturbed flow.

To first order in perturbed quantities,

$$U_0 \, \partial T/\partial x = 1 - \Delta u(x, \Delta z)/U_0(z_s) \quad \text{and} \quad U_0 \, \partial T/\partial z = -2\Delta w(x, \Delta z)/U_0(z_s)$$

For a symmetric hill, at the hill creast $\Delta w = 0$ and the strain tensor γ_{ij} reduces to

$$\gamma_{ij}(0, \Delta z) = \begin{bmatrix} 1 + \dfrac{\Delta u(0, \Delta z)}{U_0(z_s)} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 - \dfrac{\Delta u(0, \Delta z)}{U_0(z_s)} \end{bmatrix}$$

Thus at the top of a symmetric hill there is a simple compression of streamlines with no rotation of vortex elements. We can estimate the changes in turbulence quantities at a given height above the hill top by applying the results of Batchelor and Proudman (1954) who have calculated the changes in σ_u, σ_v and σ_w for a simple contraction of the mean flow and isotropic upstream turbulence. These show a decrease in σ_u and increases in σ_v and σ_w. Townsend (1976, p. 72) has shown that to first order in $(c-1)$, where c is the contraction ratio (in our case $c = 1 - \Delta u/U_0$) Batchelor and Proudman's result is

$$\frac{\sigma_u^2(x, z)}{\sigma_u^2(z_s)} = 1 - \frac{4}{5} \frac{\Delta u(x, z)}{U_0(z_s)}, \qquad (3.3)$$

and

$$\frac{\sigma_w^2(x, z)}{\sigma_w^2(z_s)} = 1 + \frac{4}{5} \frac{\Delta u(x, z)}{U_0(z_s)}.$$

Goldstein and Durbin (1979) show that the effect of a non-zero $\partial T/\partial z$ on σ_u^2 and σ_w^2 for a simple contraction is second order in $(c-1)$. We can therefore apply these results over the entire hill.

The measured r.m.s. turbulent velocity parallel to the local mean velocity, u_s', non-dimensionalized with U_δ is plotted in Fig. 4(a) as a function of x, with $\Delta z/\delta$ as a parameter. At a given height Δz above the hill the longitudinal turbulence velocity *decreases* as the top of the hill is approached. Beyond the top of the hill very large turbulence levels are observed both near the separated shear layer and out to $\Delta z/\delta < 0{\cdot}40$. The shear layer had reattached at $x = 1{\cdot}15$ m.

Again due to the relative height of the roughness elements no measurements of turbulence intensity could be made in the inner layer. However, Bradley (1980) in a well-documented set of field measurements has obtained profiles of all three turbulence components at the top of Black Mountain, Australia, the cross-section of which can be approximated by equation (2.1) with $h = 170$ m, $L = 275$ m. The height of the inner layer, is then 28 m. Close to the surface of the hill σ_u, σ_v and σ_w were all approximately doubled over the values measured upwind. This gives $\Delta\sigma^2/\sigma_\infty^2 \simeq 3.0$ compared with an estimate of $\Delta\sigma^2/\sigma_\infty^2 \simeq 2{\cdot}5$ given by Eq. (3.1).

The measured change in streamwise turbulence intensity $\Delta\sigma_u^2/(\sigma_u^2)_\infty$ along *streamlines* (estimated from potential theory) at the hill top from the wind tunnel experiment, is plotted against $\Delta z/L$ in Fig. 4(b). The measurements ($\Delta z > l$) show a *decrease* in turbulence intensity

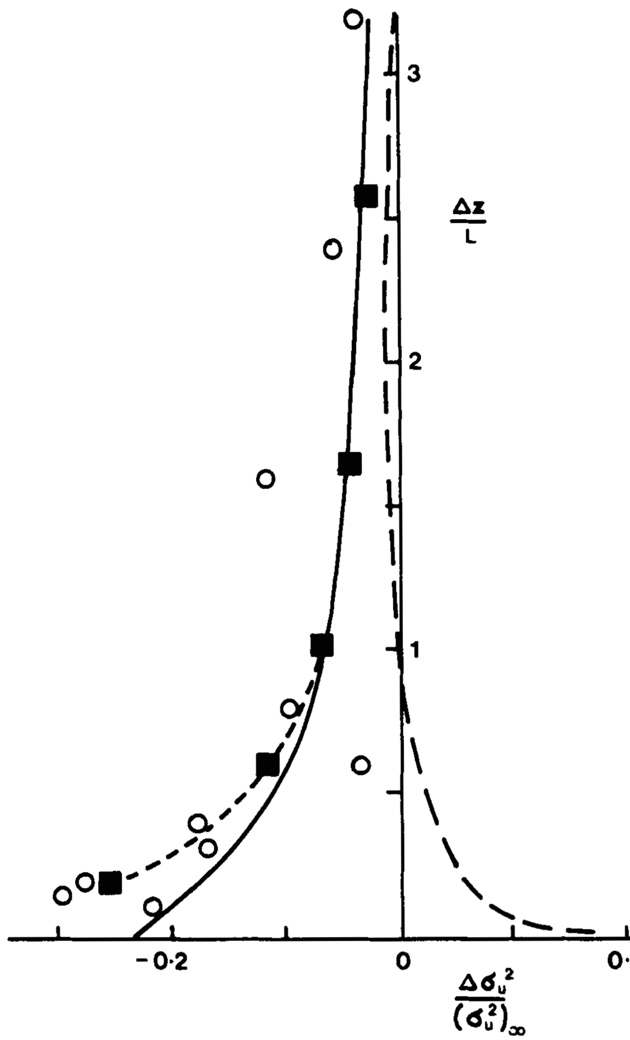

Figure 4(b). Changes in streamwise turbulence intensity, $\Delta\sigma_u^2$, at hill top, normalized by the upwind values.
○ measured
——— rapid distortion theory using potential theory mean velocity perturbation Δu
-- ■ -- rapid distortion theory using measured Δu
— — — Taylor eddy viscosity model

from that upstream. Also shown are the prediction of rapid distortion theory (3.3), with both the measured Δu and that estimated from potential flow theory, and the eddy viscosity approach of Taylor (1977), who assumes that σ_u^2 is related to the local velocity gradients by $\sigma_u^2 = \frac{1}{3}\sigma^2 - K\left(\frac{\partial U}{\partial x} - \frac{\partial W}{\partial z}\right)$ where K is an eddy viscosity.

The predictions of the two theories are very different. The eddy viscosity approach predicts an increase in σ_u^2 for $\Delta z < L$ with a slight decrease above this level, whereas rapid distortion theory predicts a decrease in σ_u^2 whose magnitude increases towards the hill surface. Rapid distortion theory appears to provide a useful approximation for the turbulence changes in the region $\Delta z > l$ where the turbulence is primarily controlled by its upstream history. Towards the surface the prediction using the measured Δu is better than using Δu estimated from potential theory. At $\Delta z = 3l$ the theory (using the measured Δu) predicts $\Delta\sigma_u^2/(\sigma_u^2)_\infty = -0{\cdot}19$ compared with a measured value of approximately $-0{\cdot}18$.

Rapid distortion theory predicts an *increase* in the vertical component of the turbulence intensity σ_w. For the hill used in the present wind tunnel studies it predicts $\Delta\sigma_w^2/(\sigma_w^2)_\infty = 0{\cdot}16$ at $\Delta z/L = 0{\cdot}4$. Bradley's (1980) field measurements show that in the outer layer at a height $\Delta z/L = 0{\cdot}3$, although σ_u was increased by $\Delta\sigma_u^2/(\sigma_u^2)_\infty = 0{\cdot}2$ the relative increase in σ_w, $\Delta\sigma_w^2/(\sigma_w^2)_\infty = 1{\cdot}25$, was significantly greater, thus supporting the qualitative features of the theory.

Taking our own and other wind tunnel experiments (e.g. Bouwmeester 1978; Huber *et al.* 1976) and field experiments by Bradley (1980) and Bowen (1979) it now appears that the rapid distortion changes in turbulence occur above a distance l_T from the surface where

$$l_T \simeq l \quad . \quad . \quad . \quad . \quad (3.4)$$

and is given by Eq. (2.2a).

4. Conclusions

From the comparison between theory and wind tunnel experiments presented in this paper the following remarks can be made.

(a) *Mathematical modelling*

The effects of modelling the shear stress in the outer layer are negligible in determining the perturbed mean flow even though this layer is still well within the boundary layer. This is because (as suggested by JH) the perturbation to the mean flow is essentially inviscid. The modelling of the shear stress only becomes important in an inner layer close to the surface.

Linearizing the momentum equations exaggerates the speed-up near the surface for moderately sloped hills. For a hill with $h/L = 0{\cdot}4$, ΔS is over-predicted by 25%.

The wake flow is poorly modelled even when there is no mean flow separation and suggests that a higher order closure scheme is necessary.

In the case of combined roughness and elevation changes a linear superposition of the two effects can be approximately justified at or upwind of the hill top. In the lee of the hill with an associated change in roughness from rough to smooth the accelerating shear layer due to the roughness change stabilizes the flow and significantly alters the flow. Here a linear superposition is not appropriate.

To model the changes in turbulent properties of the flow an eddy viscosity approach is only appropriate in an inner layer whose thickness is similar to that of the inner shear stress layer. Outside this layer, where the history of the turbulence becomes important, rapid distortion theory provides a simple and approximate method of estimating the changes to the turbulence.

(b) *Wind tunnel modelling*

Because of the need in some wind tunnels to use relatively large roughness elements in comparison with the size of model hills in order to obtain a fully turbulent flow over the surface, the height of these roughness elements may have to be out of scale with the depth of the inner shear layer. Or, in other cases where large roughness elements occur at full scale, l/k may be correctly modelled, but then l may be of the same order as k. In our experiment l/k is only 1·25. In either case it means that reliable wind tunnel measurements of the inner layer are difficult to obtain. We are not aware of any yet.

(c) *Practical and meteorological consequences*

The marked changes in the anisotropy and in the vertical structure of turbulence over hills shown here may need to be considered in practical problems such as air pollution, dispersion over hills (as indicated by Hunt *et al.* (1979)) or the wind loading of structures on hills. The effects of roughness changes may, by influencing the wake structure, have a significant effect on the drag of hills which in turn may have effects on air flows at higher levels. This effect on the aerodynamic drag of water waves has been recognised by Gent and Taylor (1976).

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