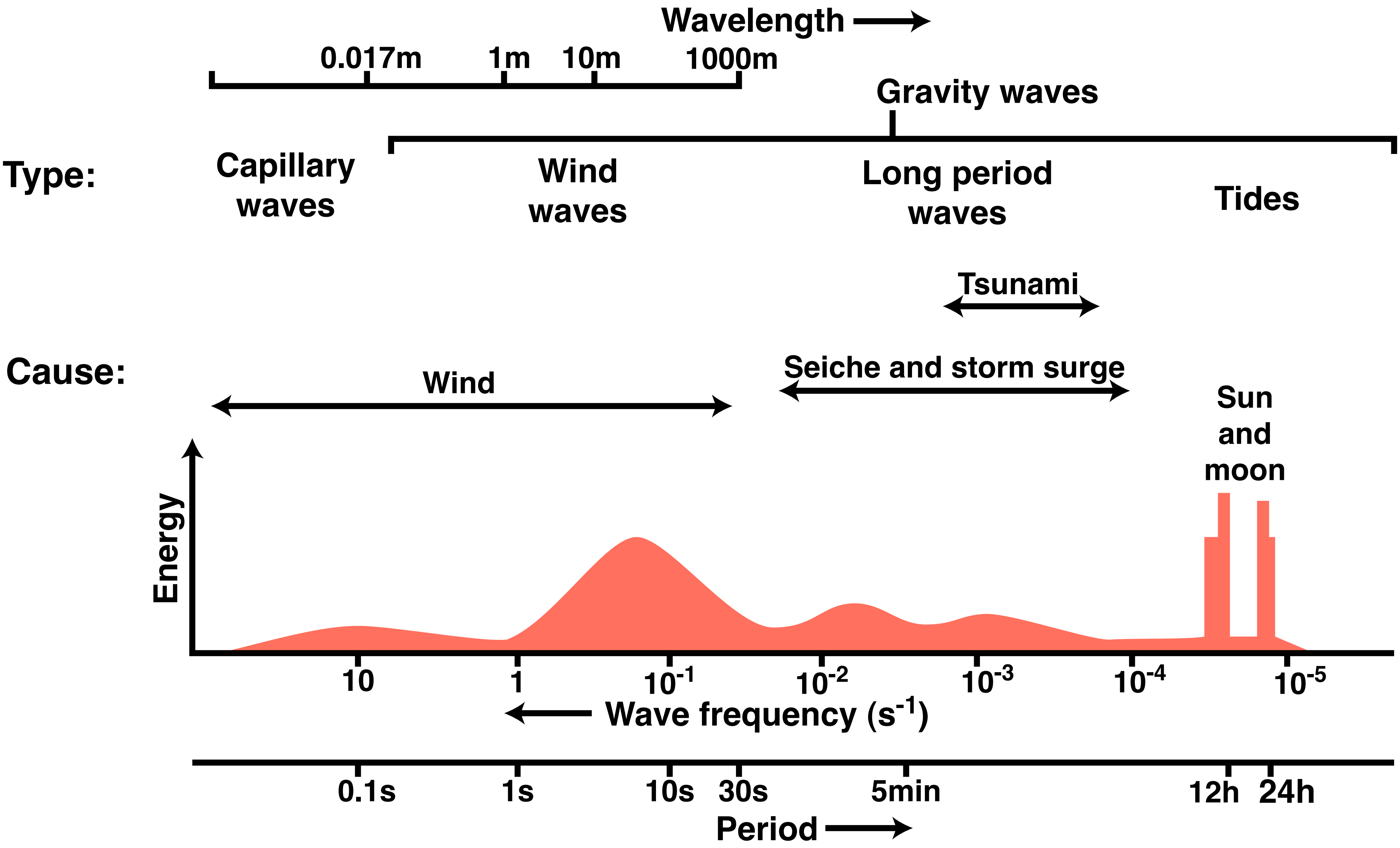
What are waves?

- A wave transfers a disturbance / energy from one part of a material to another.
- The energy is propagated through the material without substantial overall motion of the material.
- The energy is propagated without any significant distortion of the wave form and at constant speed.
- Can be either on the surface or within the medium.


Wavelength →
0.017m
1m
10m
1000m
Gravity waves
Type:
Capillary waves
Wind waves
Long period waves
Tides
Tsunami
Cause:
Wind
Seiche and storm surge
Sun and moon
Energy
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
← Wave frequency (s^{-1})
0.1s
1s
10s
30s
5min
12h
24h
Period →

Types - Initial forcing

- Wind
- Gravity from astronomical bodies (Tides)
- Anything that causes a discontinuity in the ocean surface
  - Earthquakes
  - Landslides
  - Raindrops

Types - Restoring Force

- Restoring force acts on a water particle displaced from its equilibrium position.
- Restoring force causes the water particle to ‘overshoot’, setting up an oscillation.
- Two possible restoring forces for ocean surface waves:

1. Surface tension (capillary waves)
2. Gravity (surface gravity waves)

Types - Period

< 0.2 s	Capillary waves
1 - 10 s	Locally generated wind waves, 'chop'
10 - 25 s	Remotely generated wind waves, 'swell'
25s - 20min	Infragravity waves and Tsunamis
~12h +	Tides

Gravity wave theory

- Approximations
  - Periodic in time and space.
  - The waves shapes are sinusoidal.
  - The wave amplitudes are small compared to wavelength and depth.
  - Viscosity, surface tension, and the earth's rotation can be ignored.
  - Freely propagating, and uniform depth.

Dispersion relation

The dispersion relation gives the frequency (ω) associated with a particular wavenumber (k).

For surface gravity waves

$$\omega = \sqrt{gk \tanh(kH)}$$

$g = \text{gravitational acceleration}$

$H = \text{water depth}$

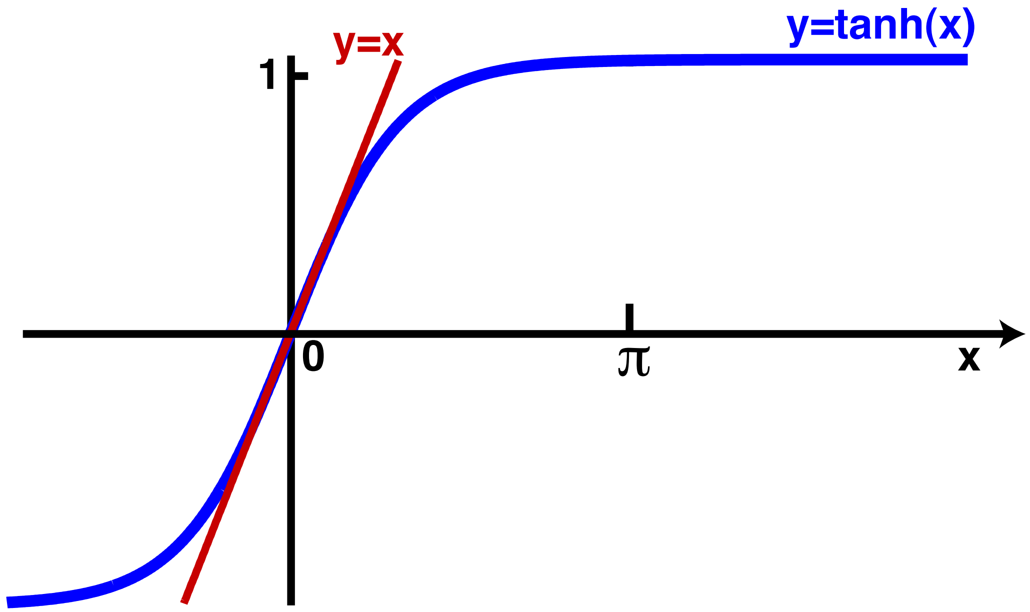
Phase speed

Speed that wave crests travel.

The phase speed for surface gravity waves is

$$c = \frac{\omega}{k} = \sqrt{\frac{g}{k}\tanh(kH)} = \sqrt{\frac{gL}{2\pi}\tanh\left(\frac{2\pi H}{L}\right)}$$

Phase speed



- Two limiting cases:
  - When x is small, tanh(x) ~ x
  - When x is greater than pi, tanh(x) ~ 1

Deep water limit

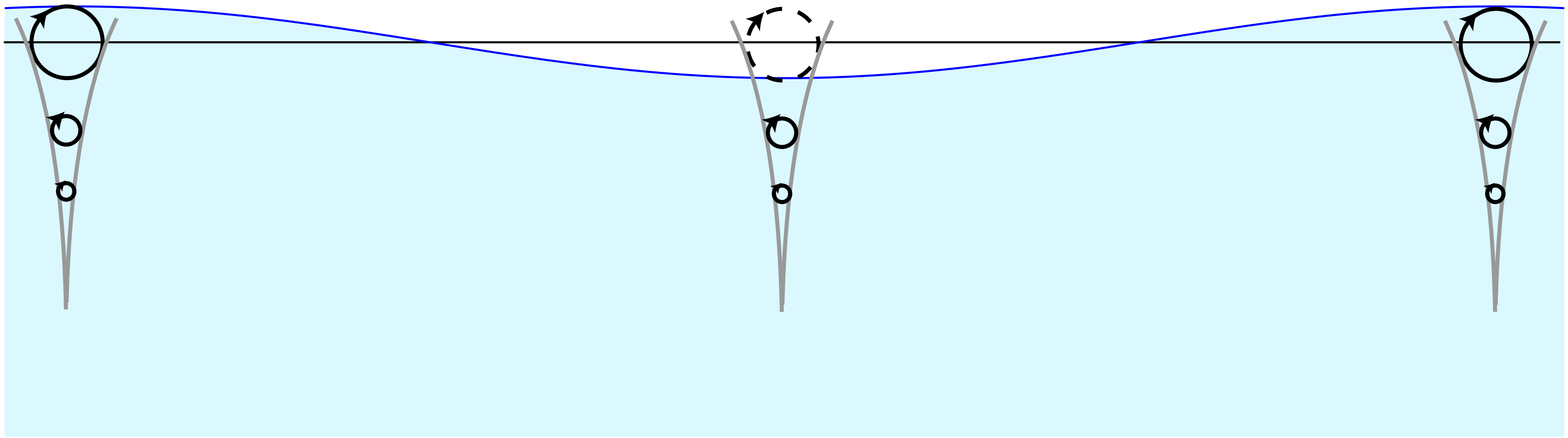
$$c = \frac{\omega}{k} = \sqrt{\frac{g}{k}\tanh(kH)} = \sqrt{\frac{gL}{2\pi}\tanh\left(\frac{2\pi H}{L}\right)}$$

$$\tanh(kH) \sim 1 \Rightarrow kH > \pi \Rightarrow H > \frac{L}{2}$$

$$\boxed{c = \sqrt{\frac{g}{k}} = \sqrt{\frac{gL}{2\pi}}}$$

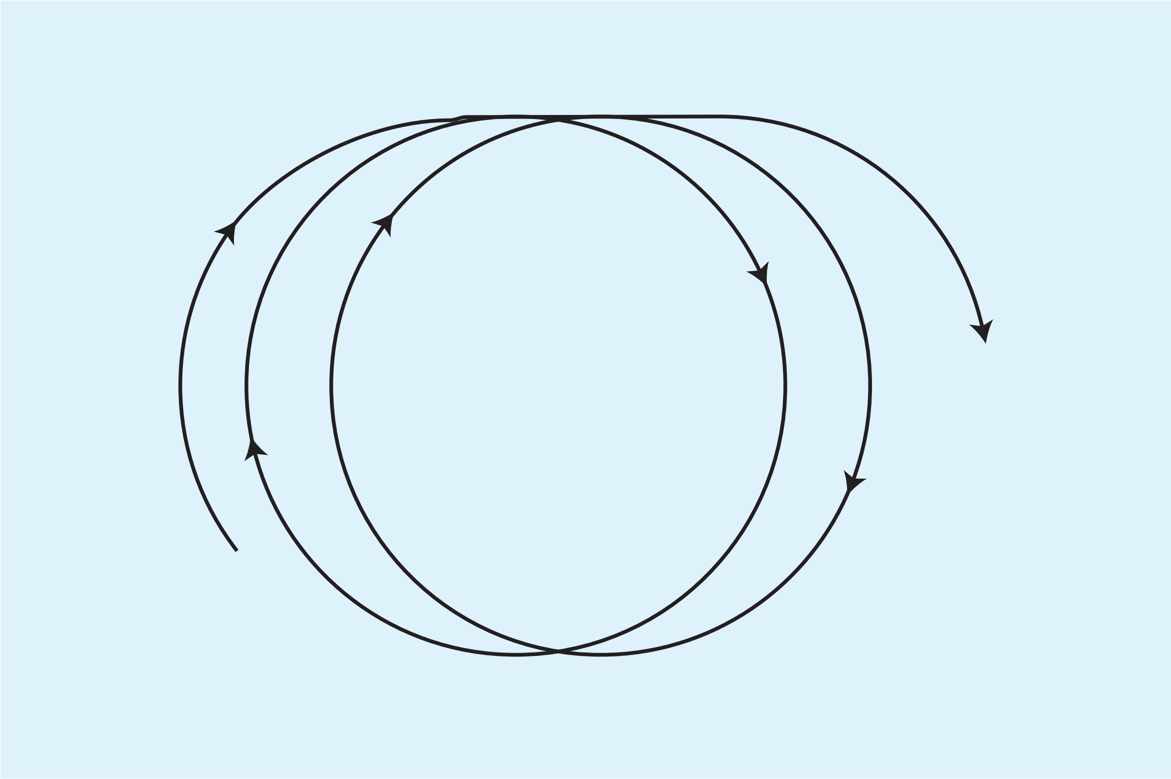
Particle motions

- Particles move in a nearly circular path.
- Orbital diameter decreases exponentially.
- Near zero displacement by depth = L/2.



Wave (Stokes) drift

- Displacement at the top of the 'circle' is greater that the negative displacement at the bottom.



Shallow water limit (long)

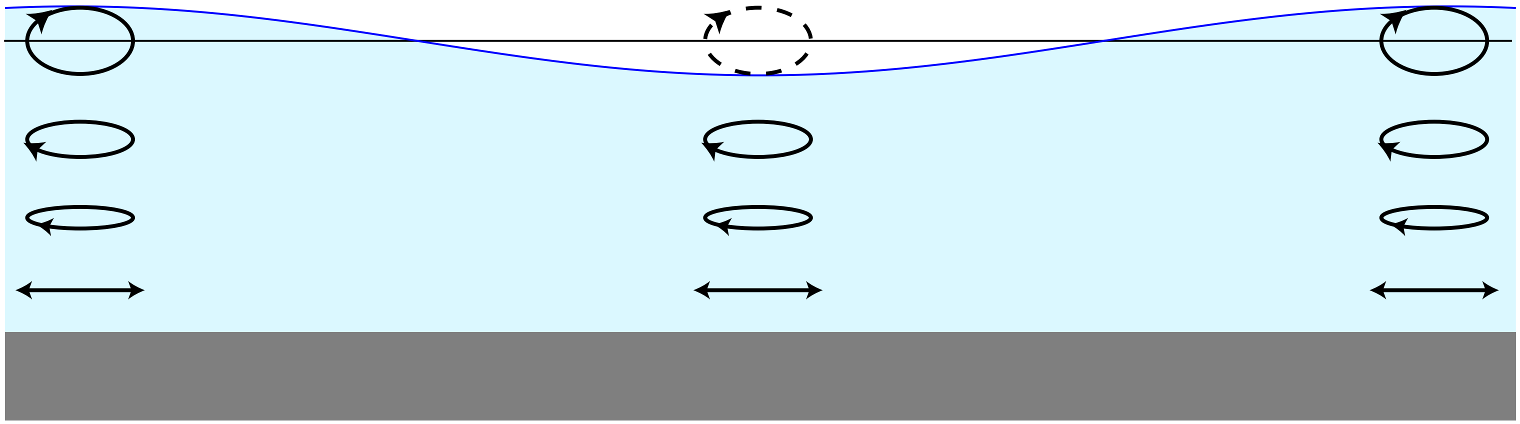
$$c = \frac{\omega}{k} = \sqrt{\frac{g}{k}\tanh(kH)} = \sqrt{\frac{gL}{2\pi}\tanh\left(\frac{2\pi H}{L}\right)}$$

$$\tanh(kH) \sim kH \Rightarrow kH \ll 1 \Rightarrow H \lesssim \frac{L}{20}$$

$$\boxed{c = \sqrt{gH}}$$

Particle motions

- Waves ‘feel’ the bottom.
- Particles paths are ellipses, which get progressively flatter with depth.
- Near bottom flows are rectilinear.



Dispersive waves

Waves are dispersive if their speed depends on wavenumber.

Deep water waves are dispersive.

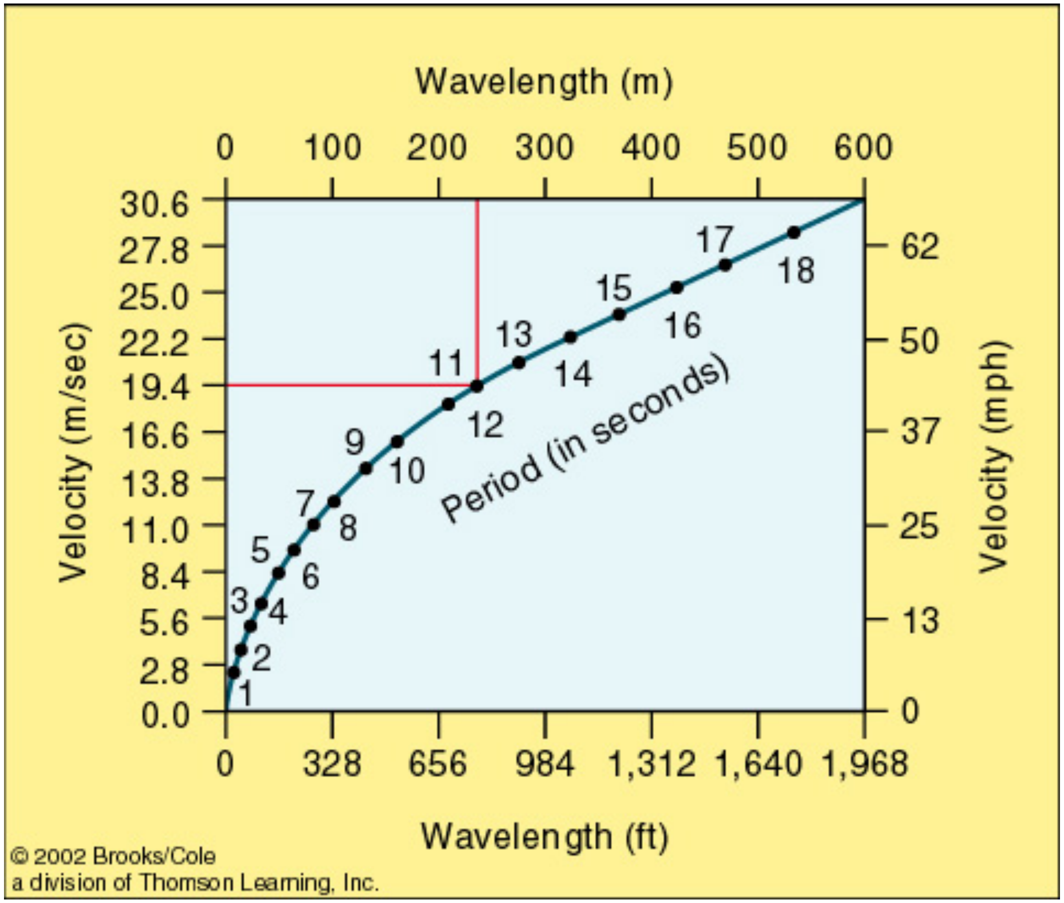
$$c = \sqrt{\frac{g}{k}} = \sqrt{\frac{gL}{2\pi}}$$

Longer wavelength (and period) waves travel faster.

Shallow water waves are NOT dispersive.

$$c = \sqrt{gH}$$

Dispersive waves



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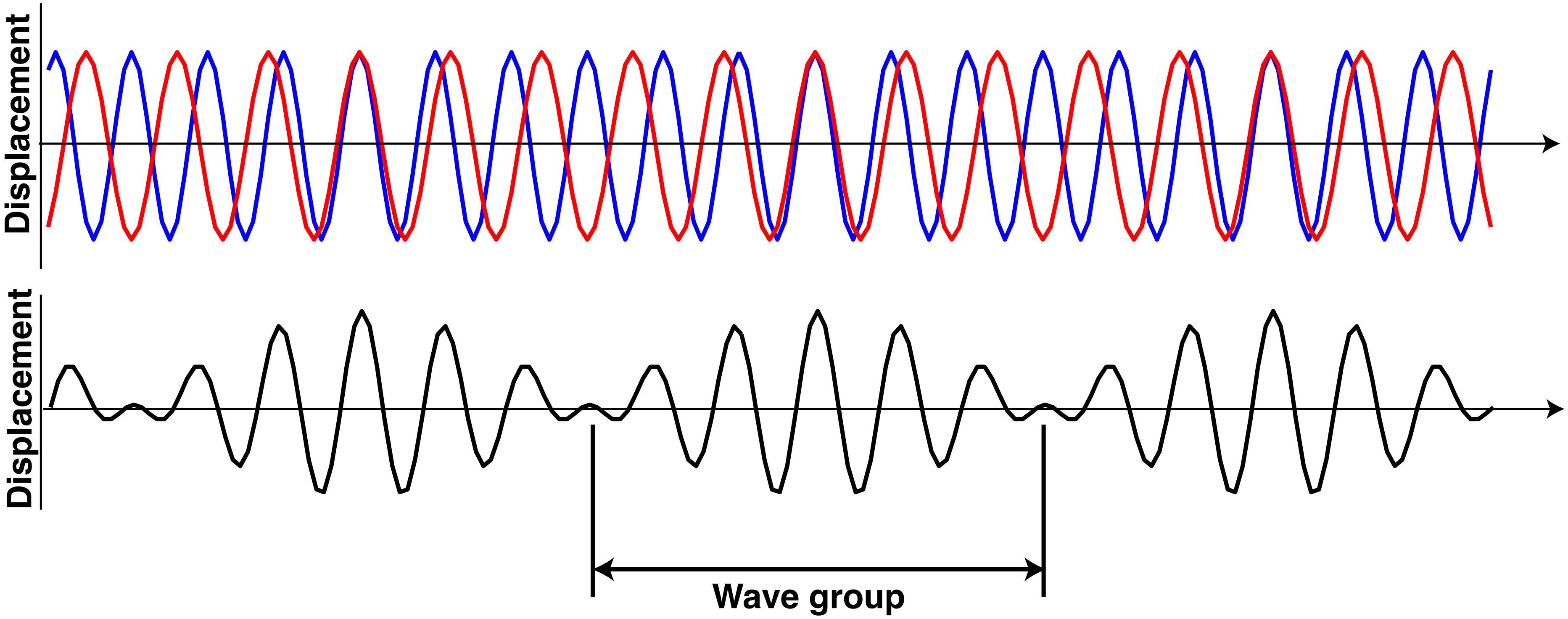
Wave groups

- Ocean not made up of a single frequency wave.
- Add another frequency wave.



Wave groups

$$\begin{aligned} \eta_1 &= a_1\cos(k_1 x - \omega_1 t) \\ &\quad + \\ \eta_2 &= a_2\cos(k_2 x - \omega_2 t) \end{aligned}$$

Let

$$\omega_1 = \overline{\omega} + \frac{\Delta\omega}{2}, \quad \omega_2 = \overline{\omega} - \frac{\Delta\omega}{2} \qquad k_1 = \overline{k} + \frac{\Delta k}{2}, \quad k_2 = \overline{k} - \frac{\Delta k}{2}$$

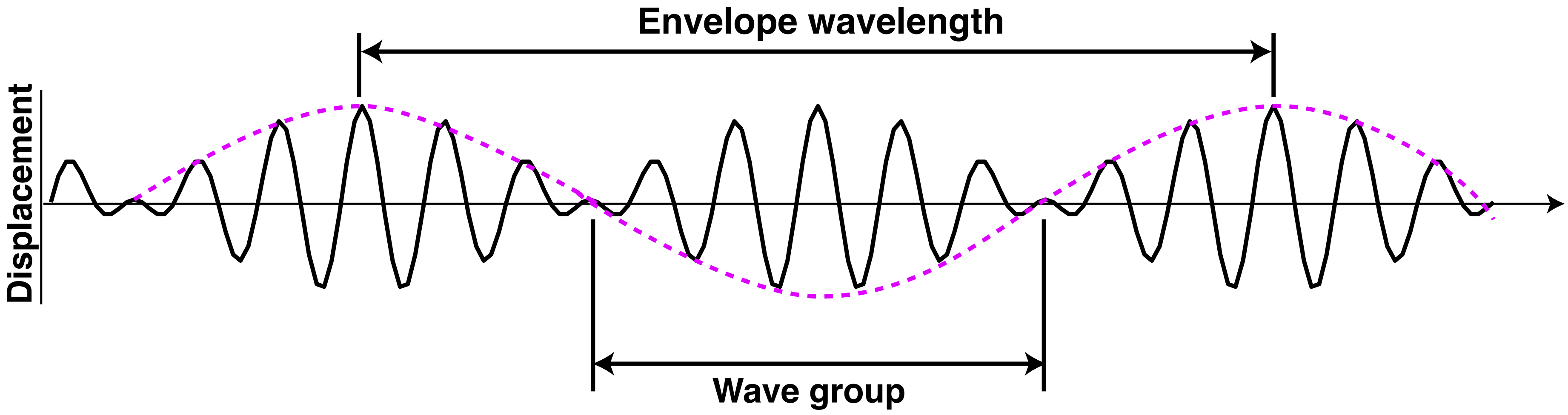
Trigonometric identity

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\eta_3 = a_3\cos(\bar{k}x - \bar{\omega}t)\cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right)$$

Wave groups

$$\eta_3 = a_3 \cos(\bar{k}x - \bar{\omega}t) \cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right)$$



$$c = \frac{\omega}{k} \quad \rightsquigarrow \quad c_g = \frac{\Delta\omega}{\Delta k}$$

Wave groups

Taking the limit gives

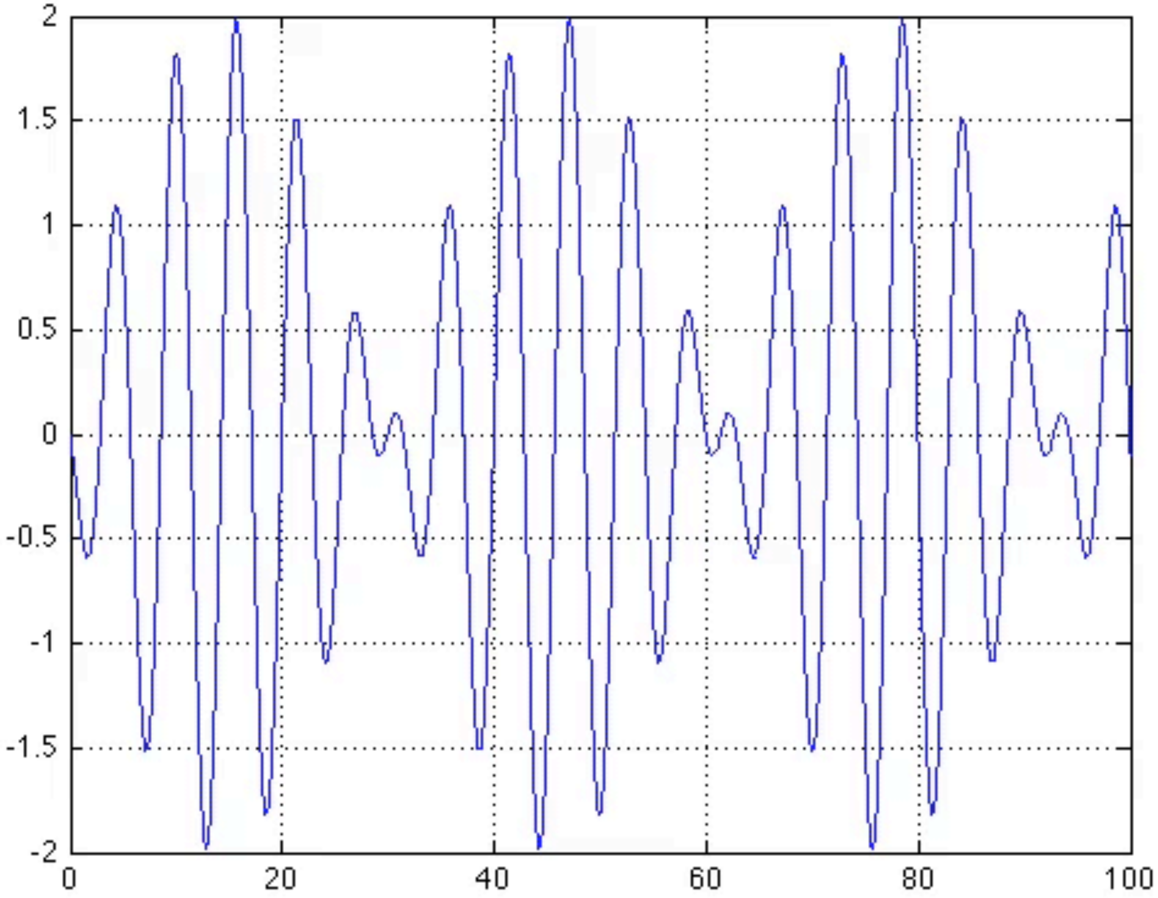
$$c_g = \frac{d\omega}{dk}$$

Shallow water limit

$$\begin{aligned} \omega &= \sqrt{gHk^2} \\ c_g &= \frac{d\omega}{dk} = \sqrt{gH} \\ &= c \end{aligned}$$

Animation: c = c_g

$$\sin(x-t)+\sin(1.2x-1.2t)$$



Group speed

Taking the limit gives

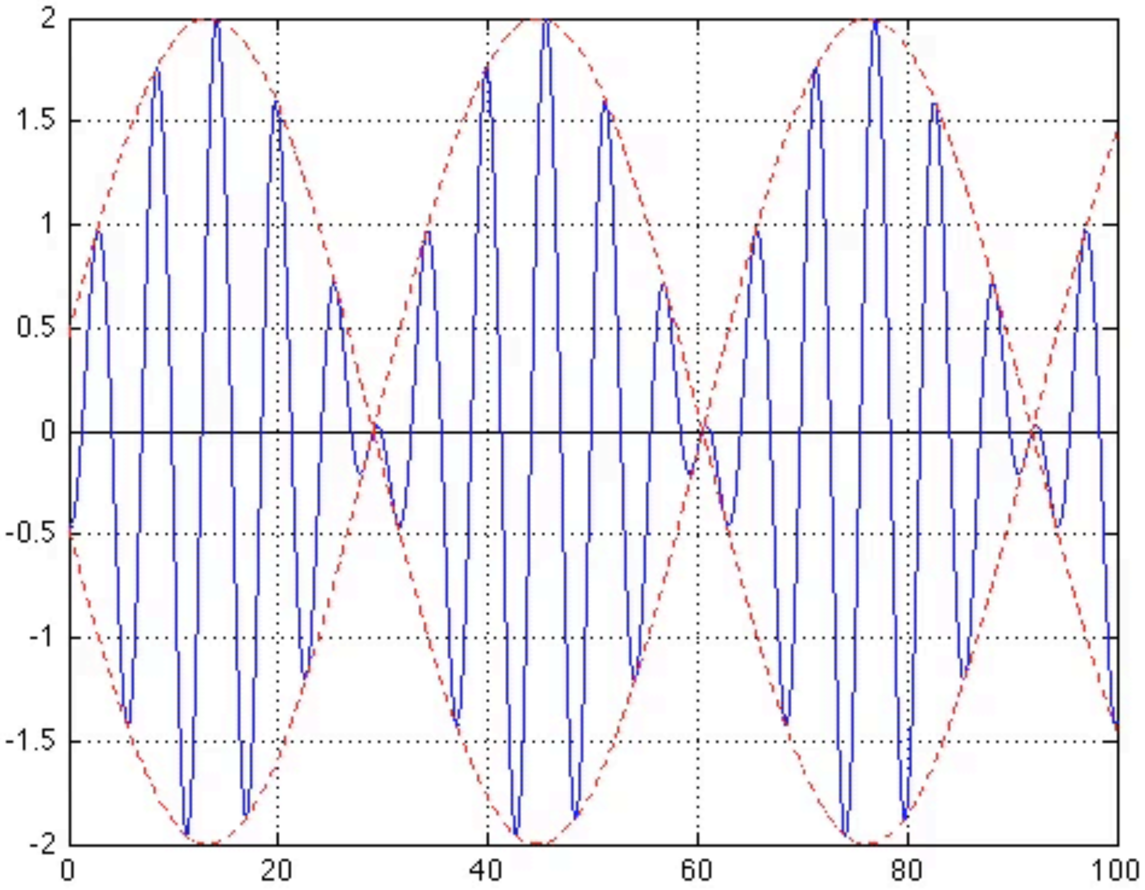
$$c_g = \frac{d\omega}{dk}$$

Deep water limit

$$\begin{aligned} \omega &= \sqrt{gk} \\ c_g &= \frac{d\omega}{dk} = \frac{1}{2}\frac{g}{\sqrt{gk}} \\ &= \frac{1}{2}\sqrt{\frac{g}{k}} \\ &= \frac{1}{2}c \end{aligned}$$

Animation: $c > c_g$

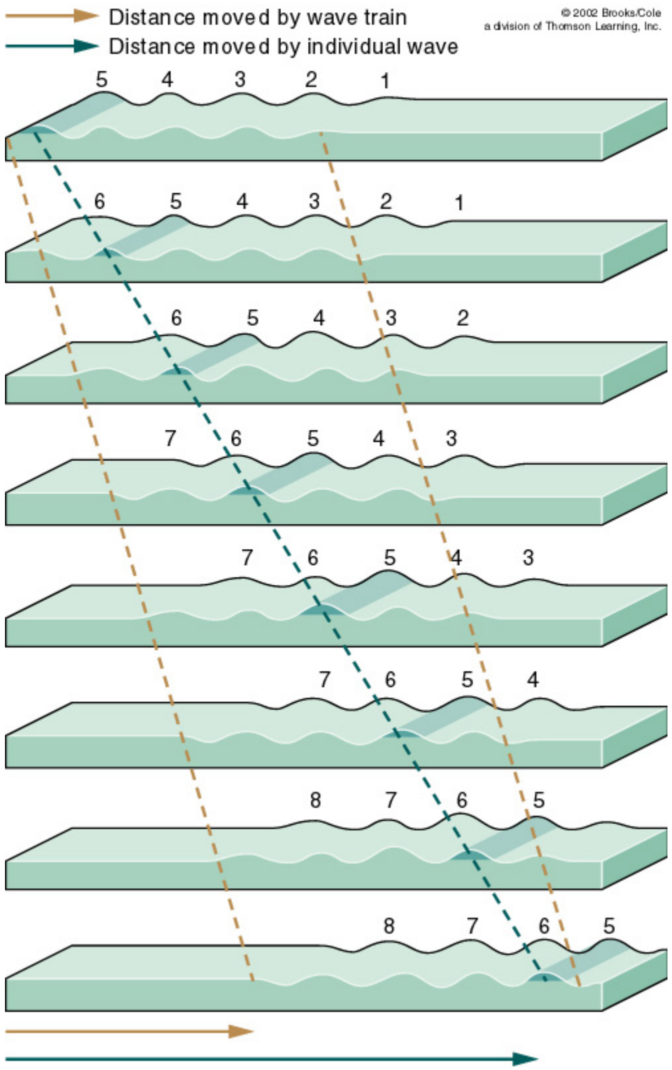
$$\cos(x - t) + \cos(1.2x - 1.1t)$$



Group speed

Taking the limit gives

$$c_g = \frac{d\omega}{dk}$$



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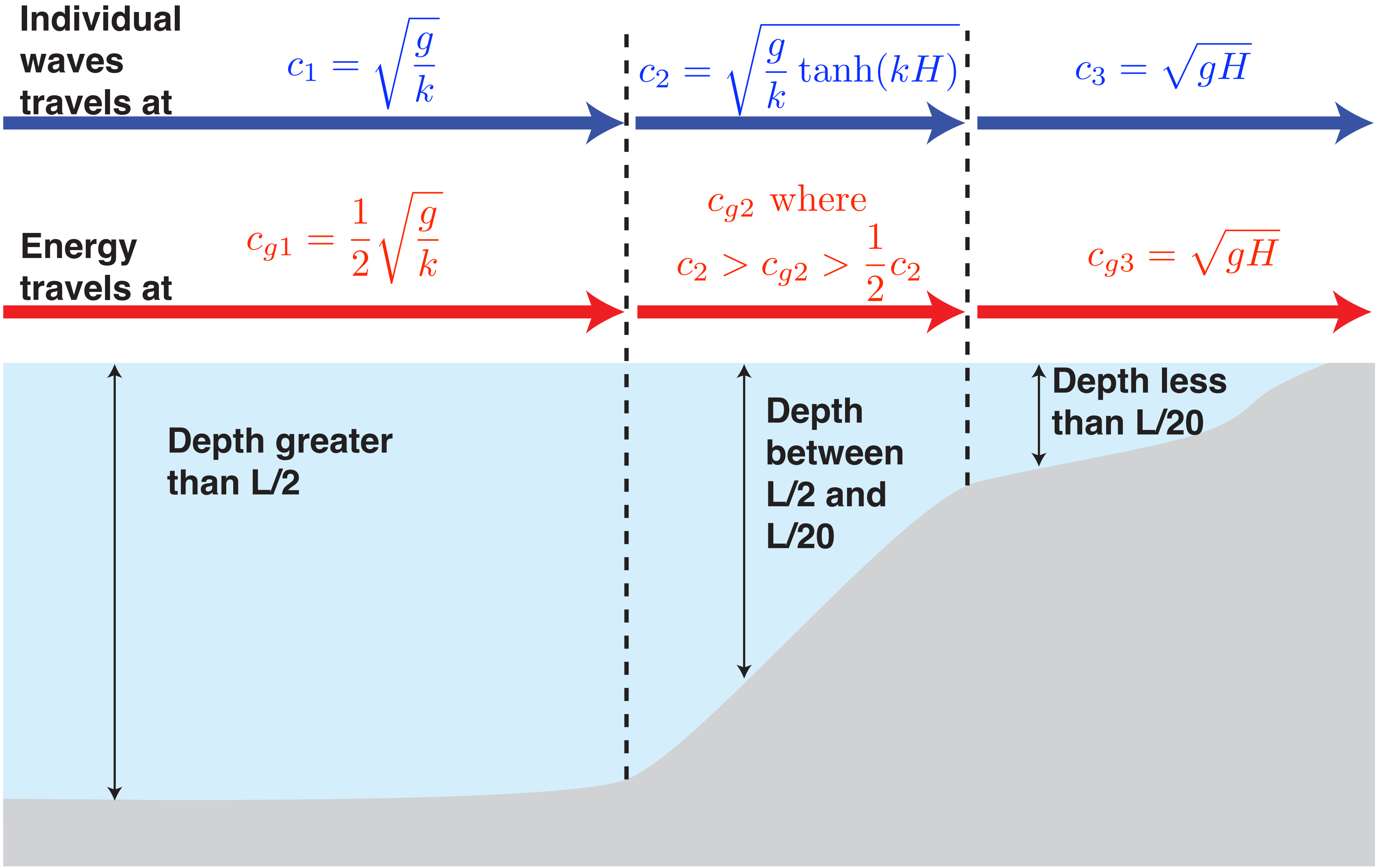
Deep water limit

$$\begin{aligned} \omega &= \sqrt{gk} \\ c_g &= \frac{d\omega}{dk} = \frac{1}{2}\frac{g}{\sqrt{gk}} \\ &= \frac{1}{2}\sqrt{\frac{g}{k}} \\ &= \frac{1}{2}c \end{aligned}$$

Recap



Individual waves travels at

$c_1 = \sqrt{\frac{g}{k}}$

$c_2 = \sqrt{\frac{g}{k}\tanh(kH)}$

$c_3 = \sqrt{gH}$

Energy travels at

$c_{g1} = \frac{1}{2}\sqrt{\frac{g}{k}}$

c_{g2} where $c_2 > c_{g2} > \frac{1}{2}c_2$

$c_{g3} = \sqrt{gH}$

Depth greater than L/2

Depth between L/2 and L/20

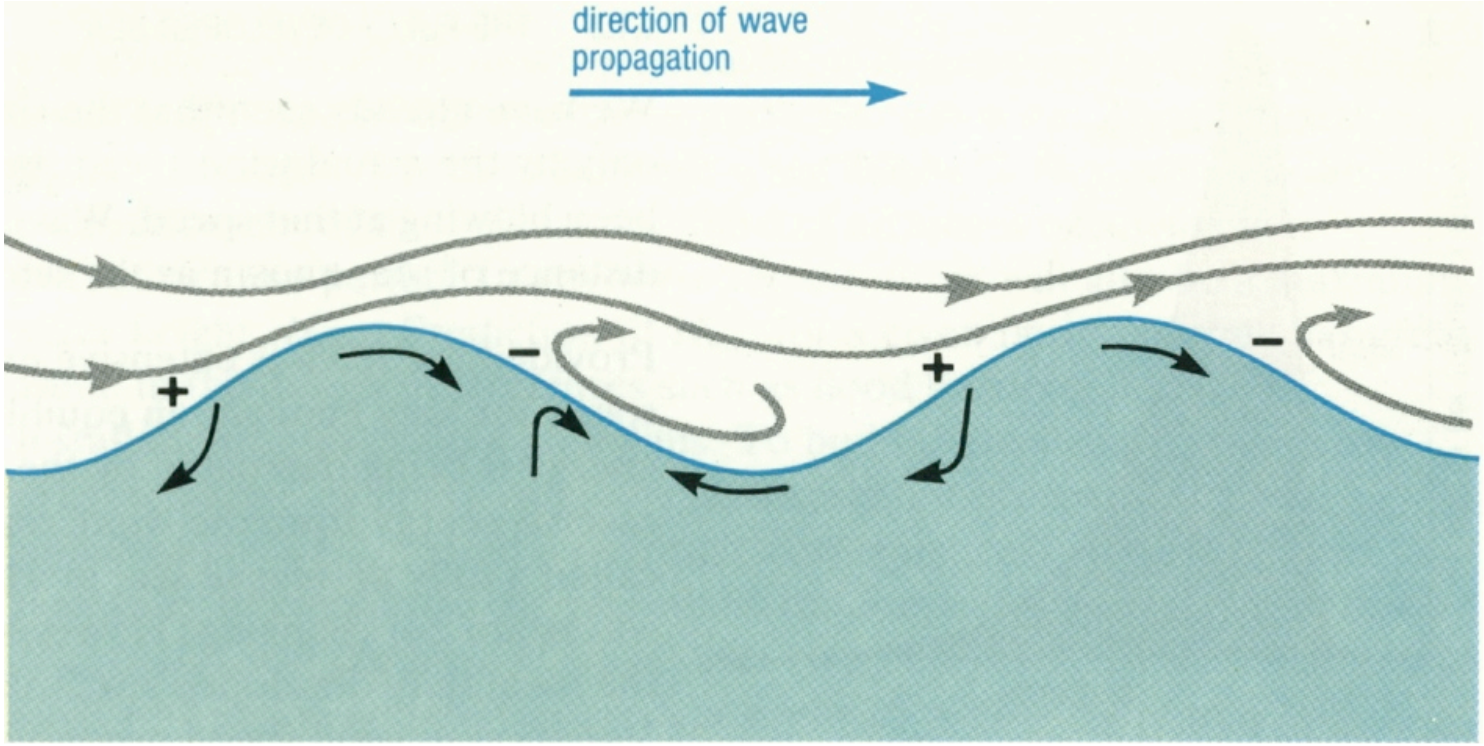
Depth less than L/20

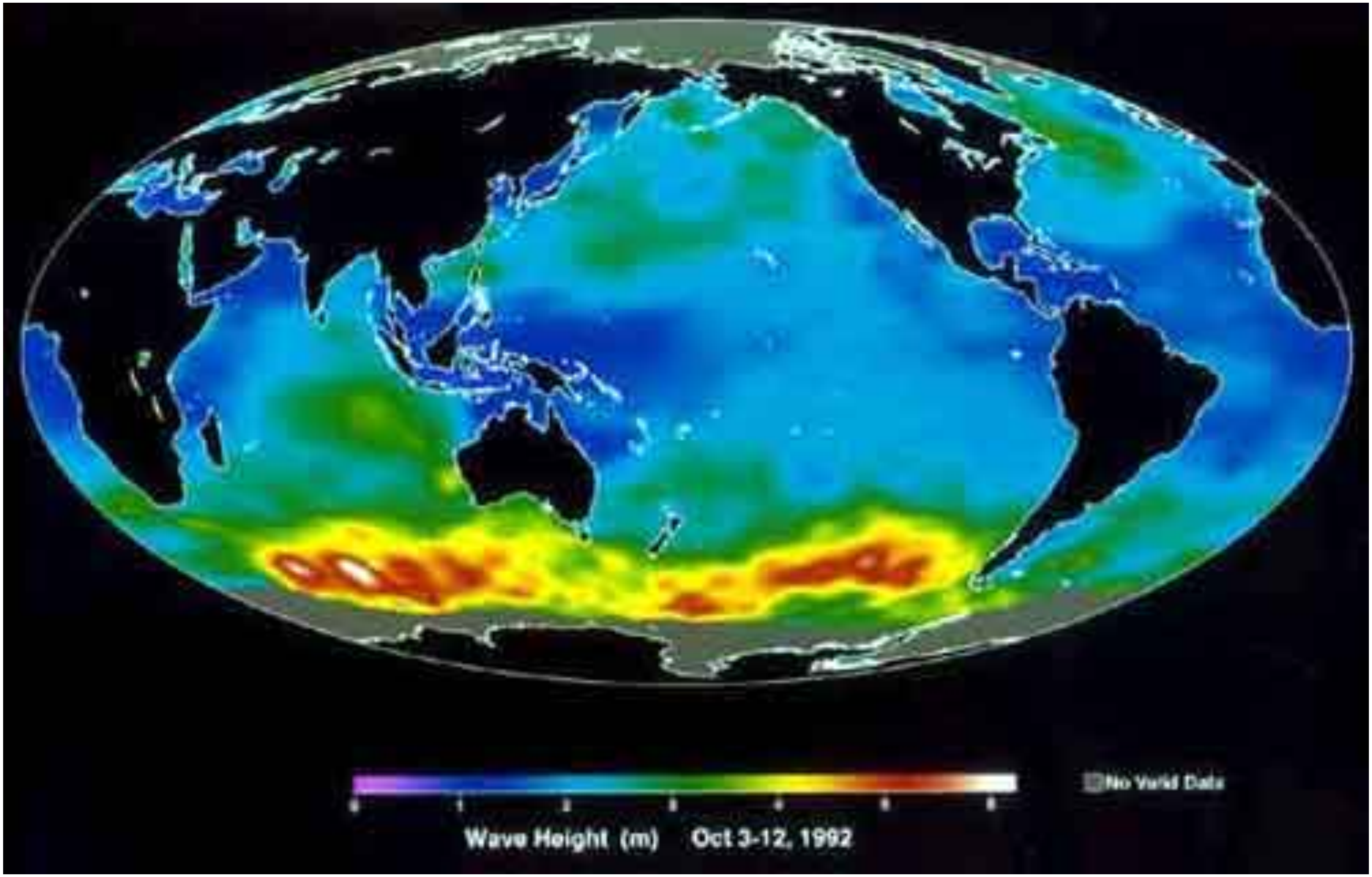
Wind wave generation

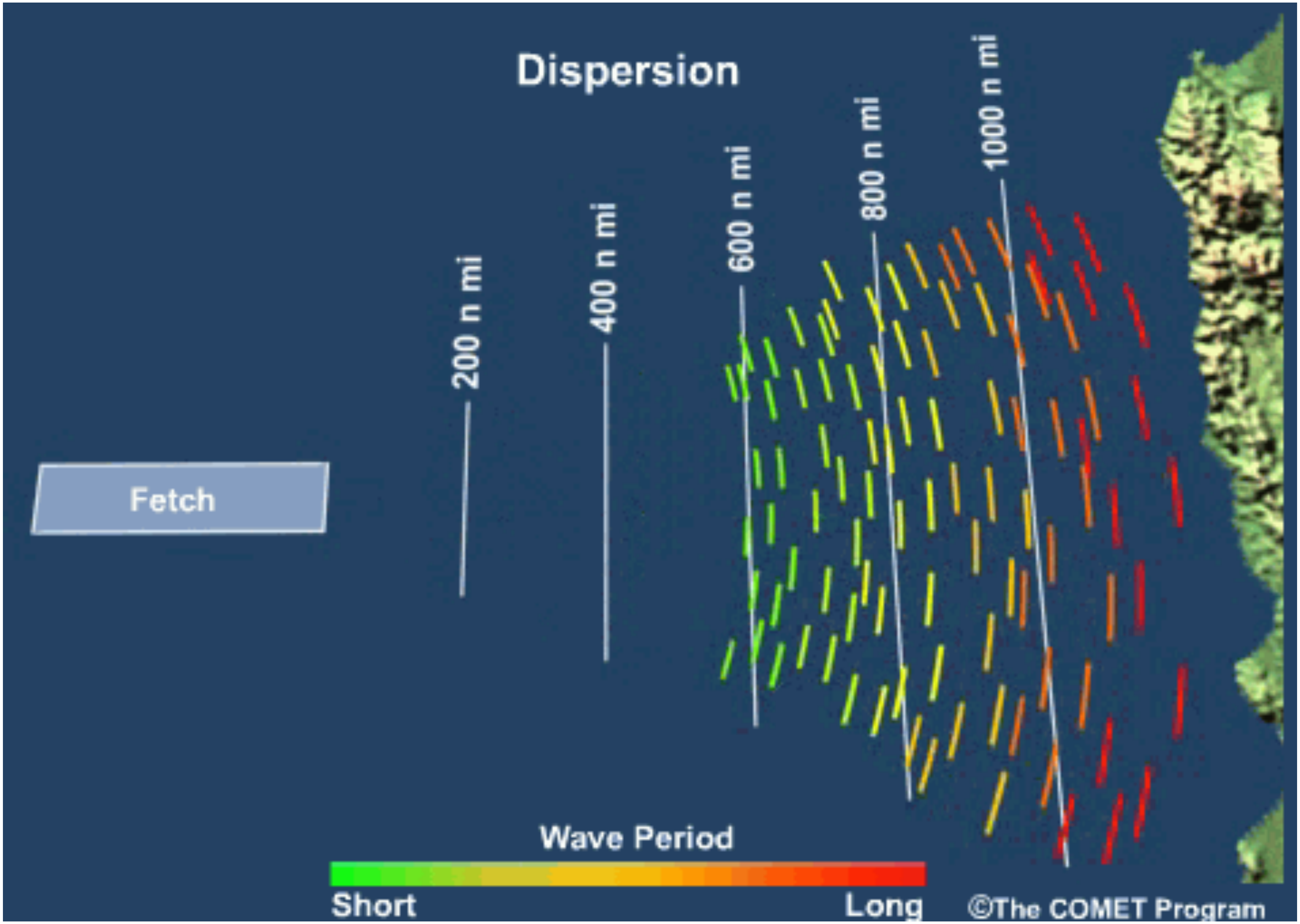
- Factors affecting wind wave development:
  - **Wind strength** - wind speed exceeds wave speed, and greater than one m/s
  - **Wind duration**
  - **Fetch** - the uninterrupted distance over with the wind blows without changing direction

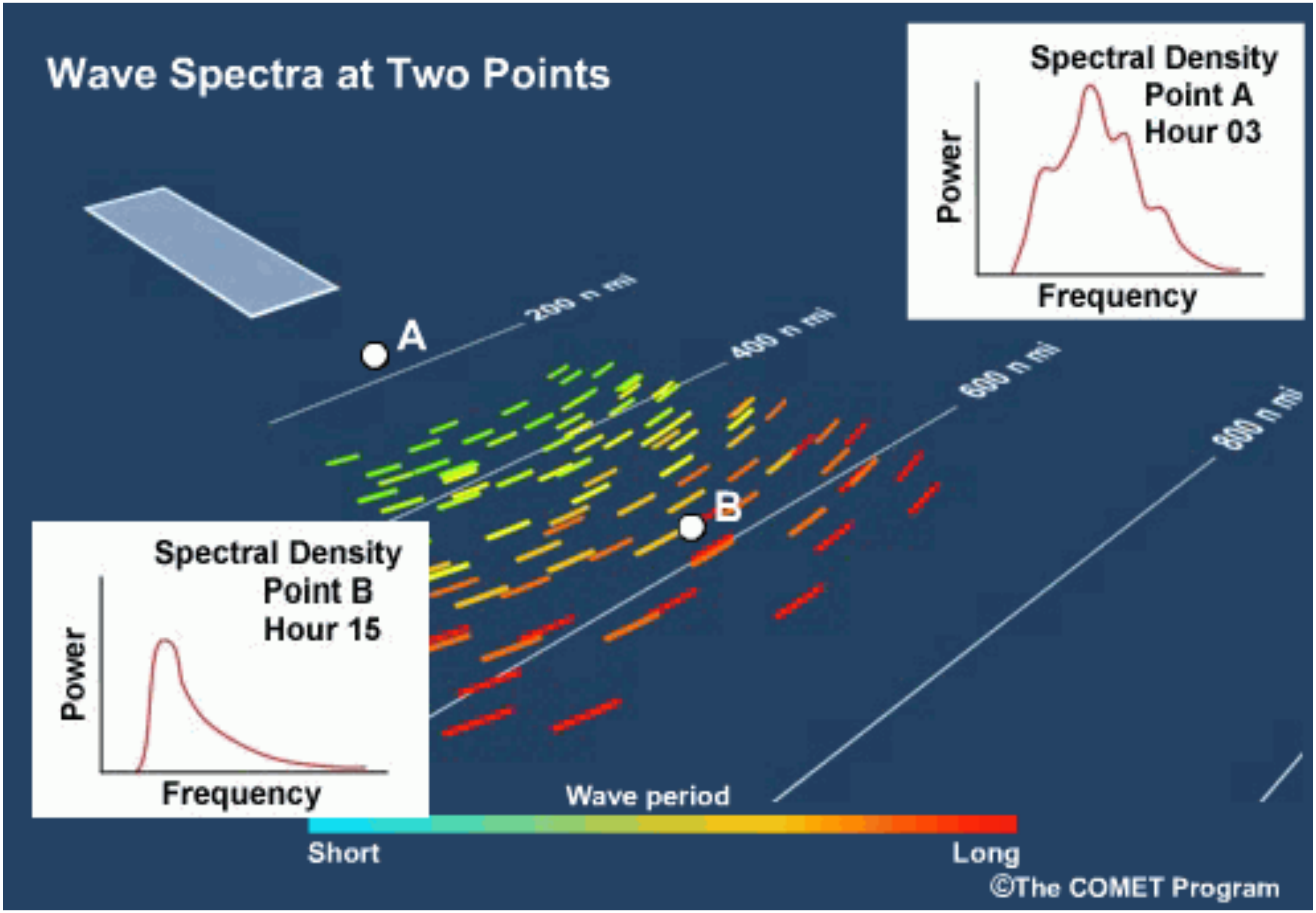
Wind wave generation

1. Turbulence in wind causes small waves.
2. Wind blowing over waves produces pressure differences resulting in growth.



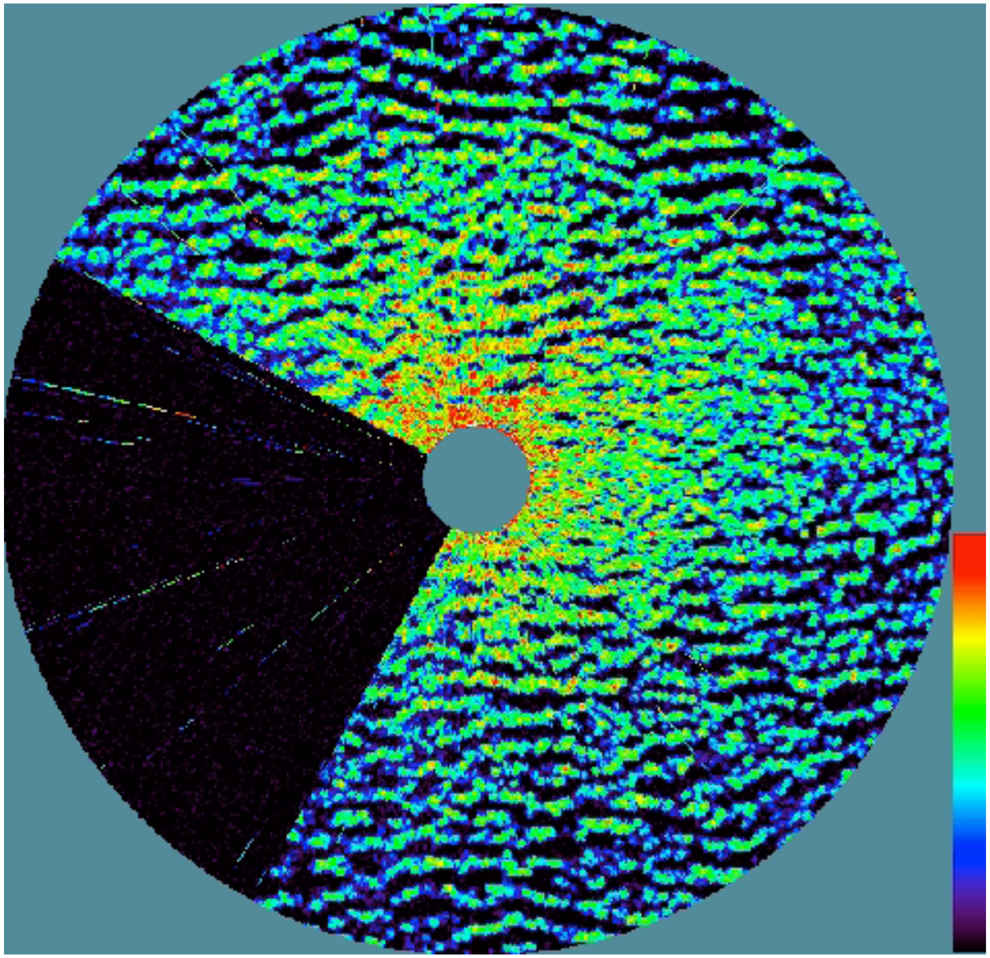

0
1
2
3
4
5
6
No Valid Data
Wave Height (m) Oct 3-12, 1992

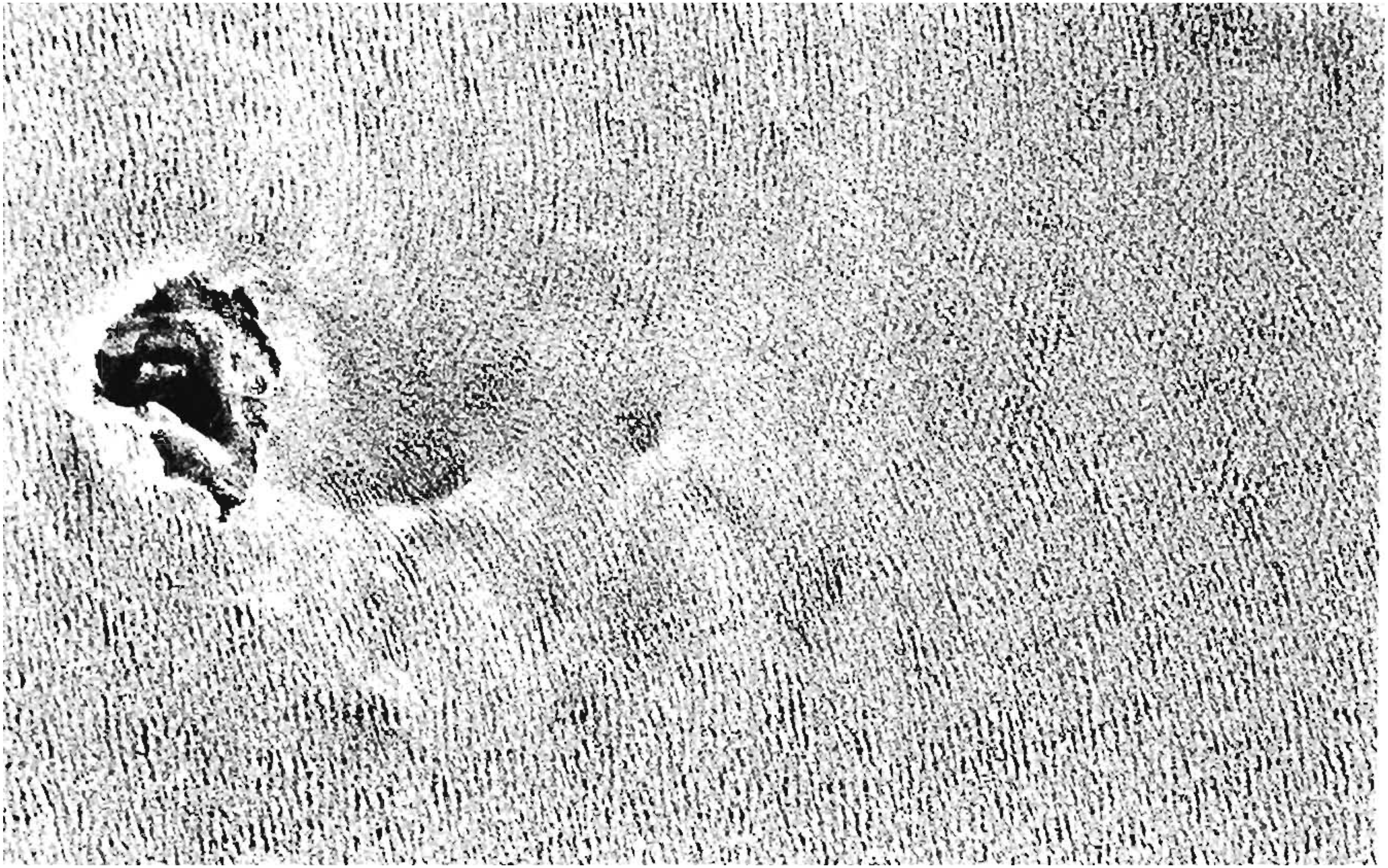

Dispersion
Fetch
200 n mi
400 n mi
600 n mi
800 n mi
1000 n mi
Wave Period
Short
Long
©The COMET Program


Wave Spectra at Two Points
Spectral Density
Point A
Hour 03
Power
Frequency
A
B
200 n mi
400 n mi
600 n mi
800 n mi
Spectral Density
Point B
Hour 15
Power
Frequency
Wave period
Short
Long
©The COMET Program

Wave measurements

- Direct
  - Wave buoys
  - Pressure sensors
  - x-band radar
- Indirect
  - SAR images (synthetic aperture radar)





Energy and energy flux

- The total energy per unit area is

$$E = \frac{1}{2}\rho g a^2 \quad [J\, m^{-2}]$$

- The rate at which energy is supplied to a particular location is the energy flux or wave power.

$$F = c_g E \quad \text{per unit length of wave crest}$$

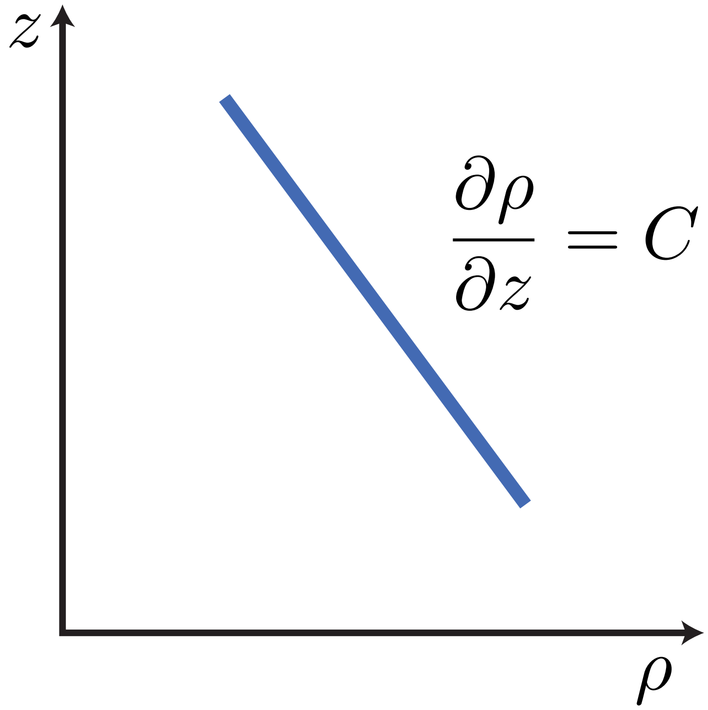
Attenuation

- Loss or dissipation of wave energy, resulting in a reduction of amplitude.

1. White-capping.
2. Viscous attenuation (capillary waves).
3. Air resistance.
4. Non-linear wave-wave interactions.

Internal waves

Buoyancy frequency



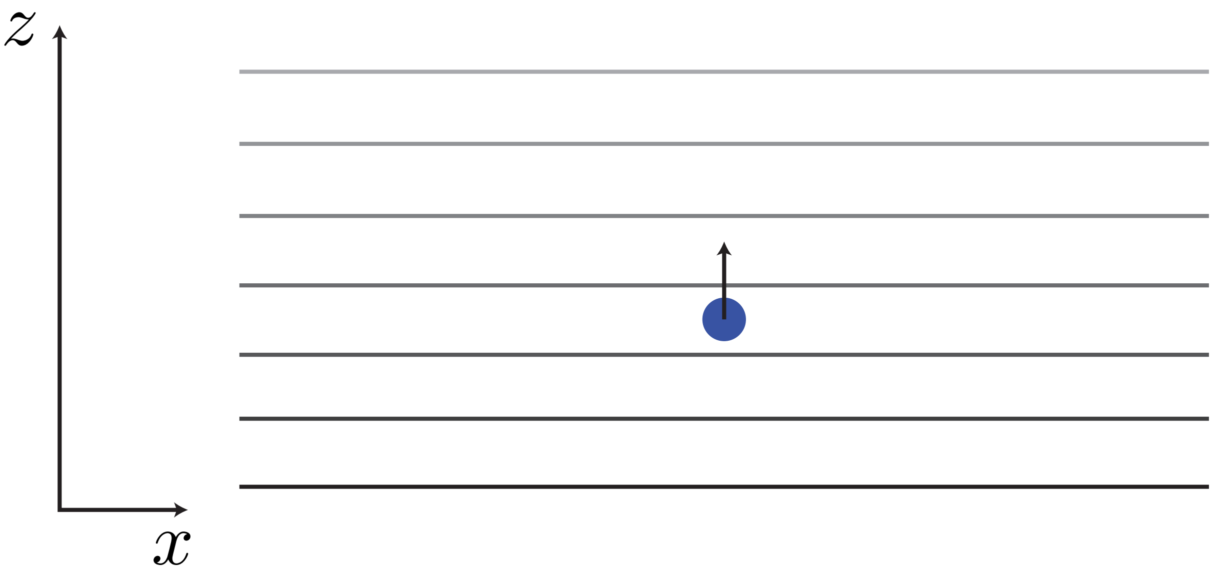
$$\frac{\partial \rho}{\partial z} = C$$

for incompressible fluid

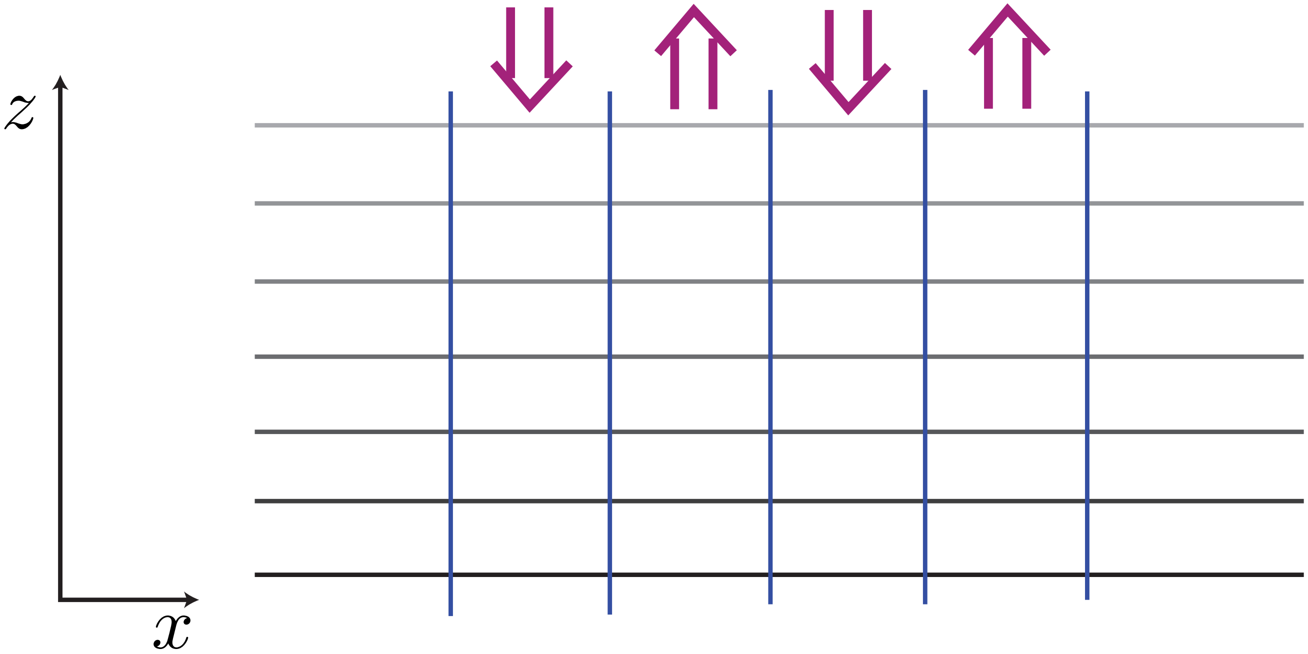
$$N^2 = -\frac{g}{\rho}\frac{\partial \rho}{\partial z}$$

$$[s^{-2}] \qquad \left[\frac{m\ s^{-2}}{kg\ m^{-3}}\frac{kg\ m^{-3}}{m}\right]$$

Internal waves

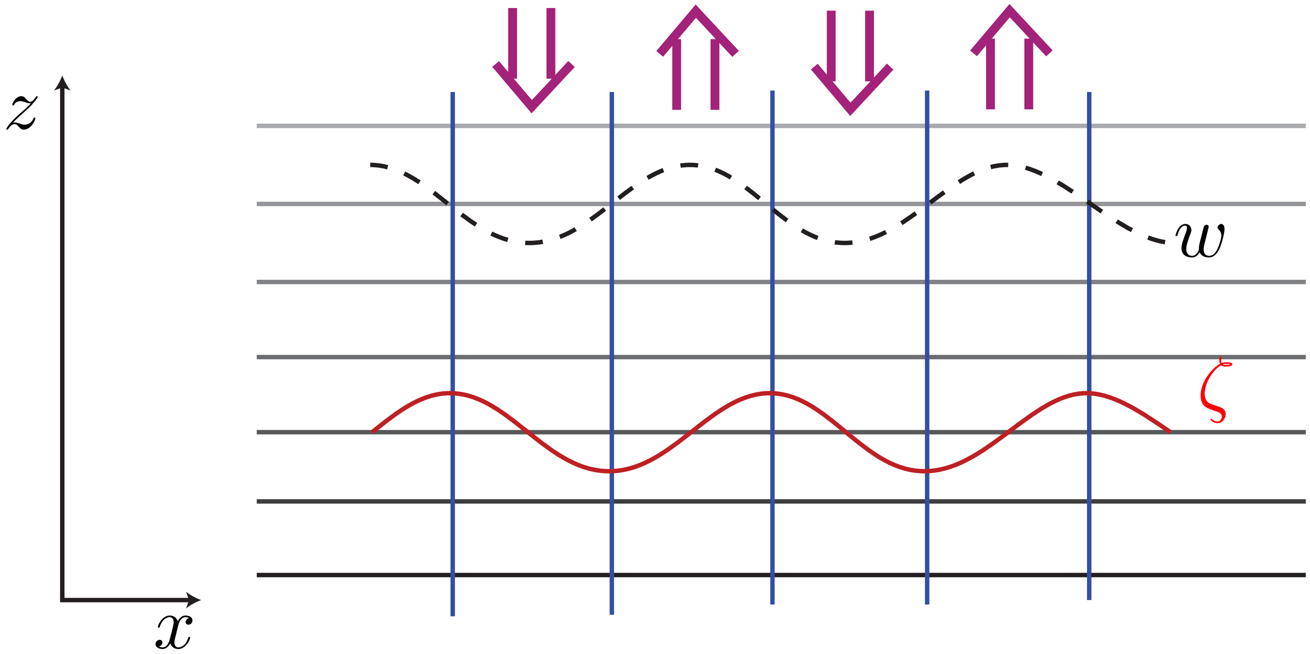


Internal waves



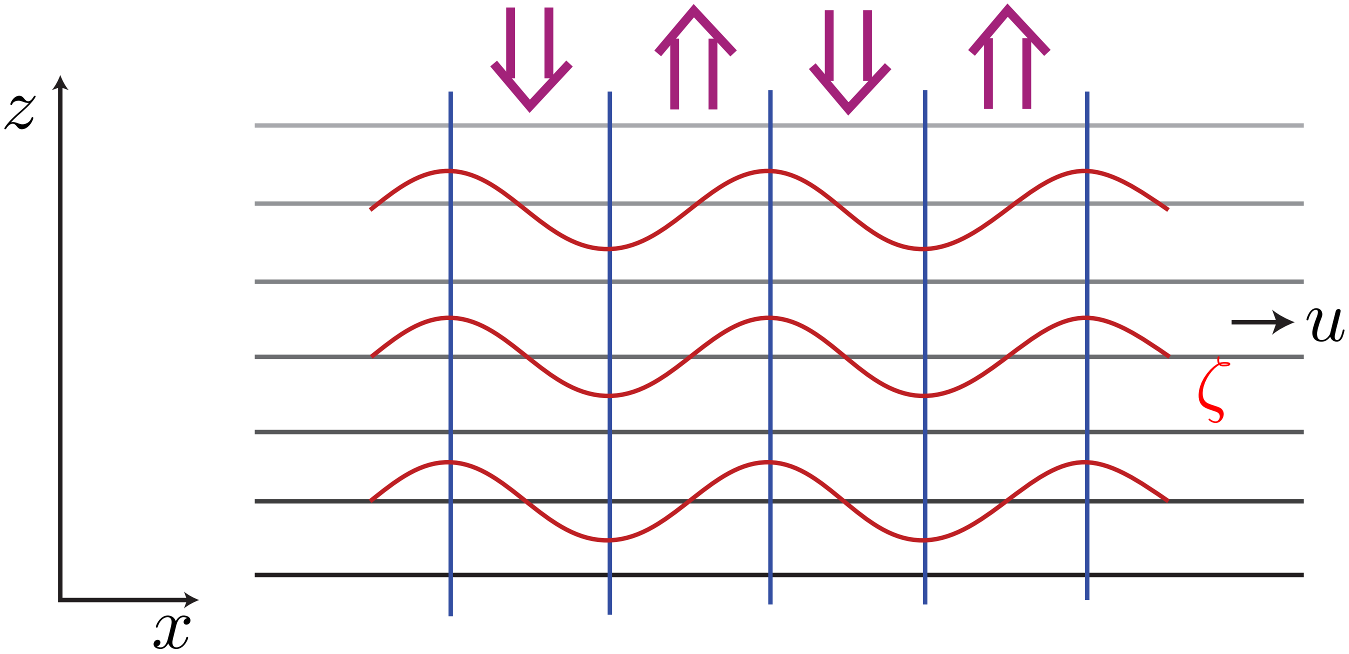
- Mass conservation will prevent moving just a single particle.
- Consider a ‘infinite ocean’.

Internal waves

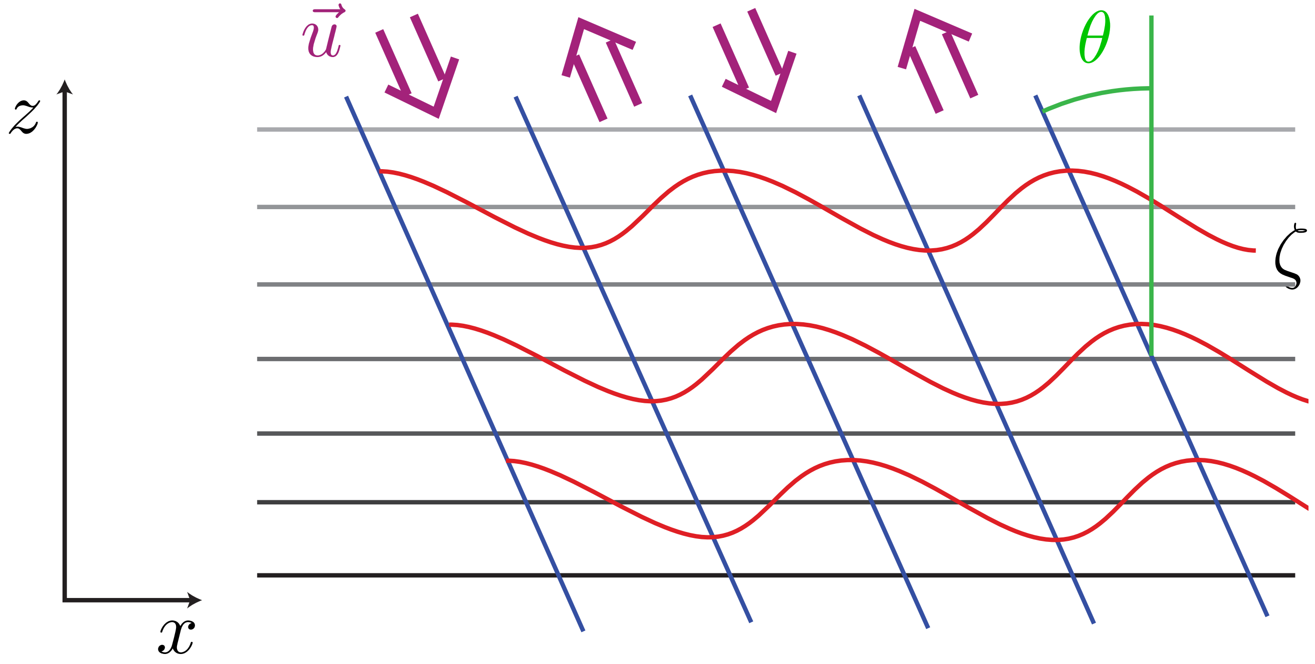


ζ

Internal waves



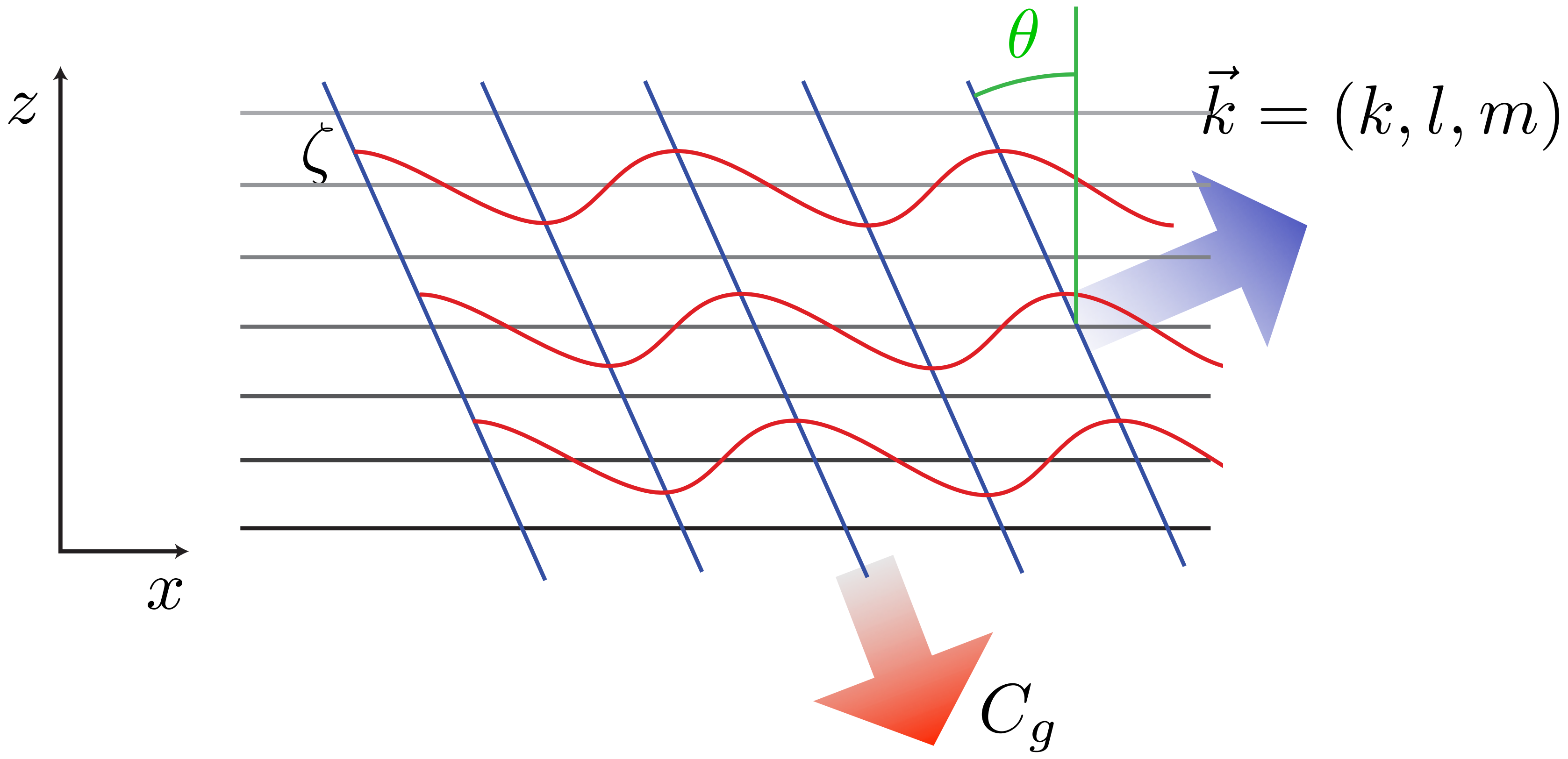
Internal waves



- No reason to limit the particles to vertical displacement.
- Restoring force reduced as at angle to gravity

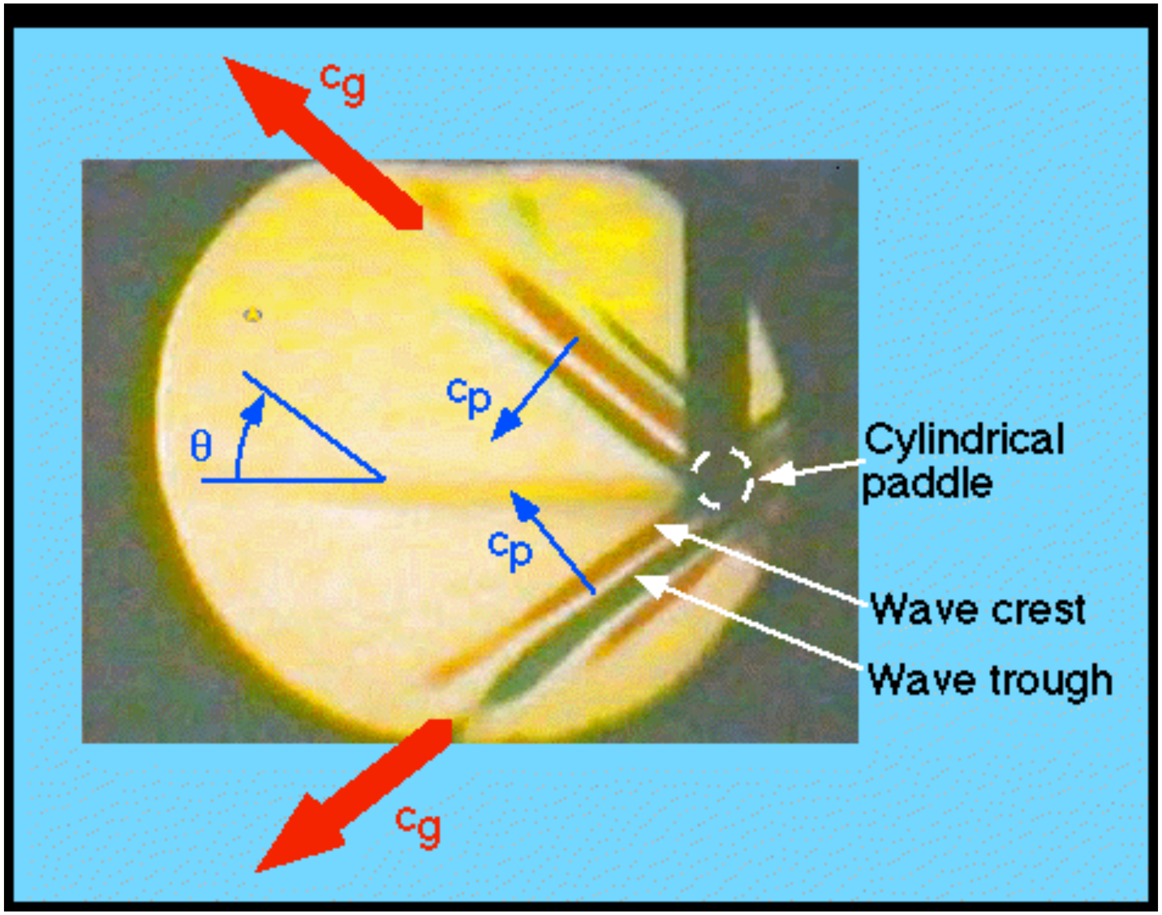
$$\omega = N \cos\theta$$

Internal waves

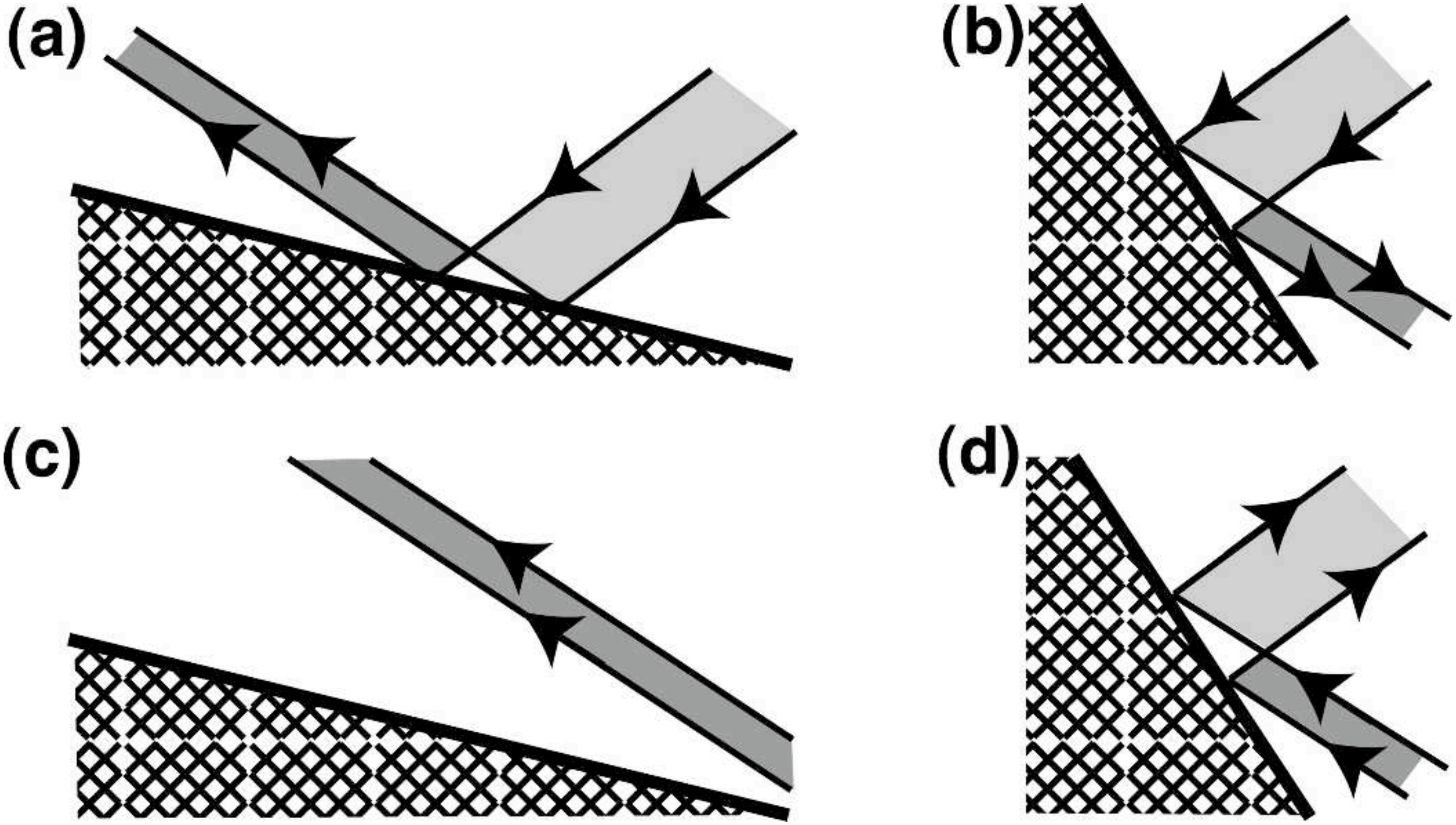


- Energy propagates perpendicular to phase
- Vertical components have opposite sign.

Internal wave animation



Reflection



Carter et al. 2006

Internal waves

IW frequency due to gravity (θ is angle to vertical)

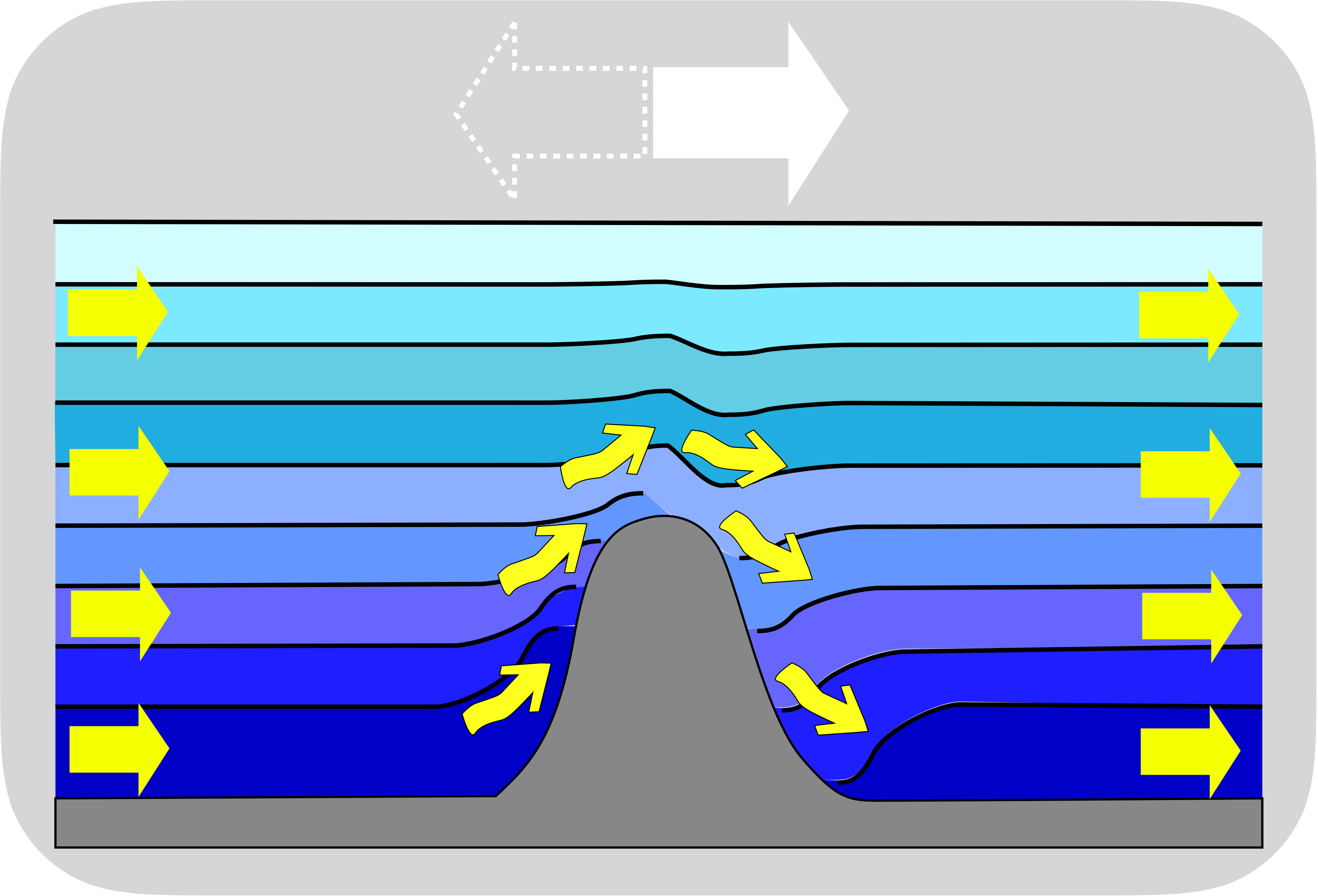
$$\omega = N \cos \theta$$

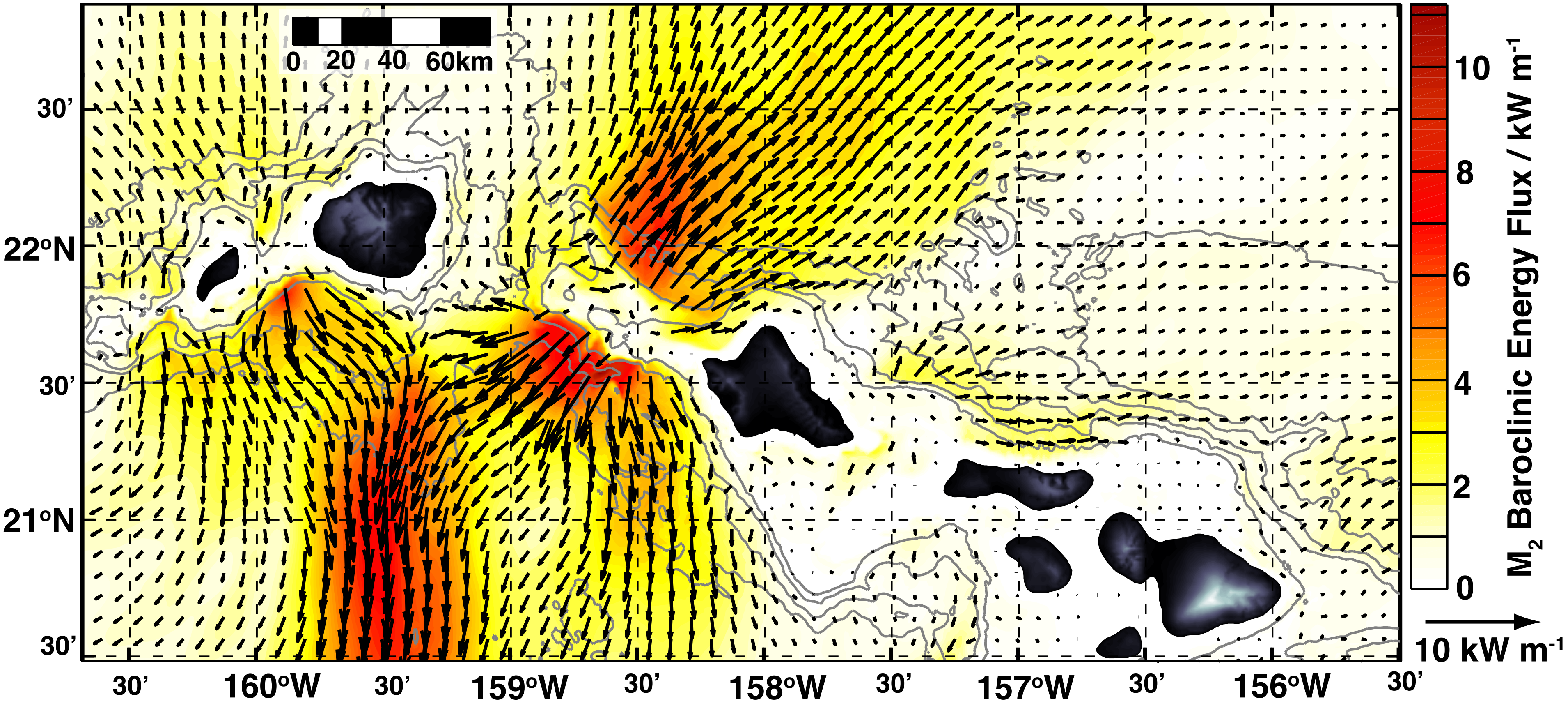
But gravity is not the only force acting on IWs in the ocean --- also rotation (which is maximum in the horizontal)

$$\omega = N \cos \theta + f \sin \theta$$

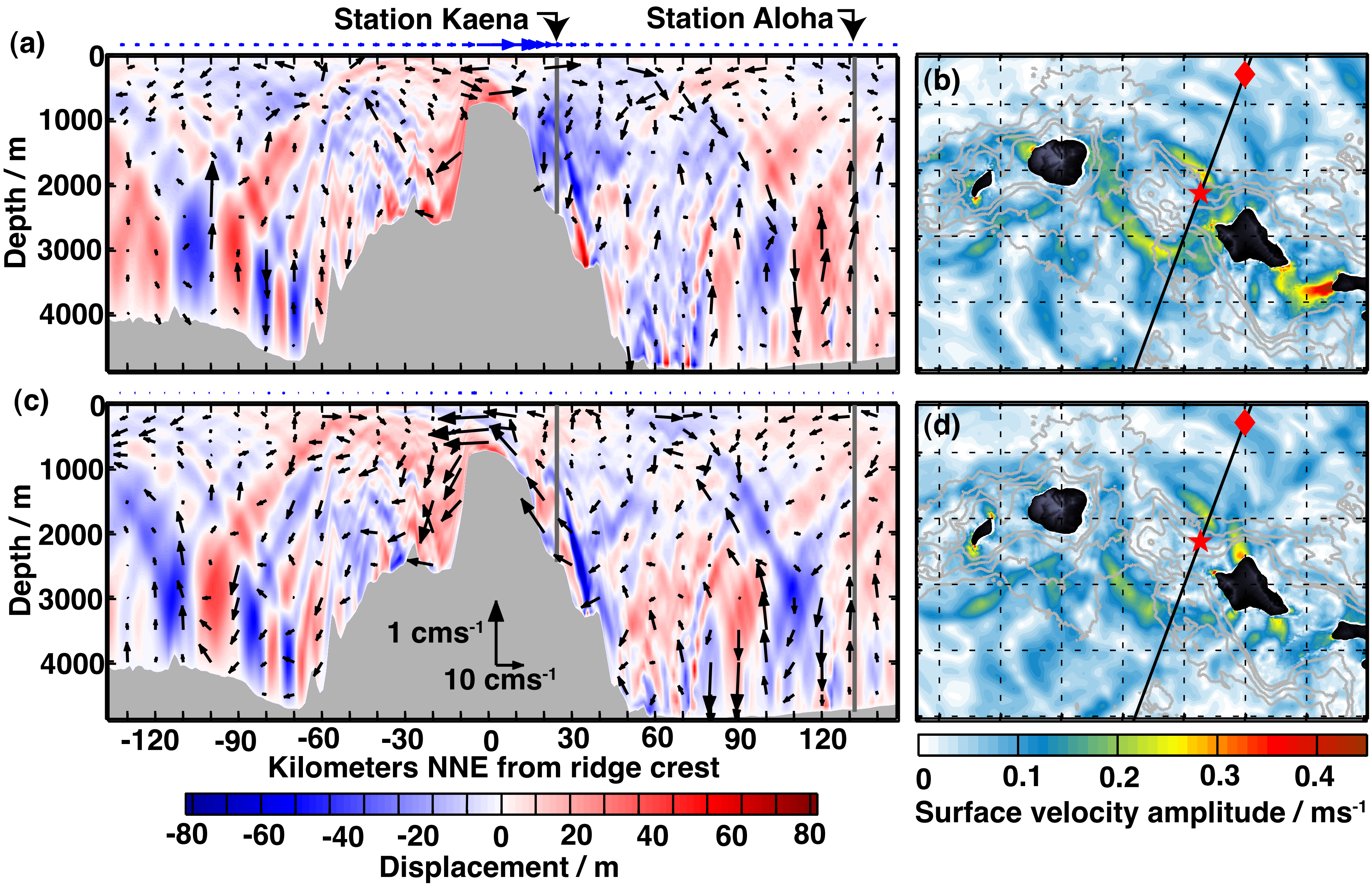
So

$$f < \omega \leq N$$

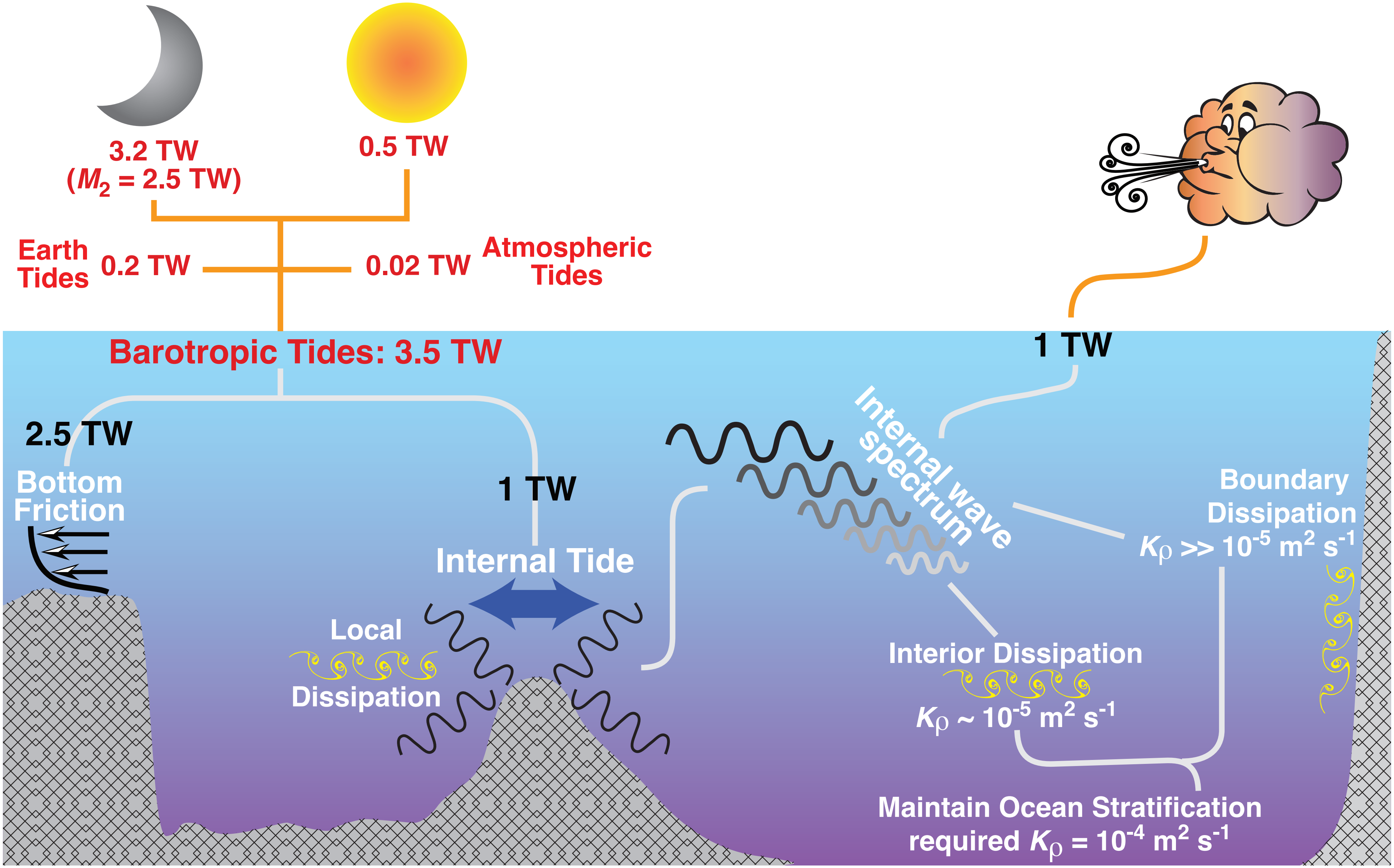




Carter et al. 2008



Carter et al. 2008



3.2 TW
($M_2 = 2.5$ TW)

0.5 TW

Earth Tides 0.2 TW

0.02 TW Atmospheric Tides

Barotropic Tides: 3.5 TW

2.5 TW

Bottom Friction

1 TW

Internal Tide

Local Dissipation

1 TW

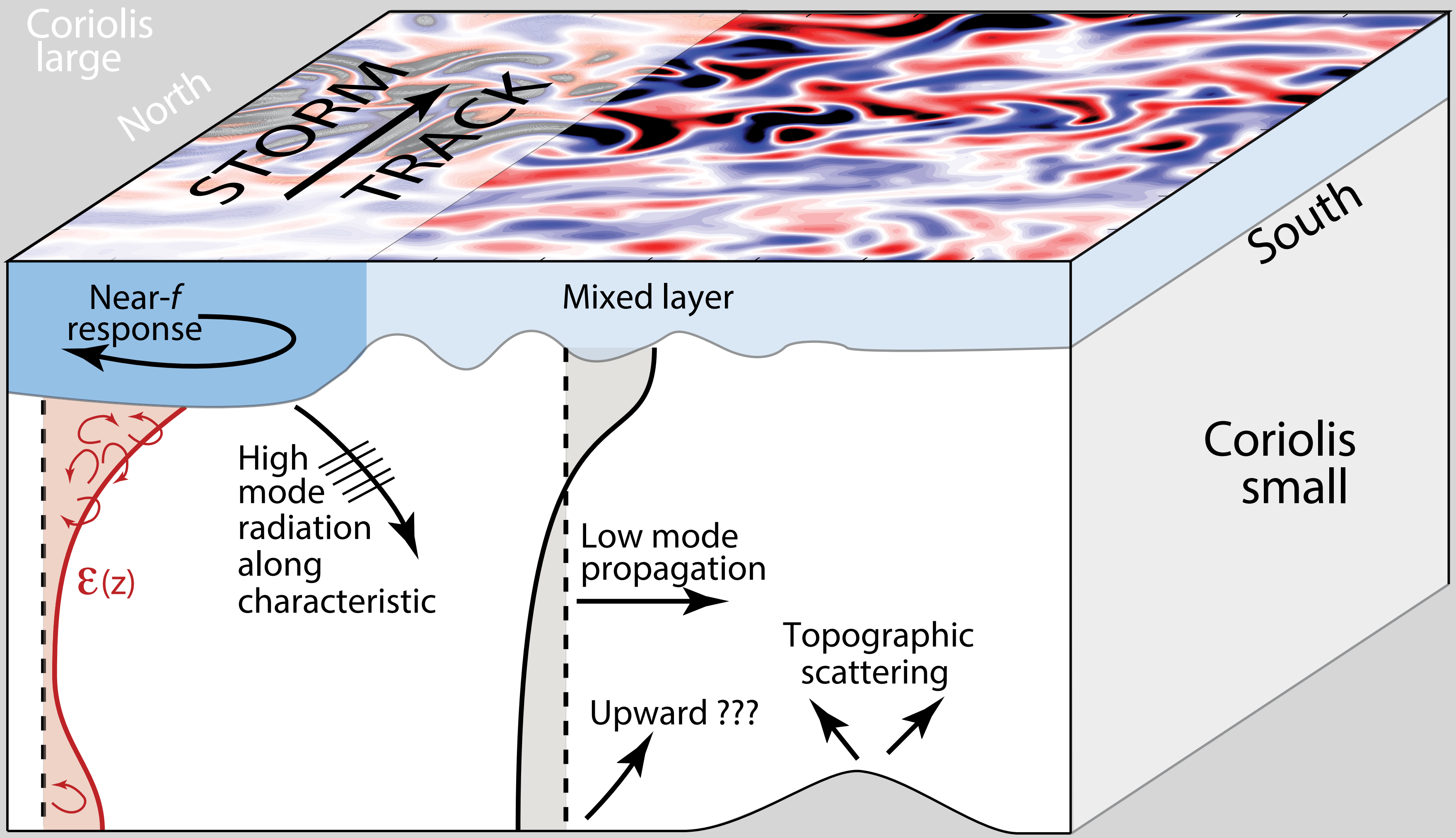
Internal wave spectrum

Boundary Dissipation
$K_\rho \gg 10^{-5}$ m^2 s^{-1}

Interior Dissipation
$K_\rho \sim 10^{-5}$ m^2 s^{-1}

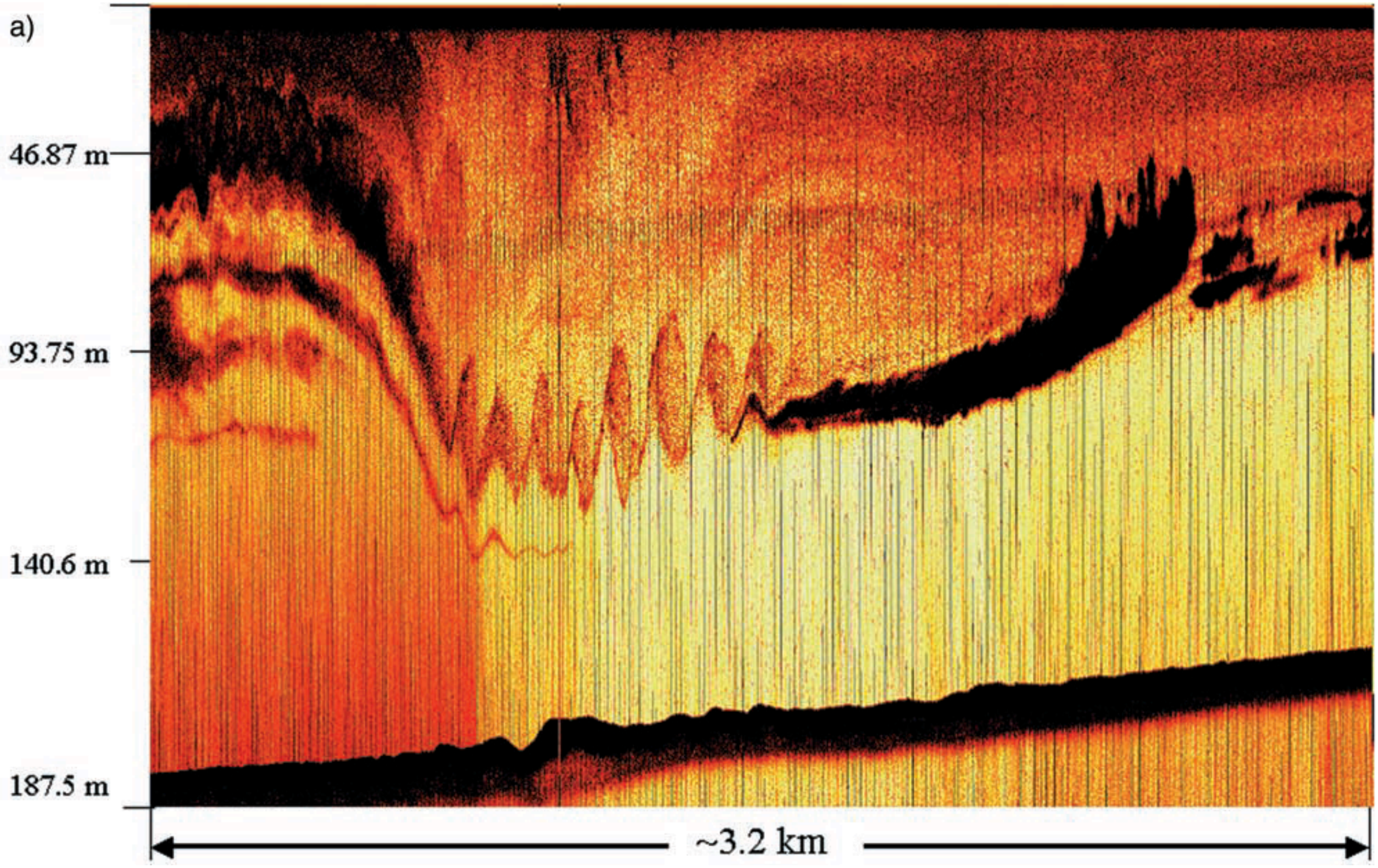
Maintain Ocean Stratification
required $K_\rho = 10^{-4}$ m^2 s^{-1}

Carter et al., 2012



Simmons et al., 2012

Nonlinear internal waves



Orr and Mignerey 2003

Nonlinear internal waves