A one-dimensional model of the seasonal thermocline

II. *The general theory and its consequences*

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ABSTRACT

A theory of the layer formation due to surface processes is presented, which is more general than that used in the preceding paper I. Convection due to heating at depth and cooling at the surface is included, as well as the mechanical stirring due to wind action. The theory is applicable to arbitrary forms of heating, including intermittent or continuous processes, and could be used to investigate diurnal as well as seasonal effects. A detailed application is made to the case treated approximately in I, for which a solution is now obtained in analytic form.

The results obtained allow a quantitative, as well as qualitative, comparison with the ocean. It is found that reasonable layer depths are predicted using measured heating rates, and a value of the turbulent kinetic energy input to the water deduced from the mean surface stress. The effects of heating at depth can be comparable with wind stirring, even when the temperature of the upper layer is increasing. During the winter, convection due to surface cooling dominates the processes which deepen the layer.

1. Introduction

Among the various recent theories of the well-mixed surface layer of the ocean there are two which are one-dimensional in nature. Both Kitaigorodski (1960) and Kraus & Rooth (1961) consider steady-state models which imply different kinds of balances in the vertical. Kitaigorodski computes the depth of the stirred layer from a balance between the mean work of the wind stress and the work needed to mix heat downward from the surface. This theory breaks down when there is an upward flux of heat—at night or during autumn, for example. It also ignores changes in layer temperature, which cannot remain constant with a constant positive heat input and invariant depths.

Kraus and Rooth, on the other hand, consider the effect of a net heat loss from the surface by evaporation, conduction and infrared radiation. This can be compensated by the absorption of visible solar radiation through a finite depth, so there will be an upward convective heat flux between the compensation level and the surface. Kinetic energy generated by this upward flux of heat can be transformed into potential energy by penetrative convection, that is, by the overshooting of sinking water parcels below the compensation level, causing a downward flux of heat between that level and the bottom of the stirred layer. It should also cause an increase in layer depth due to entrainment as described in the preceding paper by Turner & Kraus (1966), which will be referred to here as paper I. The process is analogous to the rise of atmospheric inversions due to surface heating, as discussed by Ball (1960). Kraus & Rooth keep the layer depth steady by stipulating that there should be a balance between the tendency for the layer thickness to increase and the upwelling of cold water from below.

This theory has features which one must retain in a more general treatment, but it too is not applicable realistically to a wide range of conditions. The model takes no account of mechanical stirring and of the (relatively rare) occasions when heat flows downwards from the surface. Because of the assumption of upwelling it is generally inapplicable to ponds, lakes, or to those regions of the oceans where no upwelling takes place. Upwelling will not be considered in the following treatment. It can undoubtedly influence the depth and temperature of daily or seasonal thermoclines, but it is not essential for their formation and persistence as is the case for the main thermocline. (This latter clearly could not persist throughout the year without a cold upwelling, for then penetrative convection would gradually extend the depth of the mixed layer down to the sea bottom.)

The most important omission from both these earlier models is any discussion of *time dependent* processes. It seems that fluctuations in time do play an essential role in the formation and maintenance of thermoclines. Radiational heat losses during the night or during autumn will be associated with penetrative convection which mixes heat from near the surface into lower layers. The depth of the thermocline can be influenced by the amplitude of daily, synoptic and seasonal fluctuations of cooling and heating at the surface, as well as by variations of wind stirring.

The following treatment deals with the time dependent case of the one-dimensional model. It will appear that the formulation suggested by Kitaigorodski is just a special case of this time-dependent theory. The theory is also applied to the laboratory experiments described in paper I.

2. The energy relations

The non-adiabatic heating at a depth z below the sea surface can be described by the expression

$$Q^* = \beta S^* e^{-\beta z} + 2B^* \delta(z). \qquad (1)$$

The first term on the right represents the convergence of the penetrating component S^* of solar radiation. Below a depth of some ten centimeters, the absorption of this radiation becomes approximately isotropic and exponential, with a scale length β^{-1} of about 10 to 20 meters in the open sub-tropical ocean. The last term in (1) accounts for heat exchanges associated with the flux B^* due to infrared radiation, sensible heat and evaporation. Numerical values for this quantity were presented by Kraus & Rooth (1961). The Dirac delta function $\delta(z)$ expresses the fact that these processes are concentrated at the surface. Its integral is

$$\int_0^h \delta(z)\,dz = \tfrac{1}{2}$$

for all values of $h > 0$.

It is convenient to use new variables in the further computations, defined by the substitutions

$$Q^*,\ S^*,\ B^* \equiv \varrho c(Q,\ S\ B).$$

As the density ϱ of sea water and the specific heat c are both close to unity by definition, the new variables are nearly identical numerically with the old ones, though they do have different dimensions.

From (1) it follows that a surface heat loss should give rise to a convective upward flux of heat through a depth of the order

$$d = \beta^{-1} \ln(1 + B/S). \qquad (2)$$

With T representing the horizontal mean of the temperature, T' the deviation from the mean and W' the corresponding vertical velocity, the thermal energy equation can be written in the form

$$\frac{dT}{dt} + \frac{\partial}{\partial z}(\overline{W'T'}) = Q. \qquad (3)$$

Within the stirred surface layer, the temperature T_s is nearly constant with depth. Integration of (3) with the use of (1) gives therefore

$$\frac{dT_s}{dt} z + (\overline{W'T'})_z = S + B - Se^{-\beta z}. \qquad (4)$$

The isothermal layer will become shallower if the penetrative convection does not reach down through its full depth h. On the other hand it will get thicker by entrainment of water from below if the downward heat flux is greater than zero at the level h. The entrained water will then be heated from its temperature T_h below the layer to the temperature T_s of the layer. Formally:

$$(\overline{W'T'})_h = \Lambda (T_s - T_h) \frac{dh}{dt}, \qquad (5)$$

where Λ is the Heaviside unit function, defined to be

$$\Lambda \equiv \Lambda\left(\frac{dh}{dt}\right) = \begin{matrix} 1 \\ 0 \end{matrix} \quad \text{for} \quad \frac{dh}{dt} \gtrless 0.$$

The application of (4) to the whole layer depth h gives therefore

$$h\frac{dT_s}{dt} + \Lambda (T_s - T_h)\frac{dh}{dt} = S + B - Se^{-\beta h} \approx S + B, \qquad (6)$$

the final step being possible because the penetration of radiation below the level h is likely to be small ($\beta h > 1$). Precipitation and evaporation do produce changes in salinity, but the resulting density variations are relatively small. If they are neglected, the transformation of potential energy into kinetic energy by convection within the layer is given by

$$W^* = -g\alpha\varrho \int_0^h \overline{W'T'}\,dz, \quad (7)$$

where α is the coefficient of expansion

$$\alpha = \varrho^{-1} d\varrho/dT.$$

Let D^* denote the dissipation within the layer and G^* the kinetic energy input from the wind. The mechanical energy balance is then expressed by

$$W^* + G^* - D^* = 0. \quad (8)$$

From equations (4) and (6) we obtain:

$$W = \frac{W^*}{g\alpha\varrho} = -\int_0^h \overline{W'T'}\,dz$$

$$= \frac{1}{2}\frac{dT_s}{dt}h^2 - (S+B)\,h + \frac{S}{\beta}$$

$$= -\frac{1}{2}\frac{dT_s}{dt}h^2 - \Lambda(T_s - T_h)\,h\frac{dh}{dt} + \frac{S}{\beta}. \quad (9)$$

The introduction of (9) into (8) gives, after division by $g\alpha\varrho$ and rearrangement, a mechanical energy equation of the form

$$\frac{1}{2}\frac{dT_s}{dt}h^2 + \Lambda(T_s - T_h)\,h\frac{dh}{dt} = G - D + \frac{S}{\beta}, \quad (10)$$

where $G = G^*/g\alpha\varrho$ and $D = D^*/g\alpha\varrho$. (11)

The first term in (10) represents the potential energy change associated with the change in temperature of the layer, and the second the potential energy change due to entrainment, when this exists. The terms on the right hand side are related to the mechanical stirring, dissipation, and convection due to internal heating.

The two equations (6) for the thermal energy balance and (10) for the mechanical energy permit the computation of layer depth h and temperature T_s as a function of time for quite general external energy inputs, provided the dissipation is known. In the following section we will explore some general properties of these solutions before specializing further to a particular example.

3. Calculation of layer depth and temperature

Following the arguments presented in paper I, it will be assumed that vertical mixing within the thermocline region is weak, and that it has no significant effect on the daily or seasonal temperature structure below the stirred surface layer. This means that strata of fluid will be heated to a temperature $T_h(z)$ as the depth of the mixed layer decreases during the heating season, and then left behind unchanged until the stirred layer reaches them again. The temperature $T_h(z)$ therefore corresponds to the surface temperature T_s at the time when the stirred layer reached down to the level z.

With this assumption we have

$$\left.\begin{aligned} &\int_{t_0}^{t_h} T_h \frac{dh}{dt}\,dt = \int_\infty^h T_h\,dh \\ \text{and} \quad &\int_{t_0}^{t_h} T_h h \frac{dh}{dt}\,dt = \int_\infty^h T_h h\,dh. \end{aligned}\right\} \quad (12)$$

The first expression is proportional to the internal energy and the second to the potential energy of the water below the stirred layer at a time t_h when the depth of the layer is given by any specific h. At the beginning of the heating when $t = t_0$ the layer depth $h_0 = \infty$, and the temperature is taken to be $T_0 \equiv 0$ by definition.

When the interface is descending again ($dh/dt > 0$), the integrals of equations (6) and (10) can therefore be written

$$T_s h + \int_h^\infty T_h\,dh = \int_{t_0}^{t_h} (S+B)\,dt \quad (6')$$

and

$$\tfrac{1}{2}T_s h^2 + \int_h^\infty T_h h\,dh = \int_{t_0}^{t_h} (G - D + S/\beta)\,dt. \quad (10')$$

These equations simply express the facts that the internal and potential energies of the whole water column are equal to the integrated heat and mechanical energy inputs in the past, and they could have been written down directly in this form. The use of these equations, with D and S/β neglected, is implicit in the step-by-step calculation described in paper I, and they will be used again for the corresponding computations in this paper.

For a more general discussion, it is convenient to transform (6) and (10) into

$$\frac{dT_s}{dt} = \frac{2}{h^2}\left[(S+B)h - \left(G - D + \frac{S}{\beta}\right)\right] \quad (13)$$

and

$$\Lambda\frac{dh}{dt} = \frac{1}{(T_s - T_h)h}\left[2\left(G - D + \frac{S}{\beta}\right) - (S+\beta)h\right]. \quad (14)$$

When the thermocline is rising, $\Lambda = 0$ and the numerator of the right hand side of (14) must also be zero. During such periods it therefore follows that

$$h = 2\frac{G - D + S/\beta}{S+B}. \quad (15)$$

The introduction of (15) into (13) then gives

$$\frac{dT_s}{dt} = 2\frac{G - D + S/\beta}{h^2} = \frac{1}{2}\frac{(S+B)^2}{G - D + S/\beta}. \quad (16)$$

Differentiation of (15), with the reasonable assumption that the daily or seasonal variations of the solar radiation S are significantly larger than the corresponding variations of the other variables on the right hand side, results in

$$\frac{dh}{dt} \approx \frac{1}{(S+B)}\left(\frac{2}{\beta} - h\right)\frac{dS}{dt}. \quad (17)$$

For most seasonal thermoclines $h > 2/\beta$, so (17) shows immediately that h and $-S$ are in phase, and the layer depth is decreasing while the heating rate is increasing. The layer depth h therefore has a minimum at the time of the summer solstice when $dS/dt = 0$. Equation (16) shows however that the temperature is still increasing at that time, and that in fact the warming should be most rapid when h is close to its minimum. The surface temperature T_s attains its maximum later in the season.

After the summer solstice the layer begins to deepen. As can be seen from equation (14) this implies

$$2(G - D + S/\beta) - (S+B)h > 0. \quad (14')$$

At first the deepening must be slow, because $(S+B)$ remains positive during summer and the difference between the first and the second terms in (14′) will remain relatively small. The temperature contrast at the bottom of the layer is steepest after the heating maximum so the term $(T_s - T_h)$ in the denominator of (14) is large and this also will contribute to keeping dh/dt relatively small during that period.

When the heat input becomes negative ($(S + B) < 0$) later in the season, say after the equinox, both terms in (14′) will contribute to an increase of layer depth, which therefore should deepen rapidly during autumn.

These general predictions from the theory are clearly in agreement with the oceanic observations and with the results of the experiments described in paper I (see especially Figs. 2 and 4 in paper I).

4. Application of the theory to the experiment with constant stirring and a saw-tooth heating function

In this section we will specialize further, and treat in more detail an example which corresponds closely to the case considered in paper I. That is, we put $D = 0$ and $\beta = \infty$, and suppose that the net heating at the surface $(S + B)$ follows a symmetrical saw-tooth form described by

$$S + B = \begin{cases} Kt \\ K(P/2 - t) \\ K(t - P) \end{cases} \text{when} \begin{cases} 0 \leq t \leq P/4 \\ P/4 \leq t \leq 3P/4 \\ 3P/4 \leq t \leq P. \end{cases} \quad (18)$$

The beginning of the heating has been taken as the zero of time, and P is the period of the heating and cooling cycle. We will also assume that the rate of mechanical working or G is constant throughout the cycle.

During the period of increasing heating, we have from (15) and (18)

$$h = 2G/Kt. \quad (19)$$

Introduction of the explicit heating function (18) into (16) and integration gives

$$T_s = T_h = \frac{1}{6}\frac{K^2}{G}t^3. \quad (20)$$

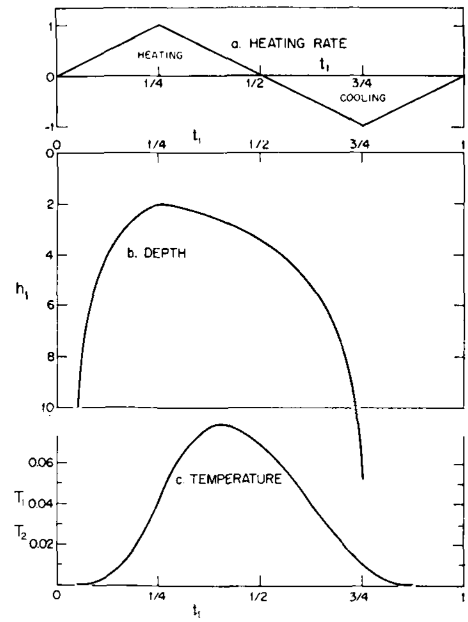

FIG. 1. The theoretical calculation of the time-dependent mixed layer behaviour, using constant stirring and continuous heat input. This plot is in non-dimensional form. (*a*) The assumed saw-tooth heating function. (*b*) Depth of the well-mixed surface layer. (*c*) Temperature of the well-mixed layer. These curves are directly comparable (except for a change of scale) with those deduced in Fig. 7 of paper I by a step-by-step calculation.

When (19) is substituted in (20) we obtain T_h as a function of depth

$$T_h = \frac{4}{3}\frac{G^2}{K} h^{-3} = ah^{-3} \text{ say,} \qquad (21)$$

where a is a constant.

Equation (21) is the starting point for the calculations involving the descending interface. The integral heat and energy balance equations (6′) and (10′) now become

$$T_s h + \tfrac{1}{2} ah^{-2} = \int_0^t (S+B)\,dt \qquad (22)$$

and

$$\tfrac{1}{2} T_s h^2 + ah^{-1} = Gt. \qquad (23)$$

The right hand side of (22) is a known function of t which can be written down from (18).

It will be convenient to put the equations (19), (20), (22) and (23) into non-dimensional form before they are solved. A suitable set of parameters with which to effect this consists of G, P and the maximum rate of heating $(S+B)_{max} = R$ say (where $R = \frac{1}{4} KP$ for the saw tooth form used here). In terms of these and corresponding dimensionless variables T_1, T_2, h_1 and t_1, the physical variables are:

$$T_h,\ T_s = (G^{-1}R^2P)\,T_1,\ T_2, \quad h = GR^{-1}h_1, \quad t = Pt_1. \qquad (24)$$

The equations (19) and (20) describing the rising interface become

$$h_1 = (2t_1)^{-1} \qquad (19')$$

and

$$T_1 = \tfrac{8}{3} t_1^3. \qquad (20')$$

For the descending interface, the introduction of (24) into (22) and (23) gives after some rearrangement:

$$4t_1 = h_1^{-1} + \tfrac{1}{2} F h_1 \qquad (22')$$

$$4T_2 = Fh_1^{-1} - \tfrac{2}{3} h_1^{-3}. \qquad (23')$$

Here F is a function of the non-dimensional time which can be shown using (18) to be

$$F = -1 + 8t_1 - 8t_1^2 \quad \text{when} \quad \tfrac{1}{4} \leq t_1 \leq \tfrac{3}{4}$$

$$\text{or} \quad F = 8 - 16t_1 + 8t_1^2 \quad \text{when} \quad \tfrac{3}{4} \leq t_1 \leq 1. \qquad (25)$$

For this special form of heating function, the behaviour of the upper mixed layer as a function of time has therefore been reduced to the solution of a set of algebraic equations. The solution curves are shown in Figs. 1 and 2, where h_1 and T are plotted first against time and then against each other. These correspond precisely to Figs. 7 and 6 of paper I. This comparison between the two methods of calculation, one based on a step-by-step procedure corresponding to intermittent heating and the other on a continuous model shows that there is no essential difference, at least on a seasonal time scale and with surface heating. Both methods in effect use the same integral heat and energy equations.

With these exact forms of h_1 and T now available, we should emphasize again the qualitative features which are believed to be significant, and in good agreement with observation. There is a rising interface, with a minimum depth at the time of maximum heating, followed by a slow and then a much faster deepening. The phase relationships are realistic too; the maximum heating and minimum depth occur first (at 0.25 P), then the time of maximum temperature at $0.40\,P$, and zero cooling at $0.50\,P$. A similar calculation for sinusoidal heating (which will not be presented in detail here) provides few surprises. The only differences are that the layer spends longer with a shallow depth and hence becomes warmer for a given total heat input, and its temperature is an even more symmetrical function of time, with the maximum later at $0.44\,P$.

We can also obtain important information about the system by examining the scaling equations (24). Several features described in the preceding section can now be exhibited more clearly using this specific example, and are worth further comment. The temperature of the layer is proportional to the *square* of the maximum rate of heating, and increases linearly with the period over which heating occurs. It is inversely proportional to the rate of kinetic energy input from the wind. The depth of the layer, on the other hand, is directly proportional to the mechanical stirring rate and inversely proportional to the heating. The depth scale does not depend at all on the period, though of course both the depth and temperature depend on time through the solutions presented in Fig. 1.

5. The comparison with oceanic observations

Finally we will compare some numerical deductions from this theory with the observations. The data presented by TABATA & GIOVANDO (1963) for Ocean Weather Station P show a mixed layer depth of about 20 meters during the height of summer. Similarly, Fig. 2 in paper I shows minimum layer depths of 20–50 meters. The seasonal ocean surface temperature range in middle latitudes is about 6–10°C. To be valid, a theory has to account for these values in terms of realistic inputs of mechanical and thermal energy. It will be shown that the effects of mechanical stirring and of convection resulting from distributed heating may each be important at different times; neither can be neglected at the expense of the other.

We must first obtain an estimate of the kinetic energy input to the water due to a surface stress τ. The stress τ of the wind works on all the fluid below a level z at a rate τu_z, where u_z is the mean velocity at this level. At the top of the viscous sublayer in the air, the mean wind velocity is of order $(\tau/\varrho_a)^{\frac{1}{2}} = u_*$,

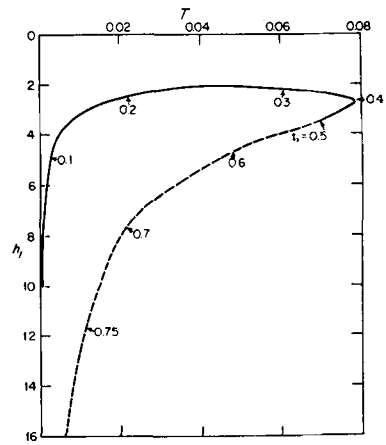

FIG. 2. The layer depth plotted against its temperature for the theoretical results of Fig. 1. Marked on the curve are values of the non-dimensional time t_1, or fractions of the total heating and cooling period.

where ϱ_a is the air density, so the work done by the mean stress at this level is approximately τu_*. Most of this energy input is not transmitted to the body of the water below, however; it is dissipated within a double viscous sublayer, on both sides of the interface. In exactly the same way, a friction velocity v_* say, can be defined for the water, which is characteristic of the water velocity at the edge of the sublayer in the water. As the stress is constant across the interfacial region we have

$$\tau = \varrho_a u_*^2 = \varrho v_*^2$$

and therefore

$$v_* = u_* \sqrt{\varrho_a/\varrho} \approx 0 \cdot 035 u_*. \tag{26}$$

The mechanical energy input into the water below the viscous sublayer is then of order

$$G^* = \tau v_* = \varrho_a u_*^3 \sqrt{\varrho_a/\varrho},$$

and for the quantity G defined by (11) we obtain:

$$G = \frac{G^*}{g\alpha\varrho} = \left(\frac{\varrho_a}{\varrho}\right)^{\frac{3}{2}} \frac{u_*^3}{g\alpha}. \tag{27}$$

The introduction of (26) into (15) with the assumption $D=0$, $\beta=\infty$ gives

$$h = 2\frac{u_*^3}{g\alpha(S+B)}\left(\frac{\varrho_a}{\varrho}\right)^{\frac{3}{2}}. \tag{28}$$

This is essentially the same formula as the one derived by Kitaigorodski on the basis of dimensional reasoning. It is of interest to note that Kitaigorodski found empirically a proportionality constant 2.0 for his dimensional formula, which agrees exactly, though perhaps fortuitously, with the theoretically derived factor 2 on the right hand side of (28). Formula (28) is not applicable to periods when the thermocline deepens. From equation (16) it follows also that it must be associated with an increasing surface temperature T_s. These consequences are not apparent in Kitaigorodski's approach.

Numerical values typical of conditions over the ocean can be found as follows. The drag coefficient C_{10} at the ten meter level over water, by definition given by

$$u_*^2 = C_{10} U_{10}^2, \tag{29}$$

can be computed from Sheppard's (1958) relation

$$C_{10} = (0.80 + 0.00114\ U_{10})\ 10^{-3}. \tag{30}$$

Figures presented by Kraus (1959) suggest a geometric mean wind velocity $(\overline{u^2})^{\frac{1}{2}}$ of about 8 m/sec at anemometer height over the oceans at latitudes less than 30°. Weatherships indicate more frequent strong winds in higher latitudes, but the value of $(\overline{u^2})^{\frac{1}{2}}$ exceeds 12 m/sec only in very restricted areas. During the summer heating season values of about 7–8 m/sec are widely prevalent. From (30) we obtain $C_{10} = 1.7 \times 10^{-3}$ for $U_{10} = 800$ cm/sec. This may be compared with values in a table of C_{10} over water published by Roll (1965). He quotes results derived in different ways by various authors, ranging mostly between 1.0×10^{-3} and 2.5×10^{-3}, so the figure we have derived using (30) falls in the middle of this range.

With this numerical value, $C_{10} = 1.7 \times 10^{-3}$, we get from (26), (27) and (29)

$$u_* = 33 \text{ cm/sec}, \quad v_* = 1.2 \text{ cm/sec},$$
$$\tau = 1.3 \text{ dyne/cm}^2,$$

and so $$G = 6.0 \text{ cm}^2 \text{ sec}^{-1}\ {}^\circ\text{K} \tag{31}$$

for the mechanical energy parameter.

Next, we must consider a typical heat balance. We will assume a relatively small mean heat loss $B^* \sim -200$ cal cm^{-2}/day, and neglect the wind dependence of this quantity in our approximate calculation. Under extremely favorable conditions, on a cloudless, clear summer day, the penetrating component of the solar radiation may be as high as $S^* \sim 600$ cal cm^{-2}/day. This gives a maximum net heat input of $(S^* + B^*)_{\max} \sim 400$ cal cm^{-2}/day, corresponding to

$$R = (S + B)_{\max} = 4.6 \times 10^{-3} \text{ cm sec}^{-1}\ {}^\circ\text{K}. \tag{32}$$

From the numerical values in (31) and (32) we can derive a minimum thermocline depth of

$$h_{\min} = 2G/R = 26 \text{ metres}. \tag{33}$$

If we approximate actual conditions by the results of the model with saw-tooth heating, we obtain from (24) and Fig. 1

$$T_{\max} = 0.08\ R^2P/G = 9^\circ \text{ K}. \tag{34}$$

The results (33) and (34) both lie in the observed range.

As Kraus & Rooth (1961) noted, there is a high information content of these thermocline theories. Realistic values for h and T_s can be obtained only from a narrow range of energy inputs. For example, we would still have obtained reasonably plausible values had we used half or double the mechanical stirring rate; but we could not change this rate by an order of magnitude, or even a factor of three, without deriving values for h and T_s which are not observed in nature.

Consider now the effect of the penetrating radiation. With $\beta^{-1} = 20$ m and the value of S^* given above,

$$S/\beta = S^*/\varrho c\beta \sim 14 \text{ cm}^2 \text{ sec}^{-1}\ {}^\circ\text{K}, \tag{35}$$

which is more than twice as large as G in (31) so it certainly cannot be neglected. This would yield a layer depth

$$h_{\min} = 2\,\frac{G + S/\beta}{R} = 87 \text{ metres},$$

considerably in excess of the observed summer minimum thermocline depth. If we had used a lower value of $\beta^{-1} = 10$ m, so that the terms (31) and (35) are comparable in magnitude, we would still have an unusually deep layer of 57 metres. A lower assumed value of S would not change this result a great deal, because it would affect also the value of R in the denominator.

We conclude therefore that the penetrative radiation produces convective mixing at a rate which is of the same order as that produced by mechanical stirring and that during the height of summer the radiation effect is likely to be predominant. These conclusions, however, introduce another difficulty; the results (33) and (34) are realistic with wind stirring alone, but the two effects together result in too deep a seasonal thermocline on the basis of the above assumptions. There remains the possibility that it is not permissible to neglect dissipation in (15), and that the actual stirring rate must be *less* than was assumed above.

Dissipation will affect both the energy produced by mechanical stirring and that produced by convection. However, the characteristic length scale of the convection is likely to be of the same order as the depth h. It was argued by Ball (1960) that dissipation has little significant effect on turbulence at such a scale. The mechanical energy input on the other hand is likely to cover a broad spectrum. The high wave number part of this spectrum must be dissipated at a short distance from the surface. Only the larger eddies, that is a fraction of the total mechanically induced turbulence, can reach deep enough to affect the thermocline.

During the colder season of the year changes in mixed layer depth and temperature are controlled predominantly by surface cooling; wind stirring or penetrative radiation have relatively little effect at that time. This result could readily have been deduced earlier from equations (13) and (14), but it is appropriate to do it now. The depth h is large during winter, and the product $(S+B)h$ will always exceed the stirring terms in the square brackets of (13) and (14). Since $(S+B)$ is negative, this product will account for most of the rates of increase of layer depth and decrease of its temperature.

In the preceding discussion we have treated the seasonal heating as a smooth function. The consideration of daily heating and nocturnal cooling is unlikely to produce significantly different seasonal changes, though this would have the effect of an additional stirring which it would be interesting to investigate in a future numerical study. This should at the same time include the effects of variable wind stirring.

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ОДНОМЕРНАЯ МОДЕЛЬ «СЕЗОННОЙ ТЕРМОКЛИНЫ»

II. *Общая теория и ее следствия*

При более общих предположениях, чем в предшествующей статье (I) рассмотрена теория слоя обуславленного поверхностными процессами. Теперь учитывается как конвенция, вызванная нагревом на глубине и охлождением поверхности так и механическое движение, вызванное действием ветра. Теория применима для произвольных форм нагрева, включая непрерывные или прерывестые процессы, она может быть использована для исследования как суточных так и сезонных воздействий. Детально рассмотрен случай изучавшийся приближенно в 1-ой статье; для него теперь получено решение в аналитической форме.

Полученные результаты можно сравнить как количественно так и качественно с данными наблюдений в океане. Найдено, что разумно определенная глубина слоя может быть предсказана с использованием измеренной величины поступающего тепла и поступающей в воду турбулентной кинетической энергии, определяемой из среднего поверхностного напряжения. Действие нагрева на глубине сравнимо с движением, вызванным ветром, даже когда температура в верхнем слое увеличивается. В течении зимнего периода конвенция, вызванная охлождением поверхности, усиливает процессы, углубляющие слой.