Lecture 12. The diurnal cycle and daytime mixed layer growth

In this lecture…

- Diurnal cycle of boundary layer temperature structure
- Mixed-layer modeling of the morning growth of the convective boundary layer

Introduction

Over flat land, under clear skies and with weak thermal advection, the atmospheric boundary layer undergoes a pronounced diurnal cycle.   A schematic and an example from the Wangara experiment are shown below. This `archetypical' diurnal cycle is muted by clouds and can be entirely obscured by rapid changes in the free atmospheric conditions due for instance to the passage of a midlatitude cyclone or front. It is also highly modified by terrain or nearby land-sea contrasts. Nevertheless, it is illuminating to study the archetypical case in more depth.

During the night, the BL is stable due to surface longwave cooling, and a shallow temperature inversion of typically 100-500 m builds up. After dawn, surface heating builds up a shallow convective mixed layer, which deepens slowly and rapidly warms until it fully erodes the nocturnal stable layer. At this point, the top of the new mixed layer starts to penetrate into the *residual layer*, the remnants of the previous day's afternoon mixed layer. This layer is very weakly stratified, so the new mixed layer rapidly deepens into it, until it encounters the top of the previous day's mixed layer, which tends to be marked by a weak inversion. At this point, further BL warming occurs much more slowly, as a much deeper layer must be warmed than in the early morning. In the late afternoon, the solar heating is no longer sufficient to maintain an upward surface buoyancy flux. Within an hour (a few eddy turnover times), turbulence collapses through most of the boundary layer and becomes restricted to a shallow layer, typically 100 m deep, driven by surface drag. During the night, clear-air radiative cooling is most intense near the cold surface, enhancing the static stability of the lowest couple of hundred meters of air. Turbulent heat exchange with the cooling surface can also be important; downward heat fluxes of up to 50 W m^{-2} can occur near the surface at night under moderately strong geostrophic winds.

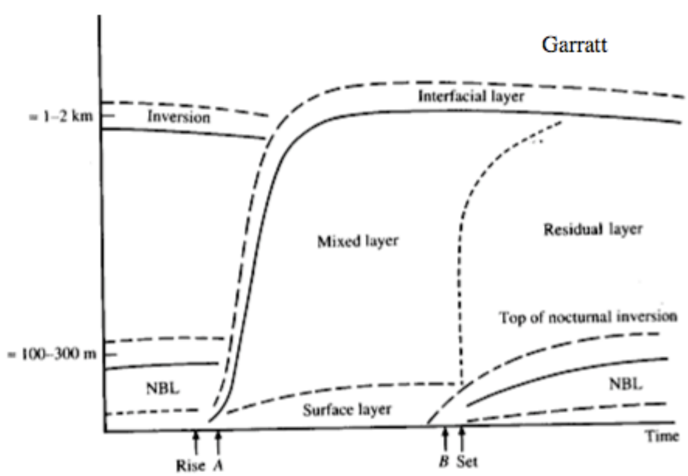

Fig. 6.1 Schematic representation of ABL evolution throughout the diurnal period over land under clear skies.

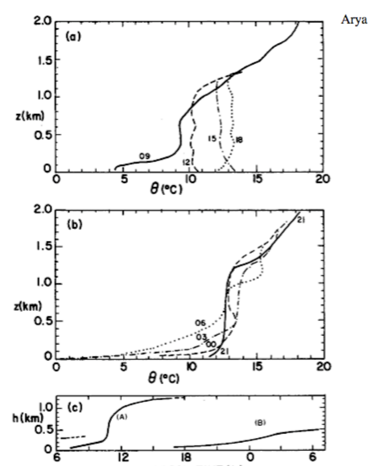

Fig. 5.2 Diurnal variation of potential temperature profiles and the PBL height during (a) day 33 and (b) days 33–34 of the Wangara Experiment. (c) Curve A, convective; Curve B, stable. [After Deardorff (1978).]

Fig. 12.1: The archetypical diurnal cycle of boundary-layer evolution over land.

Mixed layer model of morning growth of the surface-heated convective BL (Garratt 6.1)

The rate of growth of the convective mixed layer is dictated primarily by energy balance, though entrainment dynamics also play a significant role. This is a classic application of a mixed layer model (MLM).

MLMs assume that the mean vertical profiles of conserved tracers such as θ are uniform ('**well-mixed**') across the BL, and the BL is capped by a sharp inversion at which temperature and moisture have sudden jumps; similarly for the wind profile. Hence MLMs are most applicable to convective BLs and poorly represent most stable BLs. The key turbulence closure in a MLM is the **entrainment closure**, an empirically-inspired relationship between the rate that mass is entrained from above the BL and MLM-derived variables such as surface fluxes and inversion jumps.

We consider the growth of a mixed layer driven by a surface buoyancy flux B_0 into a motionless atmosphere of constant buoyancy frequency N^2. For simplicity, we will also assume no cloud and no internal diabatic heating. The air density can be described using the virtual potential temperature profile $\theta_v(z,t)$. Define θ_{v0} to be the initial virtual temperature at the ground, before convection has initiated. Then we can scale θ_v into a buoyancy $b(z, t) = g(\theta_v - \theta_{v0})/ \theta_{v0}$.

Thus the initial horizontal-mean buoyancy profile is

$$b^+(z) = N^2 z \tag{12.1}$$

and this stays constant in the free troposphere (above the BL top). We define $b_M(t)$ as the (unknown) buoyancy in the mixed layer. We let $h(t)$ be the (unknown) mixed layer top, at which there is an unknown jump $\Delta b(t) = b^+(h) - b_M$ in the buoyancy, due to convective updrafts overshooting into the overlying warmer air and eroding its base via entrainment.

The heat equation within the mixed layer can be written

$$\frac{db_M}{dt} = -\frac{\partial}{\partial z}\overline{w'b'} \tag{12.2}$$

Since the left hand side of (12.2) is height-independent within the mixed layer, the same holds for the right hand side, so the buoyancy flux is a linear function of height that can be specified in terms of its value at the surface and mixed layer top:

$$\overline{w'b'}(z) = (1-\frac{z}{h})\overline{w'b'}(0) + \frac{z}{h}\overline{w'b'}(h) \quad \Rightarrow \quad -\frac{\partial}{\partial z}\overline{w'b'} = \frac{B_0 - \overline{w'b'}(h)}{h} \tag{12.3}$$

Turbulence in the mixed layer entrains free-tropospheric air from just above the mixed layer, causing h to rise at the **entrainment rate** w_e.

$$\frac{dh}{dt} = w_e\,. \tag{12.4}$$

The entrainment deepening of the BL, in which free-tropospheric air with buoyancy b^+ of is replaced by BL air with buoyancy b_M at the rate w_e, implies an upward turbulent **entrainment flux** into the mixed layer top. This can be seen by considering the conservation of buoyancy inside a vertically thin control volume V that encompasses the moving entrainment interface and moves upward with it at rate w_e. Because V is thin, it cannot store buoyancy, so (per unit horizontal area of V):

0 = turbulent flux into V + advective flux from above – advective flux down into mixed layer

$$= \overline{w'b'}(h) + w_e b^+ - w_e b_M \quad \Rightarrow \quad \overline{w'b'}(h) = -w_e \Delta b \qquad (12.5)$$

To close (12.2-12.4), we need to specify w_e. This **entrainment closure** is the big assumption in any MLM. Since entrainment is due to turbulence, it is implicitly an assumption about the vertical structure and intensity of turbulence, and is connected to the TKE budget. In particular, overshooting convective updrafts are transporting TKE upward from its buoyant generation region near the surface into the inversion, where it is being lost to negative buoyancy fluxes associated with the overshooting.

For a dry convective boundary layer, the empirical relation in Lecture 7:

$$\overline{w'b'}(h) = -\beta B_0, \quad \beta \approx 0.2, \qquad (12.6)$$

together with (12.5), provides a suitable entrainment closure. It proves interesting to compare the solution with a realistic β to the case β = 0, as shown in Fig. 12.2. In the latter limit, called *encroachment,* convection is assumed not to be penetrative, and the mixed layer entrains air only when its buoyancy is no larger than that of the mixed layer air. The mixed layer still deepens as it warms, but less rapidly that in the entraining case.

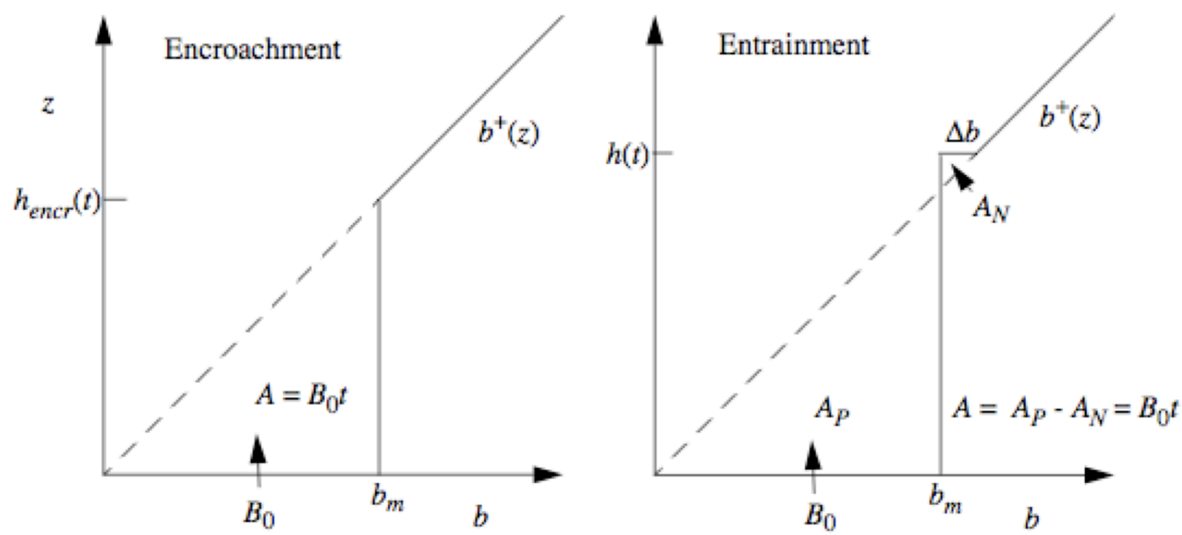

Fig. 12.2: Convective mixed layer evolution illustrating more rapid deepening if entrainment is assumed to be penetrative (β = 0.2), compared to encroachment (β = 0).

With the entrainment closure (12.6), the ML buoyancy balance (12.2) and (12.3) simplifies to

$$\frac{db_M}{dt} = \frac{(1+\beta)B_0}{h} \qquad (12.7)$$

and the boundary-layer depth equation simplifies to

$$\frac{dh}{dt} = w_e = \frac{\beta B_0}{b^+(h) - b_M} \tag{12.8}$$

which is a closed pair of equations for the mixed-layer growth.

Fig. 12.2 guides us to a solution based on considering the column-integrated buoyancy budget. Integrating from the surface up to a fixed height H above the mixed layer top:

$$\frac{d}{dt}\int_0^H b\,dz = \int_0^H \frac{\partial b}{\partial t}\,dz = -\int_0^H \frac{\partial}{\partial z}\,\overline{w'b'}\,dz = \overline{w'b'}(0) = B_0$$

Hence the surface heating through time t adds a net area to the vertically-integrated buoyancy:

$$A = B_0 t = \int_0^h (b_M - b^+(z))dz \tag{12.9}$$

If we start with the encroachment case, $\Delta b = b_M - b^+(h) = 0$, so

$$B_0 t = \int_0^h (b_M - b^+(z))dz = N^2 h^2/2 \quad \Rightarrow \quad h_{encr}(t) = \left(\frac{2B_0 t}{N^2}\right)^{1/2}, b_{encr} = N^2 h_{encr} \tag{12.10}$$

As expected, h deepens more slowly as it gets larger, since more heat must be imparted to a deeper boundary layer to raise its buoyancy by a given amount.

For the entraining ML in constant stratification, there is a 'similarity' solution in which the buoyancy profile retains the same shape as it grows, so that

$$\Delta b(t) = cN^2 h(t) \quad (c \text{ is an as yet unknown constant}). \tag{12.11}$$

One constraint on c comes from the entrainment growth equation (12.8):

$$\beta B_0 = w_e \Delta b = (dh/dt)\, cN^2 h.$$

Integrating this equation from time 0 to t, starting with $h(0) = 0$, we get

$$\beta B_0 t = cN^2 h^2/2 \tag{12.12}$$

A second constraint comes from the column buoyancy budget (12.9). Considering Fig. 12.2, we write $A = A_P - A_N$. The 'positive area' A_P is where the mixed layer buoyancy $b_M(t)$ exceeds the original environmental buoyancy and the 'negative area' A_N is where penetrative convection has reduced the buoyancy. From Fig. 2, we see that $b_M + \Delta b = b^+(h) = N^2 h$, so $b_M = (1\text{-}c)N^2 h$. The heights of the triangles making up A_P and A_N are N^{-2} times as long as their bases, so $A_P = b_M\,(b_M / N^2)/2$ and similarly for A_N. Hence (12.9) can be written:

$$B_0 t = A_P - A_N = b_M^2/2N^2 - \Delta b^2/2\, N^2 = [(1\text{-}c)^2 - c^2]\, N^2 h^2/2 = (1 - 2c)\, N^2 h^2/2. \tag{12.13}$$

Dividing (12.12) by (12.13), we see that $\beta = c/(1 - 2c)$, or that $c = \beta/(1 + 2\beta)$. It follows from (12.13) that

$$h_{entr} = \left(\frac{2B_0 t(1+2\beta)}{N^2}\right)^{1/2} \approx (1+\beta)h_{encr} \tag{12.14}$$

We conclude that entrainment adds about β = 20% to the boundary layer deepening. For a 1 km deep BL and $N^2 = 10^{-4}$ s^{-1}, the inversion strength would be $\Delta b = .14N^2 h \Rightarrow \Delta\theta_v \approx 0.4$ K, regardless of the surface buoyancy flux. Entrainment hardly changes the BL temperature.