## Lecture 16.  Mixed-layer modeling of stratocumulus-capped boundary layers

*In this lecture…*

- Mixed layer modeling of Sc
- Timescales of a Sc-capped mixed layer
- Multiple mixed layer model of diurnal decoupling of Sc

*Introduction*

Mixed-layer modeling of stratocumulus was introduced in a classic paper by Lilly (1968). and has since been used in many scientific papers about SCBLs.  It is not just useful for predictive modeling, but also for interpretation of observations and more complex models.  A mixed-layer model is only appropriate if the SCBL is indeed well-mixed, so a MLM should be able to predict when it has reached its limit of validity (see Bretherton and Wyant 1997 for a discussion of this).

There are several complications in mixed-layer modeling of stratocumulus that are not present in a dry convective boundary layer. These include internal heating and cooling of the boundary layer by condensation, evaporation and radiation.  There is also still controversy about the appropriate entrainment closure.

*Deducing the cloud properties in a stratocumulus-capped mixed layer*

The thermodynamic state of a stratocumulus-capped mixed layer is most easily specified in terms of two moist-conserved variables, for instance the moist static energy $h_M$ and the total water mixing ratio $q_{tM}$.  The mixed-layer assumption is that vigorous turbulence keeps these variables vertically uniform between the surface and the inversion height $z_i(t)$.

Quantities that are not moist-conserved, such as temperature or liquid water content, are not vertically uniform within the mixed layer; their vertical profiles must be deduced from $h_M$ and $q_{tM}$ and pressure $p(z)$. As for the dry mixed layer, we will neglect variations of density $\rho$ with height within the boundary layer. We also specify the surface pressure $p_s$.  Then the hydrostatic approximation applied to the mean state implies that

$$p(z) = p_s - \rho g z. \tag{16.1}$$

Particularly important is the cloud base height $z_b$, at which boundary layer air is exactly saturated.  It can be calculate from the equation:

$$q_{tM} = q^*(p_b, T_b) = q^*( p_s - \rho g z_b, \ [h_M - g z_b - L q_{tM}]/c_p) . \tag{16.2}$$

Here $q^*(p, T)$ is the saturation water vapor mixing ratio, and subscript '$b$' refers to the cloud base.  This nonlinear equation can be solved for $z_b$ in terms of known quantities. Although this looks complicated, it can be approximated by a simpler linear form.  We define the mixed layer air temperature at the surface $z=0$:

$$T_{Ms} = [h_M - L q_{tM}]/c_p, \tag{16.3}$$

and we define the mixed layer saturation mixing ratio at $z=0$:

$$q^*_{Ms} = q^*(p_s, T_{Ms}). \tag{16.4}$$

We can then linearize the right hand side of (16.2) in $z_b$ around this saturated state:

$$q^*(p_b, T_b) = q^*_{Ms} + z_b(dq^*/dz)_{da}. \tag{16.5}$$

Here $(dq^*/dz)_{da}$ is the rate at which saturation mixing ratio changes with height along a dry adiabat from the surface to the cloud base. This depends on the exact thermodynamic state, but for thermodynamic conditions typical of subtropical stratocumulus,

$(dq*/dz)_{da} \approx -4$ g kg$^{-1}$ km$^{-1}$. Hence, (16.2) simplifies to

$$z_b \approx (q*_{Ms} - q_{tM}) / |dq*/dz|_{da} \tag{16.6}$$

If the surface air is more subsaturated, $z_b$ will be larger. A good approximation is that if the near-surface relative humidity is 80%, the cloud base (= lifted condensation level) will be about 500 m. If the near surface RH is 60%, the cloud base will be 1 km, etc.  Above the cloud base, similar linearization gives the liquid water profile

$$q_l(z) = q_{tM} - q*(p_M(z), T_M(z)) \approx |dq*/dz|_{ma}(z - z_b), \tag{16.7}$$

where $(dq*/dz)_{ma}$ is the rate at which saturation mixing ratio changes with height above cloud base along a moist adiabat.  Typically $|dq*/dz|_{ma} \approx 2$ g kg$^{-1}$ km$^{-1}$ is about half as large as $|dq*/dz|_{da}$ in a stratocumulus layer.  We see that the liquid water content is largest at the cloud top, and that the vertically-integrated cloud liquid water content, or liquid water path, is proportional to the square of the cloud layer depth.  An adiabatic subtropical stratocumulus cloud about 300 m thick has a cloud-top liquid water content of 0.6 g kg$^{-1}$ and a liquid water path of about 100 g m$^{-2}$.

Fig. 8 (left) shows how various profiles behave in a stratocumulus-capped mixed layer.

### MLM equations

The MLM derivation and discussion of buoyancy flux and entrainment in the next few sections are mostly based on Bretherton and Wyant (1997).

Above the boundary layer, we assume known 'free-tropospheric' profiles $q_t^+(z)$, $h^+(z)$. These affect the entrainment flux into the mixed layer:

$$\overline{w'q_t'}(z_i) = -w_e \Delta q_t, \qquad \Delta q_t = q^+(z_i) - q_{tM}, \tag{16.8a}$$

$$\overline{w'h'}(z_i) = -w_e \Delta h, \qquad \Delta h = h^+(z_i) - h_{tM}. \tag{16.8b}$$

Since stratocumulus evolve slowly, we must also consider the mean vertical velocity $\overline{w}(z)$, which is often idealized as subsidence that increases linearly with height:

$$\overline{w}(z) = -Dz, \tag{16.9}$$

where $D$ is the horizontal wind divergence, typically $3\text{-}6\times10^{-6}$ s$^{-1}$ in subtropical stratocumulus regimes. Thus, at a height of 1 km, the mean subsidence rate is around 3-6 mm s$^{-1}$. This is slow but significant.

Another important boundary condition is the sea-surface temperature $T_s$, which determines the surface heat and moisture fluxes. From $T_s$, we calculate the mixing ratio within the sea-surface skin layer, $q_s = q*(p_s, T_s)$ and the sea-surface moist static energy $h_s = c_p T_s + L q_s$.  For simplicity, we will only model the thermodynamic evolution of a SCBL, not its momentum balance, so we will just specify a mixed-layer wind speed $V$, and we will use bulk aerodynamic formulas with a nondimensional transfer coefficient $C_T(V) \approx 10^{-3}$ to specify the surface fluxes:

$$\overline{w'q_t'}(0) = C_T V(q_s - q_{tM}), \tag{16.10a}$$

$$\overline{w'h'}(0) = C_T V(h_s - h_{tM}). \tag{16.10b}$$

Within the boundary layer, there will be a net upward radiative flux profile $F_R(z)$ (including both longwave and shortwave contributions) and a downward water flux profile $P(z)$ due to precipitation.  These fluxes must be diagnosed from the mixed layer properties, including the vertical structure of the cloud layer, following the ideas presented in Lecture 2.  Here we will just assume we have some algorithm for doing this.  We must also have an entrainment closure for specifying the entrainment rate $w_e$, which we'll discuss later.

Now we are finally ready to write down the governing equations for the MLM, which express conservation of mass, water, and moist static energy in the mixed layer:

$$\frac{dz_i}{dt} = w_e + \bar{w}(z_i),\tag{16.11}$$

$$\frac{dh_M}{dt} = -\frac{1}{\rho}\frac{\partial E}{\partial z},\tag{16.12}$$

$$\frac{dq_{tM}}{dt} = -\frac{1}{\rho}\frac{\partial W}{\partial z}.\tag{16.13}$$

Here, $d/dt$ is the material derivative following the boundary layer air column, which moves with the mean horizontal wind. Furthermore,

$$W(z) = \rho\,\overline{w'q_t'}(z) - P(z)\tag{16.14}$$

is the upward water flux, composed of a turbulent and precipitation flux, and

$$E(z) = \rho\,\overline{w'h'}(z) + F_R(z)\tag{16.15}$$

is the upward energy flux, composed of a turbulent and a radiative flux.

If we know $w_e$ from the entrainment closure, the MLM equations can be solved as in the dry case. Since the left hand sides of (16.12-13) are height-independent, the same must be true of their right hand sides.  Hence, the energy and water fluxes must vary linearly with height between the surface and the inversion. Defining a nondimensional height $\zeta = z/z_i$:

$$W(z) = (1-\zeta)W(0) + \zeta W(z_i),\tag{16.16a}$$

$$E(z) = (1-\zeta)E(0) + \zeta E(z_i),\tag{16.16b}$$

and

$$-\frac{\partial W}{\partial z} = \frac{W(0) - W(z_i)}{z_i},\tag{16.17a}$$

$$-\frac{\partial E}{\partial z} = \frac{E(0) - E(z_i)}{z_i},\tag{16.17b}$$

where

$$W(0) = \rho C_T V(q_s - q_{tM}) - P(0),\quad W(z_i) = -\rho w_e\Delta q_t,\tag{16.18a}$$

$$E(0) = \rho C_T V(h_s - h_{tM}) + F_R(0),\quad E(z_i) = -\rho w_e\Delta h + F_R(z_i).\tag{16.18b}$$

This completes the specification of the right-hand sides of (16.12-13), allowing the MLM equations to be marched forward in time.

The turbulent flux profiles of $q_t$ and $h$ can be recovered from the energy and water flux profiles using (16.14) and (16.15), as illustrated on the right side of Fig. 8.  A popular idealization is to assume a nonprecipitating cloud ($P(z) = 0$) with all the radiative cooling concentrated just under the cloud top as a specified flux divergence $\Delta F_R$, so that

$$F_R(z) = F_R(0) \text{ for } 0 < z < z_i,\quad \text{and } F_R(z_i) = F_R(0) + \Delta F_R.\tag{16.19}$$

*Buoyancy, buoyancy flux and liquid flux in a stratocumulus-capped boundary layer*

The buoyancy $b' = -g\rho'/\rho_0 \approx gT_v'/T_0$ where $T_0$ is a reference temperature.  The <u>virtual (or density) temperature</u> $T_v$ is defined here to include the effect of liquid water loading,

$$T_v = T(1 + \delta q_v - q_l),\quad \delta = 0.61,$$

$$\approx T + T_0(\delta q_v - q_l),$$

from which we deduce that

$$T_v' \approx T' + T_0(\delta q_v' - q_l').$$  (16.20)

The dominant contribution to buoyancy in SCBLs is from the temperature perturbations $T'$, but the vapor and liquid loading terms are also quantitatively significant.

It is instructive to look at an ideal air parcel circuit in a SCBL (Fig. 9) in which air moves adiabatically from the surface (where it has been moistened) to the inversion, where it is affected by entrainment and radiative cooling. Above a moist surface, updrafts will tend to be moister than downdrafts and will have a lower LCL (as indicated by the two wavy dashed cloud bases in the figure), so the liquid water along the circuit will vary as in (a). Above the updraft cloud base, the upward moving air is warmed by latent heating due to condensation and follows a moist-adiabatic lapse rate. It is then cooled (mainly radiatively) at the inversion, and sinks along a moist adiabat until all liquid has evaporated. One can see from this picture that in the cloud, the updrafts are warmer and more buoyant compared to the downdrafts. Hence we can correctly anticipate that the buoyancy flux will be much larger in the cloud than below the cloud; the turbulence is mainly being driven *from within the cloud* rather than from the surface. This is a very important difference from a dry convective boundary layer. Bretherton and Wyant (1997) show nice examples of buoyancy flux profiles in stratocumulus-capped mixed layers.

To recap, the updraft-downdraft buoyancy difference increases above cloud base because there is more liquid water in updrafts than in downdrafts at the same height. It follows that the cloud base jump in the buoyancy flux is proportional to the upward liquid water flux within the stratocumulus cloud layer. To see this, we express $T_v'$ in terms of a liquid-water virtual temperature $T_{vl}$ unaffected by condensation plus a contribution from liquid water.

First ignoring virtual effects for simplicity, we have

$$T' = T_l' + L q_l'/c_p,$$

where the liquid water temperature $T_l$ is conserved under phase change and hence continuous across cloud base. Recall the way this works: above cloud base, the latent heating due to condensation of liquid water ($q_l > 0$) is reflected in higher temperature with no change in $T_l$. Hence

$$\overline{w'T'} = \overline{w'T_l'} + \frac{L}{c_p}\overline{w'q_l'}$$  (16.21)

The first term is continuous across cloud base. The second term gives the additional contribution of the liquid water flux above cloud base, which we have seen can be substantial. In a Sc-topped boundary layer, the liquid flux in the cloud layer is strongly tied to the upward water flux, which in turn is proportional to the latent heat flux, since all of these are related to updrafts being moister than downdrafts (Bretherton and Wyant 1997). In a Cu-topped BL, in which downdrafts are unsaturated, (16.21) still holds but the link between liquid flux and latent heat flux is not as tight.

Repeating this calculation including virtual effects, (16.20) can be manipulated into

$$T_v' = T_l' + L q_l'/c_p + T_0(\delta[q_t' - q_l'] - q_l') = T_{vl}' + \alpha L q_l'/c_p \,.$$  (16.22)

where

$$T_{vl}' = T_l' + T_0\delta q_t' \quad (= T_v' \text{ in unsaturated air})$$  (16.23)

$$\varepsilon = c_p T_0/L \approx 0.12,$$  (16.24)

$$\Rightarrow \quad \alpha = 1-[1+\delta]\varepsilon \approx 0.81$$  (16.25)

Hence

$$\overline{w'T_v'} = \overline{w'T_{vl}'} + \alpha \frac{L}{c_p} \overline{w'q_l'} \qquad (16.26)$$

*Writing buoyancy flux in terms of conserved variable fluxes above/below Sc base*

For the entrainment closure of a MLM, we must mathematically express the buoyancy flux in terms of the MLM-calculated fluxes of $q_t$ and $h$. To do this, we express $T_v'$ in terms of the perturbations $q_t'$ and $h'$. As the previous section suggests, the formula is different below vs. within the Sc cloud.

For simplicity, we will initially do this derivation ignoring virtual effects, i. e. for the temperature perturbation $T'$ rather than $T_v'$. We start by noting

$$q_t' = q_v' + q_l',$$
$$h' = c_p T' + L q_v'.$$

**Below cloud base** in unsaturated air ($q_l' = 0$), this gives the desired relationship:

$$T' = [h' - L q_t']/c_p. \qquad \text{(below cloud base)} \qquad (16.27a)$$

**Above cloud base**, the air is saturated. Using the Clausius-Clapeyron equation,

$$q_v' = q^{*'} = (dq^*/dT)\, T' = (\gamma c_p/L)\, T',$$

where $\gamma = (L/c_p)\, dq^*/dT = 1\text{-}3$ (larger at higher temperature). Hence

$$h' \approx c_p T' + L q_v' = (1 + \gamma)\, c_p T',$$

and

$$T' = h'/[c_p(1 + \gamma)]. \qquad \text{(above cloud base)} \qquad (16.27b)$$

With a bit more algebra, one can generalize these formulas to $T_v'$ (Randall 1981):

$$T_v' = [h' - \mu L q_t']/c_p. \qquad \text{(below cloud base)} \qquad (16.28a)$$
$$T_v' = [\beta h' - \varepsilon L q_t']/c_p, \qquad \text{(above cloud base)} \qquad (16.28b)$$

where $\varepsilon$ was given in (16.24), and

$$\mu = 1 - \delta\varepsilon \approx 0.93, \qquad (16.29)$$
$$\beta = (1 + \gamma\varepsilon(1+\delta))/(1 + \gamma) \approx 0.4\text{-}0.5. \qquad (16.30)$$

The buoyancy flux is now easily computed from the fluxes of $h$ and $q_t$:

$$B(z) = \frac{g}{T_0}\overline{w'T_v'} = \frac{g}{c_p T_0}\begin{cases} \overline{w'h'} - \mu L\overline{w'q_t'}, & 0 < z < z_b, \\ \beta\overline{w'h'} - \varepsilon L\overline{w'q_t'}, & z_b < z < z_i. \end{cases} \qquad (16.31)$$

*Entrainment closure*

Unlike for the dry convective boundary layer, entrainment closure for stratocumulus-capped boundary layers is still an open topic of research and there are also other theories than the Nicholls-Turton (1986) closure we present here. All of the theories reduce to the accepted entrainment closure for a dry-convective boundary layer when there is no cloud. However, because measurements of entrainment into stratocumulus-capped boundary layers are difficult and uncertain, observations do not clearly tell us which entrainment closure is correct. The starting point for all entrainment closures is the profile of buoyancy flux $B(z)$, which is the primary source of TKE in stratocumulus-capped mixed layers. We just showed how this is derived from the turbulent energy and moisture fluxes.

From the buoyancy flux profile, we calculate the convective velocity $w_*$ as for the DCBL:

$$w_*^3 = 2.5\int_0^{z_i} B(z)dz, \qquad (16.32)$$

and then we calculate the entrainment rate as

$$w_e = A \, w_*^3 /(z_i \Delta b),$$  (16.33)

where the <u>entrainment efficiency</u>

$$A = 0.2(1 + a_2 E).$$  (16.34)

Here $E$ is a dimensionless parameter (see Fig. 10) that describes how much evaporation of cloud liquid water reduces the buoyancy of mixtures of mixed-layer and above-inversion air. $E$ ranges from 0 (when no cloud is present) to 0.2 or more (for a thick cloud with very dry overlying air or a weak capping inversion). The empirical constant $a_2$ is in the range 15-60. The width of this range reflects the large measurement uncertainties for entrainment rate, and reflects studies by Nicholls and Turton (1986), Stevens et al. (2003) and Caldwell et al. (2005).  The term $a_2 E$ reflects <u>evaporative enhancement</u> of entrainment and raises the entrainment efficiency of typical stratocumulus into the range 0.5-2, compared to its dry value of 0.2.  Lilly (2002) proposed a related entrainment closure that has some conceptual improvements over Nicholls-Turton, but probably has little practical advantage.

A complication with applying (16.20) is that the buoyancy flux, and hence $w_*^3$, depends on $w_e$. However, we can partition $w_*^3$ into a term proportional to entrainment and a 'non-entrainment' term due to other processes such as surface fluxes, radiative cooling, etc.:

$$w_*^3 = (w_*^3)_{ne} + w_e \frac{dw_*^3}{dw_e} \; .$$  (16.35)

When this is substituted into (16.20), we can solve for $w_e$.

A key conceptual advantage of the NT entrainment closure is that it directly ties the entrainment rate to the strength of turbulence.  One consequence is that the NT closure can predict entrainment rates that bring down enough warm and dry air to cause the buoyancy flux to become negative in a significant fraction of the subcloud layer.  This would in practice lead to decoupling, so the NT entrainment closure can internally predict when the BL will become decoupled, at which point the MLM ceases to be valid.  NT proposed a 'buoyancy integral ratio' test for this purpose.

Some other 'flux-partitioning' entrainment closures choose the entrainment rate such that the ratio of some measure of negative buoyancy flux below cloud base is proportional to some measure of the positive buoyancy flux elsewhere in the BL.  By construction, such entrainment closures cannot predict decoupling.  See Bretherton and Wyant (1997) for a discussion.

### *Steady states of an MLM*

Pioneering work by Lilly (1968) and Schubert et al. (1979) focused on steady-state solutions of nonprecipitating MLMs subject to constant forcing and specified time-independent cloud-top radiative cooling.  In this case, the mixed layer equations reduce to

$$dz_i/dt = w_e - Dz_i$$  (16.36a)

$$dq_{tM}/dt = \{w_e (q_t^+ - q_{tM}) + C_T V (q_{t0} - q_{tM}) \}/ z_i$$  (16.36b)

$$dh_M/dt = \{w_e (h^+ - h_M) - \Delta R_N/\rho c_p + C_T V (h_0 - h_M) \}/ z_i$$  (16.36c)

Defining the entrainment dilution fraction

$$\lambda = w_e /( w_e + C_T V) \quad (\sim 0.3 \text{ for } w_e = 0.5 \text{ cm s}^{-1} \text{ and } C_T V = 1 \text{ cm s}^{-1})$$  (16.37)

a steady state of the MLM must satisfy:

$$z_{i,eq} = w_e/D$$  (16.38a)

$$q_{t,eq} = \lambda q_t^+ + (1 - )q_{t0}$$  (16.38b)

$$h_{eq} = \lambda h^+ + (1 - )h_0 - \Delta R_N/\rho c_p(w_e + C_T V) \tag{16.38c}$$

$$\Rightarrow s_{l,eq} = \lambda s_l^+ + (1 - )s_{l0} - \Delta R_N/\rho c_p(w_e + C_T V) \tag{16.38d}$$

Encouragingly, these equilibria are physically realistic and exhibit reasonable sensitivities to the important control parameters.   In Fig. 11, the steady state inversion and cloud thickness in one such MLM are plotted vs. SST (a proxy for lower-tropospheric stability) and horizontal divergence (a measure of mean subsidence).  Weaker subsidence or a warmer SST (red) give a deeper inversion and a thicker cloud than for cold SST and strong subsidence (blue).  These steady states respect the following dominant balances:

Mass:   Entrainment $w_e$ = subsidence $Dz_i$

Moisture:  Entrainment drying $\rho \, w_e \, \Delta q_t$ = surface evaporation rate $LHF/L$

Heat:    Entrainment warming $\rho w_e \, \Delta s_l \approx$ Radiative cooling $\Delta F_R \gg$ sensible heat flux.

One insight is that the entrainment rate adjusts to maintain heat balance via feedbacks between entrainment and the buoyancy flux profile within the BL.  The heat and mass balance together imply

$$z_i \approx \Delta F_R/\rho D \Delta s_l \tag{16.39}$$

The inversion height is increased by more radiative cooling, weaker subsidence or a weaker inversion jump.  The boundary layer humidity (which controls cloud base) is also controlled by entrainment dilution.

*Timescales of a Sc-capped mixed layer*

Over land, the strong diurnal cycle guarantees that the daytime convective BL never achieves a steady state. However, marine CTBLs are closer to a steady state structure. Following a very nice paper by Schubert et al. (1979, *J. Atmos. Sci.*, **36**, 1308-1324), we consider the timescales for the BL to relax toward a steady state, by rephrasing the mixed layer equations (16.38) as follows:

$$dz_i/dt = (z_{i,eq} - z_i)/\tau_\iota \tag{16.40a}$$

$$dq_{tM}/dt = (q_{t,eq} - q_{tM})/\tau_M \tag{16.40b}$$

$$dh_M/dt = (h_{eq} - h_M)/\tau_M \tag{16.40c}$$

Here, the relaxation timescales are:

$$\tau_\iota = D^{-1} \text{ for BL depth} \tag{16.41a}$$

$$\tau_M = z_i /(w_e + C_T V) \text{ for internal thermodynamic adjustment} \tag{16.41b}$$

For typical values for subtropical Sc-topped mixed layers, $D = 5 \cdot 10^{-6} \text{ s}^{-1}$ , $w_e = 0.5 \text{ cm s}^{-1}$, $C_T V = 1 \text{ cm s}^{-1}$, we find that:

$$\tau_\iota = 2 \times 10^5 \text{ s} = 2.3 \text{ days}$$

$$\tau_M = (500 \text{ m})/(0.5 + 1) \text{ cm s}^{-1} \approx 0.4 \text{ days}$$

The internal thermodynamic state of the BL rapidly adjusts on the time $\tau_M$ to changes in SST and free-atmospheric properties. The inversion height relaxes to an equilibrium value on the much longer timescale $\tau_\iota$. During this time, slow thermodynamic changes also continue as the entrainment rate and the temperature and humidity of the entrained air adjust to the changing depth of the boundary layer – this 'slaving' of the rapidly-adjusting internal state of the PBL to the slowly evolving inversion height has been called 'slow manifold' behavior (Bretherton et al. 2010 *JAMES*).

Fig. 16.6 shows the response of a cloud-topped mixed layer to a 2 K step change in SST. Note the rapid adjustment of cloud base (i. e. the internal thermodynamic state) to the

changed SST compared with the much slower adjustment of cloud top. In this figure, the BL air column evolves as it advects over a changing surface with a fixed wind speed $V = 7$ m s$^{-1}$; 1 day's evolution corresponds to a distance of 600 km.   In reality, the conditions following a BL air column vary over periods of days, so the BL height is not close to steady state.

### Response of a Sc-capped MLM to warming SST

Fig. 16.7 shows nonprecipitating MLM simulations with constant divergence and radiative cooling but SST increasing at 2 K day$^{-1}$.  One simulation uses an NT-like entrainment closure and the other uses a flux-partitioning entrainment closure.  As the boundary layer warms, weakening the capping inversion, $z_i$ deepens similarly in both simulations.  They have similar entrainment rates because both simulations adjust into an approximate energy balance between radiative cooling (the same in both simulations) and entrainment warming.  But the entrainment differences are evident in the subcloud buoyancy flux profiles.  The NT closure develops increasing negative subcloud buoyancy fluxes as the BL deepens, exceeding its diagnostic threshold of buoyancy integral ratio for decoupling after 2.3 days (after which the MLM is no longer a good model for the vertical BL structure).  The flux-partitioning closure by construction maintains a small negative subcloud buoyancy flux throughout, so it cannot internally predict decoupling.

### Problems with simulation of diurnal cycle with MLM

Fig. 16.8 shows an MLM simulation with a diurnal cycle of insolation (Schubert 1976).  The daytime Sc absorption of sunlight cuts entrainment, lowering the inversion as observed.  It also reducing the resulting entrainment drying, thickening the Sc, which is not observed.  This failure of the MLM occurs because the well-mixed assumption is better at night (when radiative cooling is strongest) than during the day (when solar absorption cancels much of the cloudtop longwave cooling, often leading to decoupling of the surface and cloud layer turbulence).

### Multiple mixed-layer model of diurnal decoupling of Sc

Turton and Nicholls (1987) presented an elegant simulation of this process in which a mixed layer splits into two mixed layers separated by a nonturbulent stable layer if a decoupling criterion is satisfies, and the two mixed layers can recouple into a single mixed layer if the stable layer between them weakens, e.g. due to nocturnal radiative cooling of the upper layer.  While a mixed layer model (Fig. 16.9) shows almost no daytime thinning of the cloud (shaded region in upper plot), their model (left) predicts that the upper mixed layer dries by 0.5 g kg$^{-1}$ while the lower mixed layer moistens almost 1 g kg$^{-1}$, resulting in a more realistic 70% thinning of the upper Sc layer while thin scud develops atop the lower mixed layer. The lower panels show the corresponding diurnal evolution of the conserved variables in the two models.

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