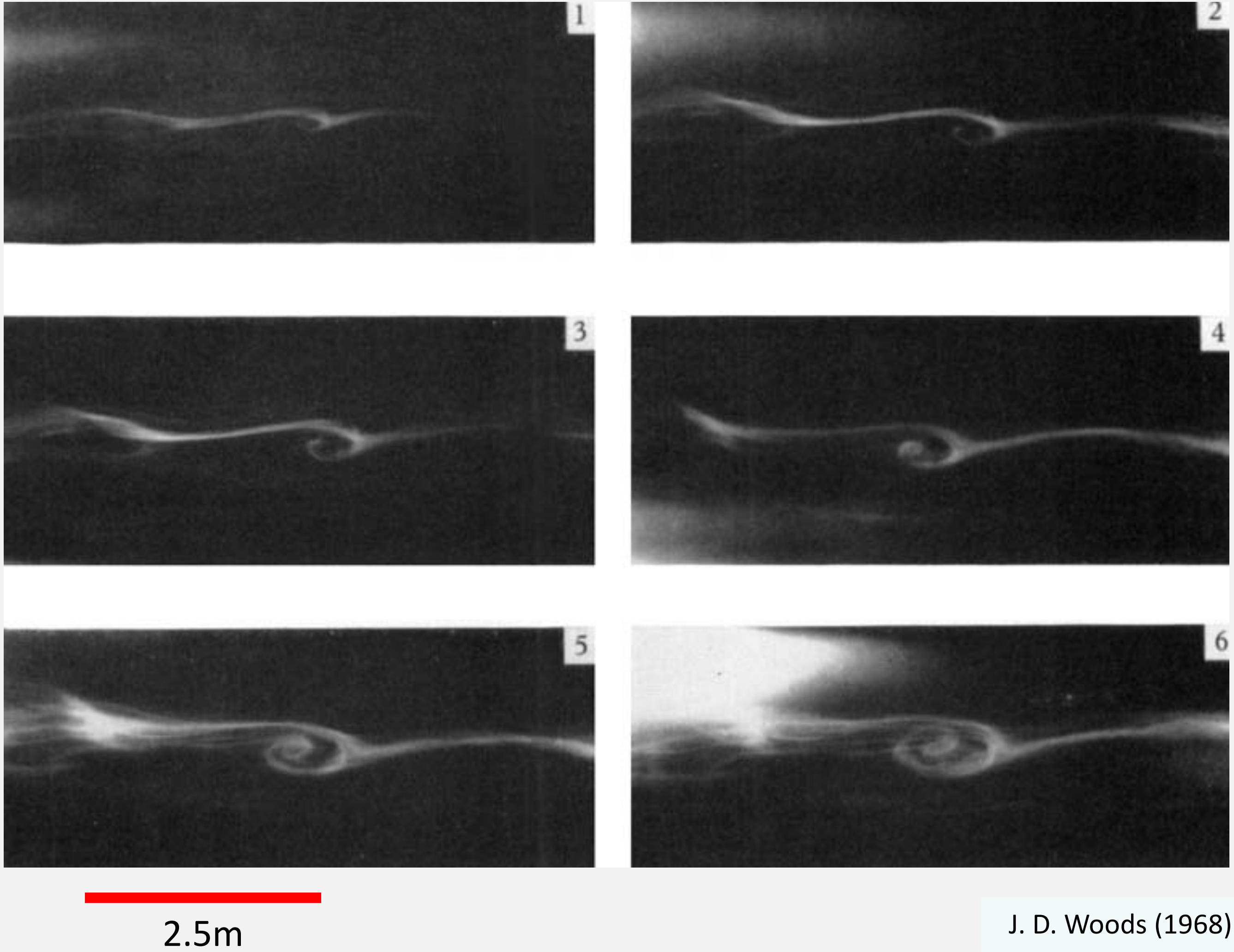
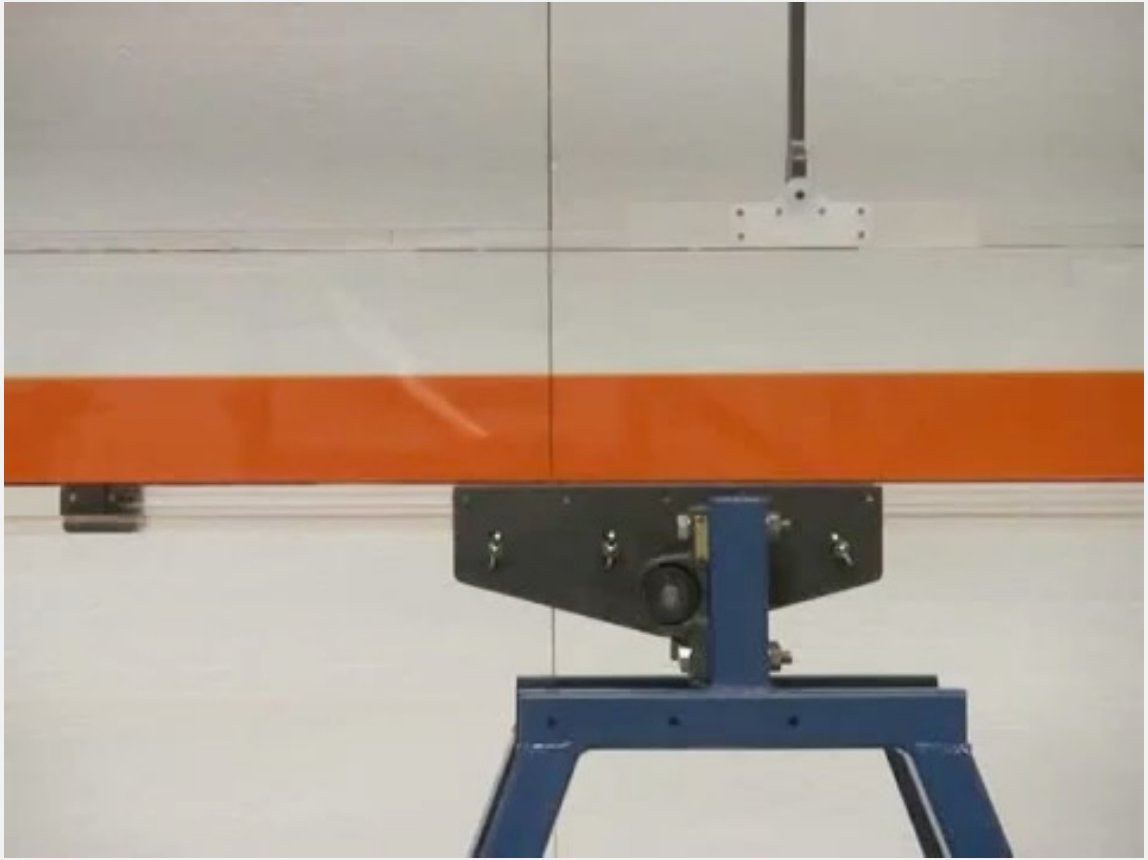
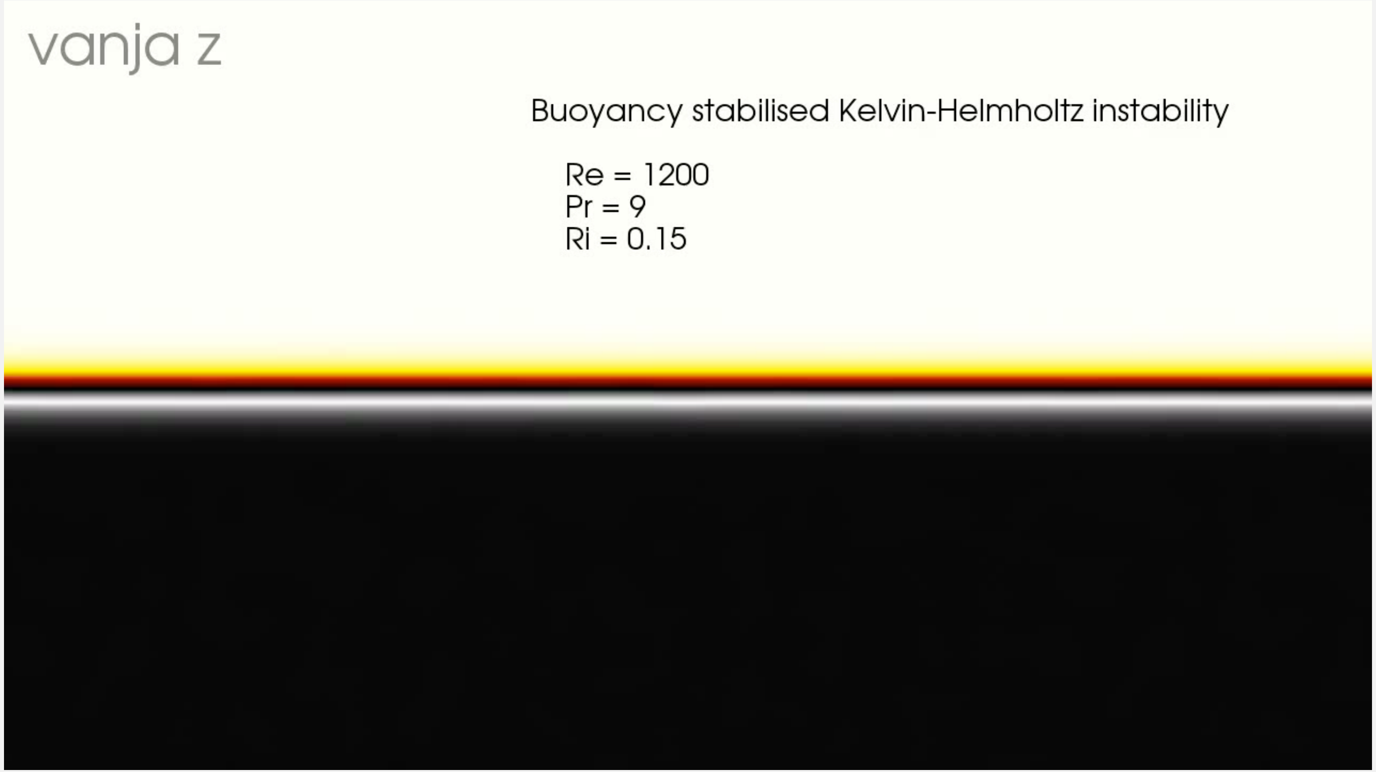


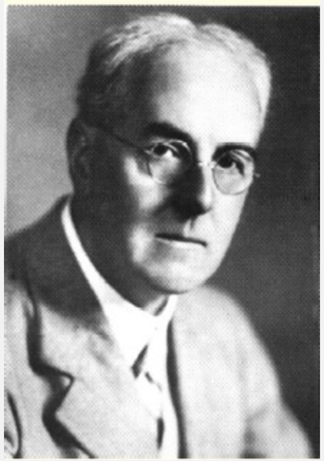

J. D. Woods (1968)



Grae Worcester, University of Cambridge




Vanja Zecevic, University of Sydney



Lewis Fry Richardson

# Richardson Number

Necessary condition for instability:

$$R_i \left(= \frac{N^2}{S^2}\right) \leq 0.25$$

somewhere in the flow
Miles (1961), Howard (1961)

$$N = \sqrt{-\frac{g}{\rho_o}\frac{\partial \rho(z)}{\partial z}}$$

Thorpe (1987, JGR 92, C5, 5231-5248



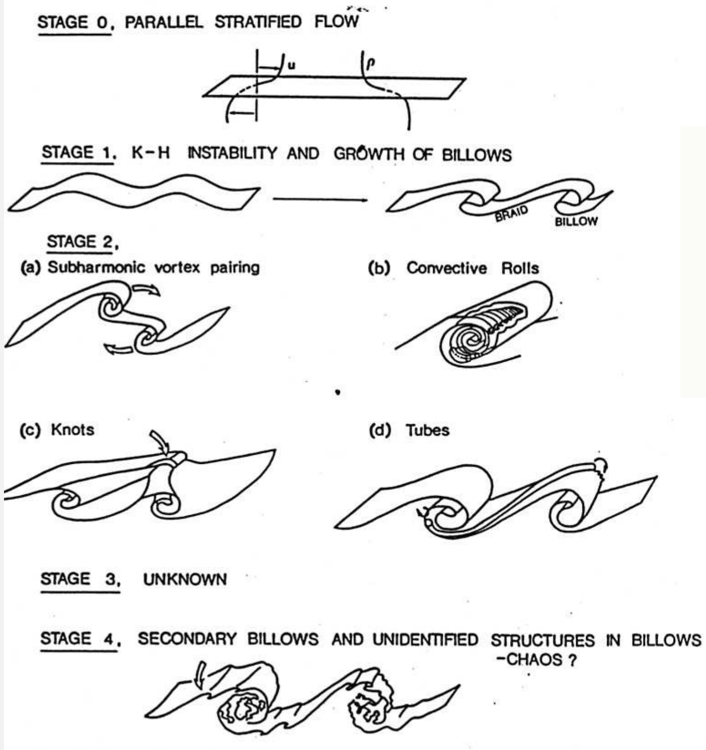
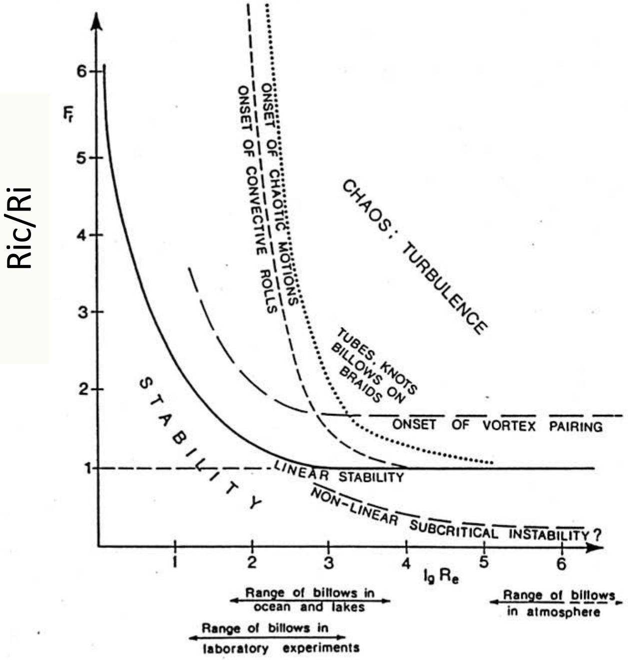



Fig. 7. Stages in the transition to turbulence of Kelvin-Helmholtz instability.



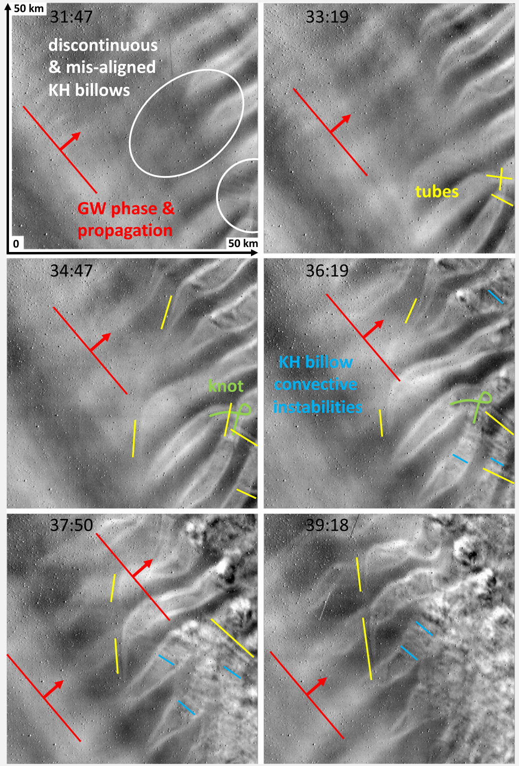



**Figure 1.** Differenced brightness images from H20 at ~90-s intervals showing emerging vorticity dynamics as KH billows that are deformed and/or misaligned evolve to larger amplitudes and exhibit various instability dynamics in an ~50 × 50-km field of view (with north at top and east at left). See the image dimensions at upper left and notation and highlights of the relevant features where they first arise and evolve thereafter.

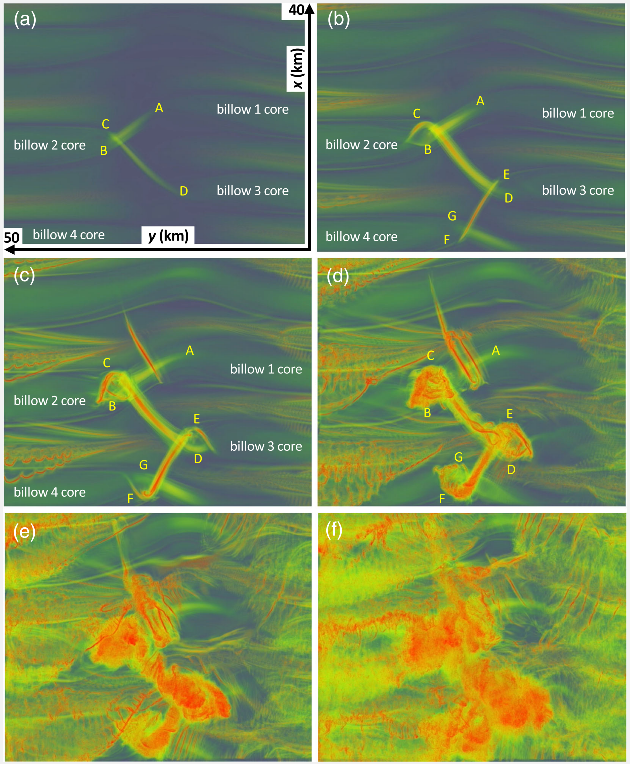


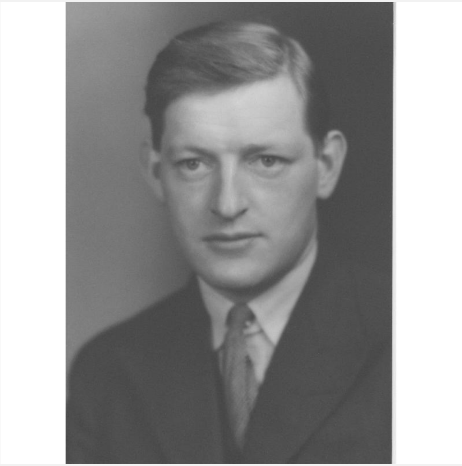

**Figure 6.** (a–f) KHI tube and knot evolution due to linking of misaligned billows spanning ~2 $T_b$ viewed from above beginning at the initial time in Figures 3 and 4. Shown are volumetric views of $\lambda_2$, with small values dark green and large values red in a subset of the full domain. Here, $x$ and $y$ increase toward top and left, respectively, and positive mean $\zeta_y$ is toward increasing $y$ (to the left). Panel (a) shows the axes.

Fritts et al 2020

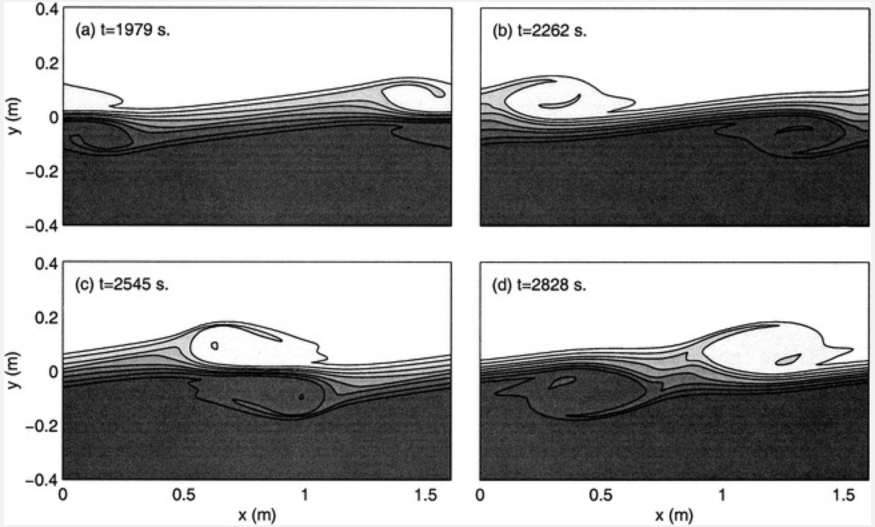
Observations: brightness

Model: rotation

# Holmboe instability

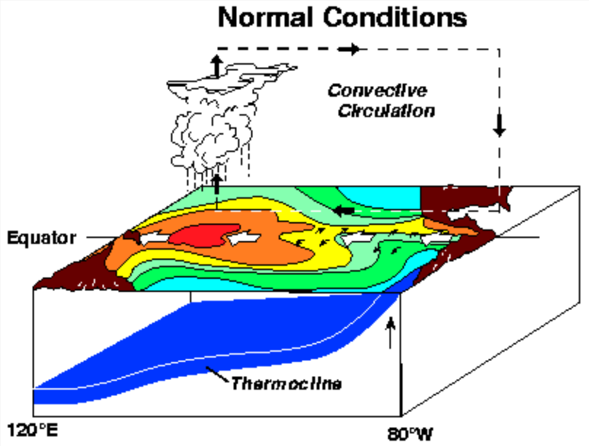


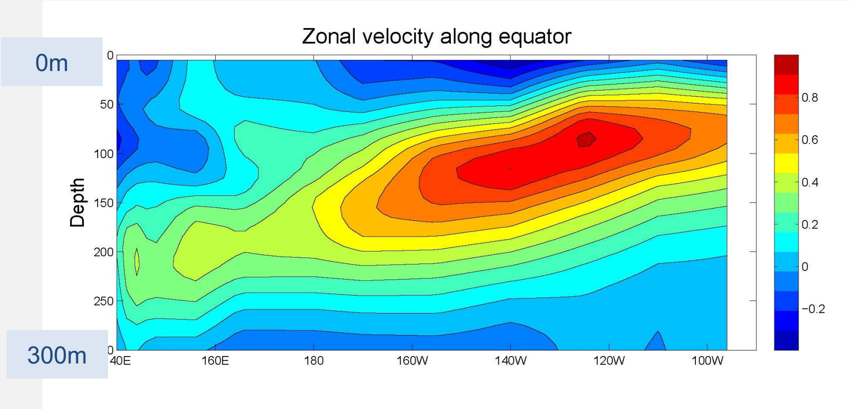
Jørgen Holmboe




Smyth and Winters JPO 2003

# Mixing in models of the Ocean and Atmosphere




## Turbulent fluxes

$$-\overline{u'w'} = \kappa_m \frac{\partial \overline{u}}{\partial z}$$

$$-\overline{w'T'} = \kappa_h \frac{\partial \overline{T}}{\partial z}$$

Scaling:

$$\epsilon = \ell_v^2 N^3 f(Ri)$$

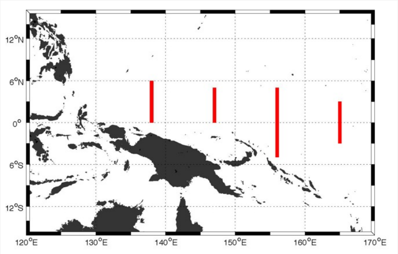
Note: if $f(Ri)=1$ then $\ell_v = \ell_o = \sqrt{(\varepsilon/N^3)}$, the Osmidov scale, and if $f(Ri)=Ri^{-3/2}$ (as $Ri \to 0$) then $\ell_v = \ell_C = \sqrt{(\varepsilon/S^3)}$, the Corrsin scale.

$$\kappa = \gamma \ell_v^2 N f(Ri)$$

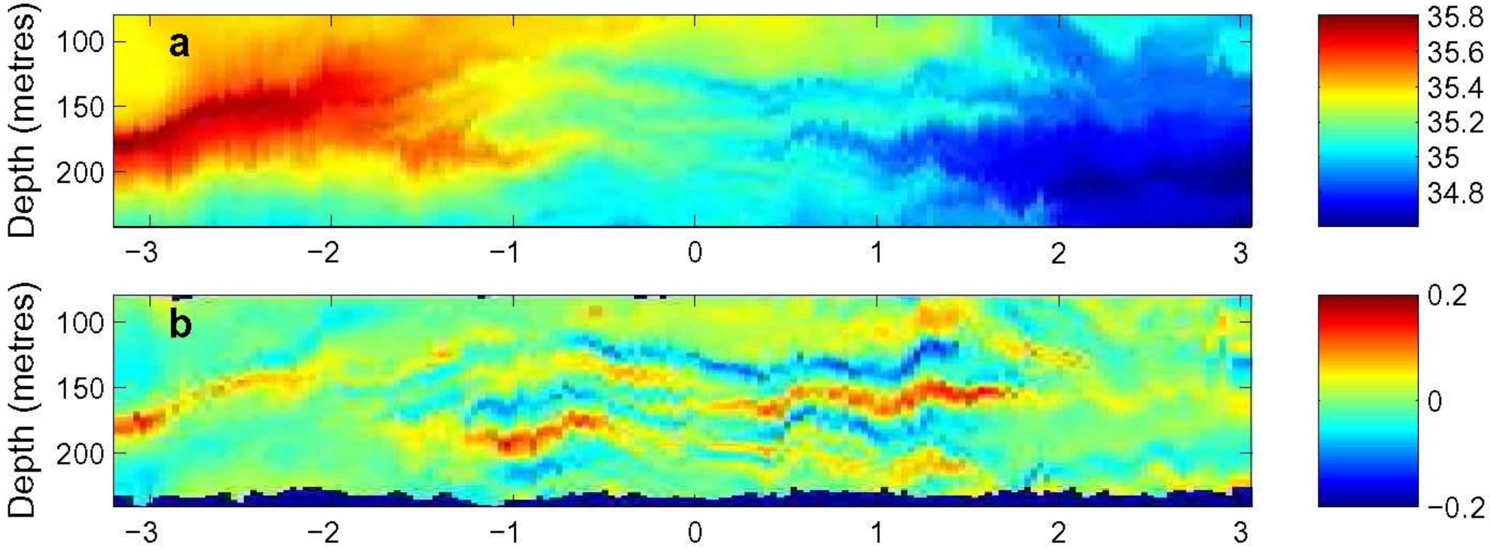
See Ijichi and Hibiya JPO(2018) for estimates of $\boldsymbol{\gamma}$

$$\kappa = \gamma \frac{\epsilon}{N^2}$$

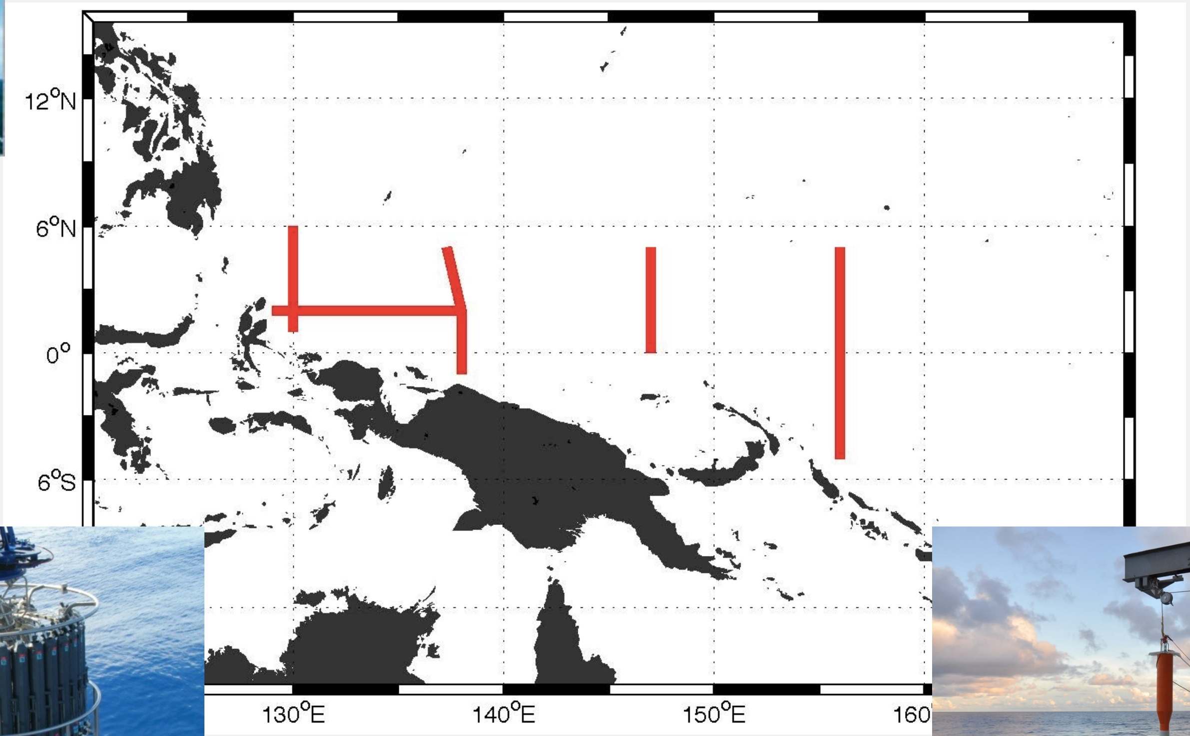
Osborn 1982

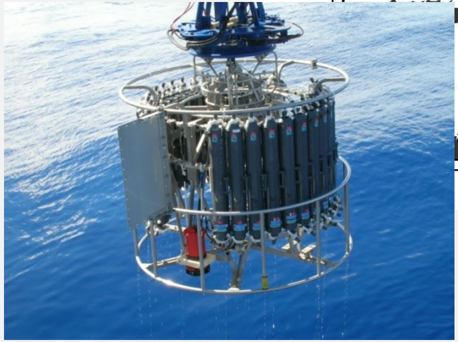



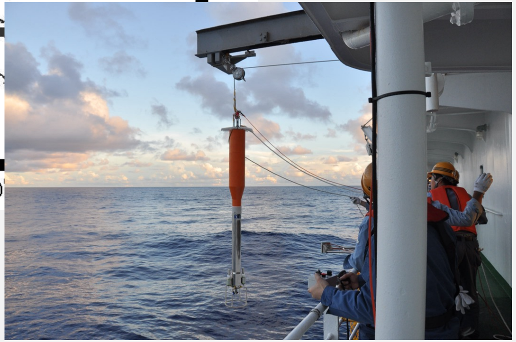

# Salinity along 165E: April 1988




TRITON

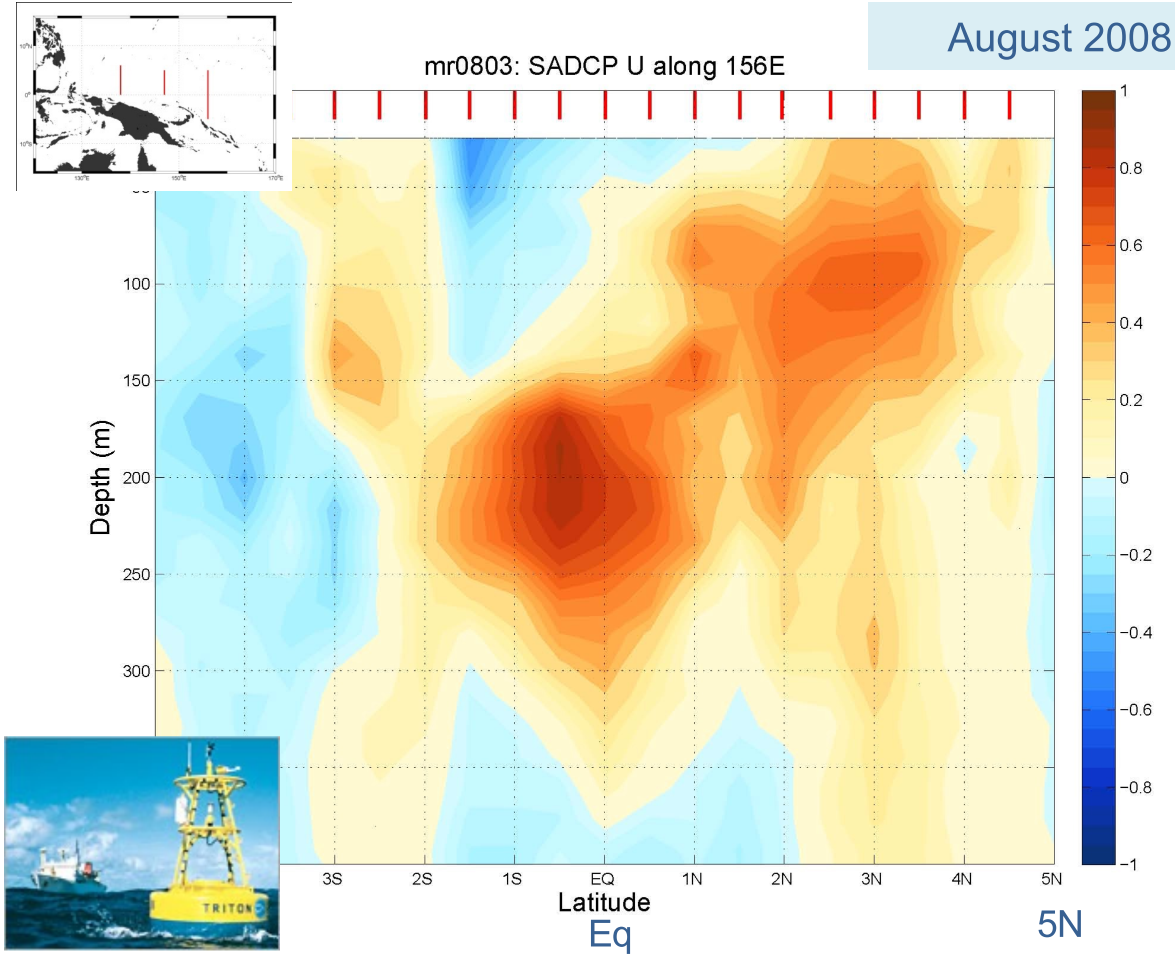

12°N
6°N
0°
6°S
130°E
140°E
150°E
160






https://www.pmel.noaa.gov/tao/drupal/disdel/

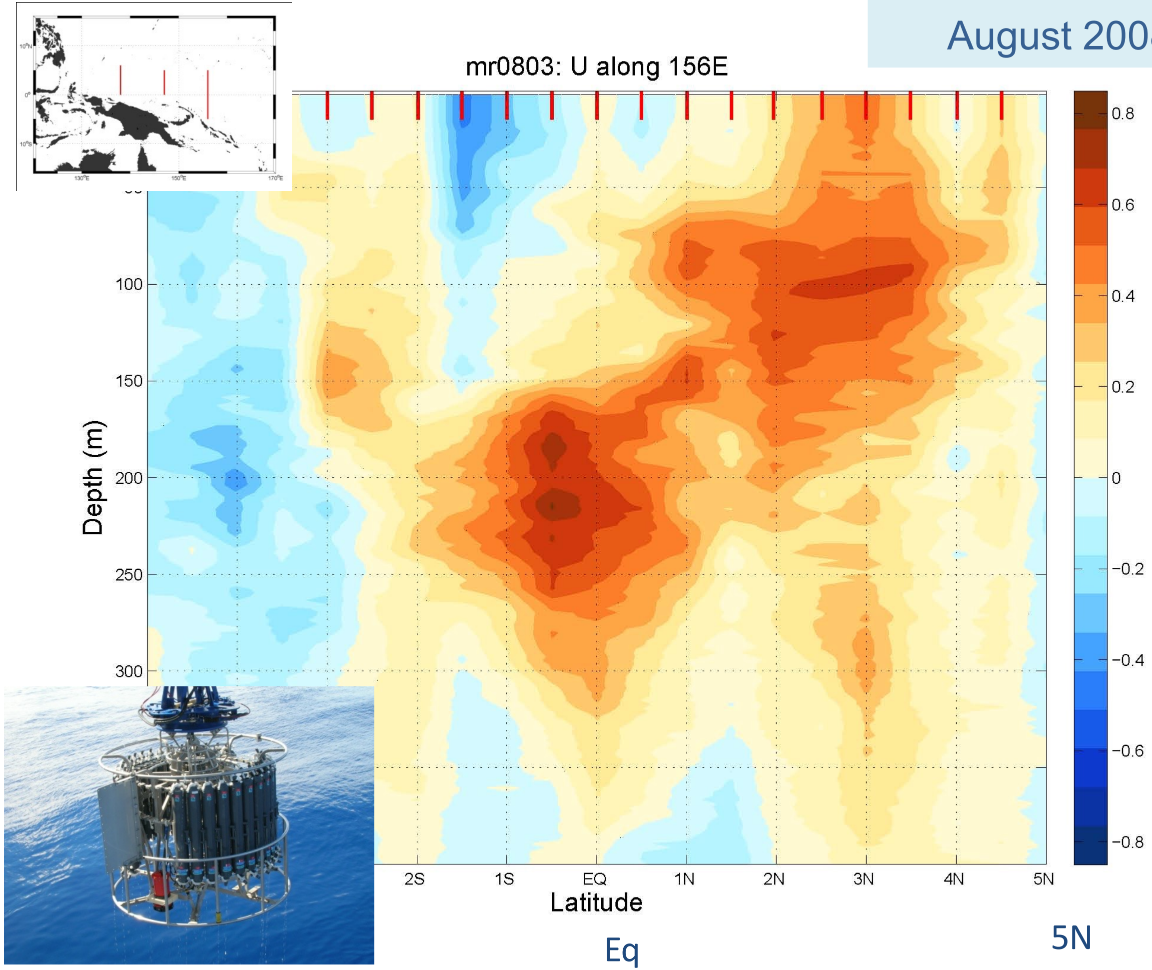
# August 2008




Eq

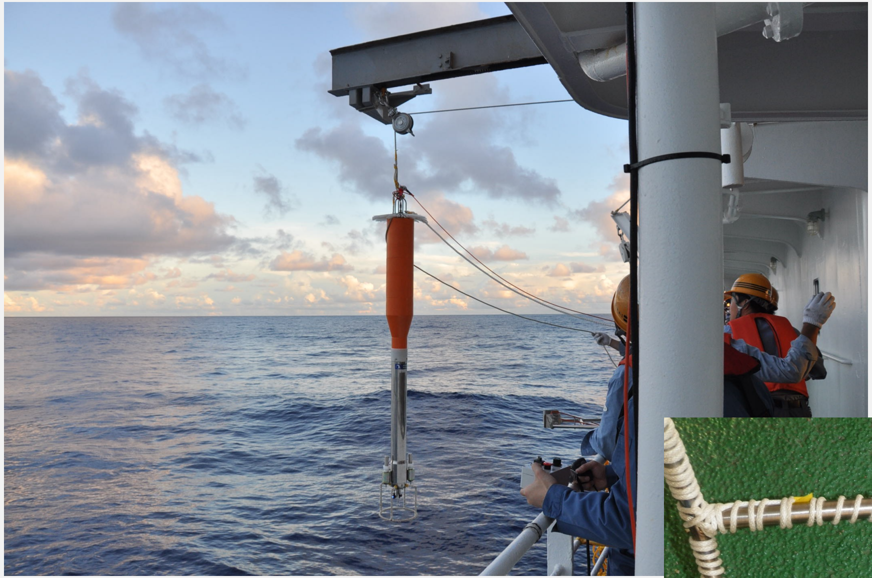
5N

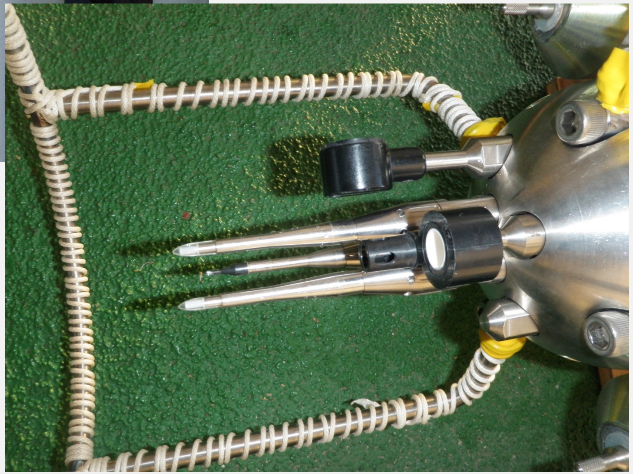
# August 2008




Eq

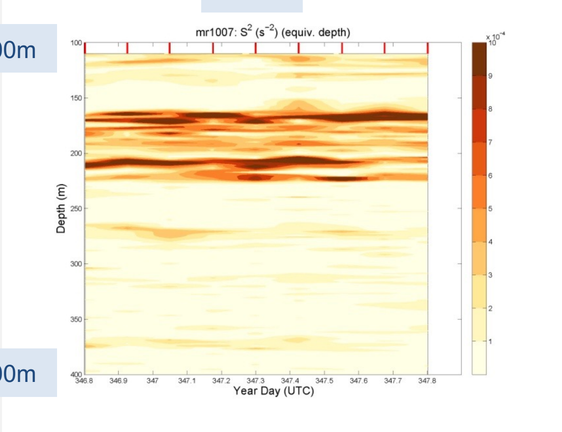
5N

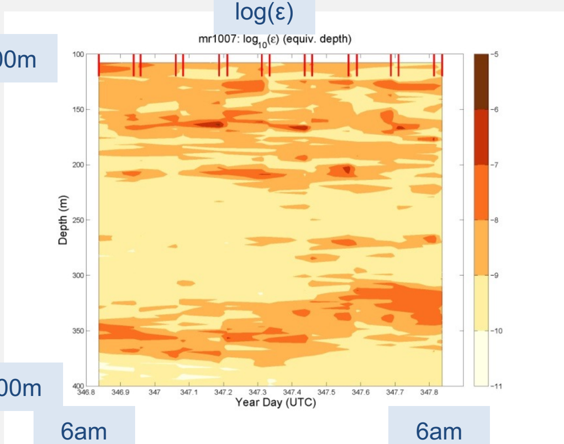


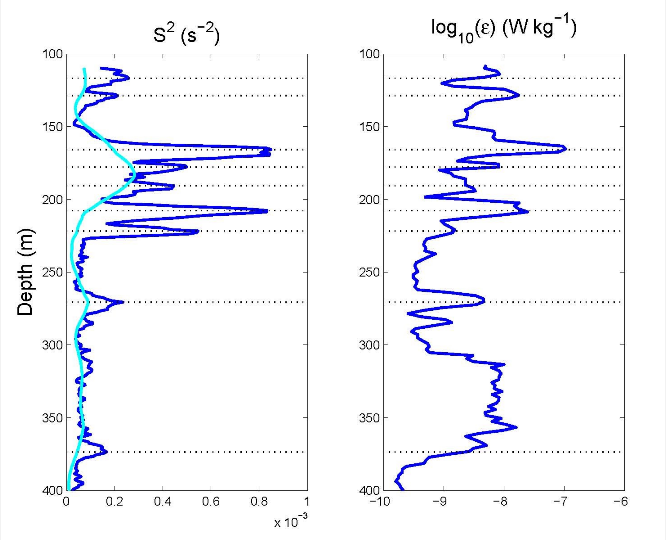


"Big whirls have little whirls that feed
on their velocity,
and little whirls have lesser whirls and
so on to viscosity"
Lewis Fry Richardson

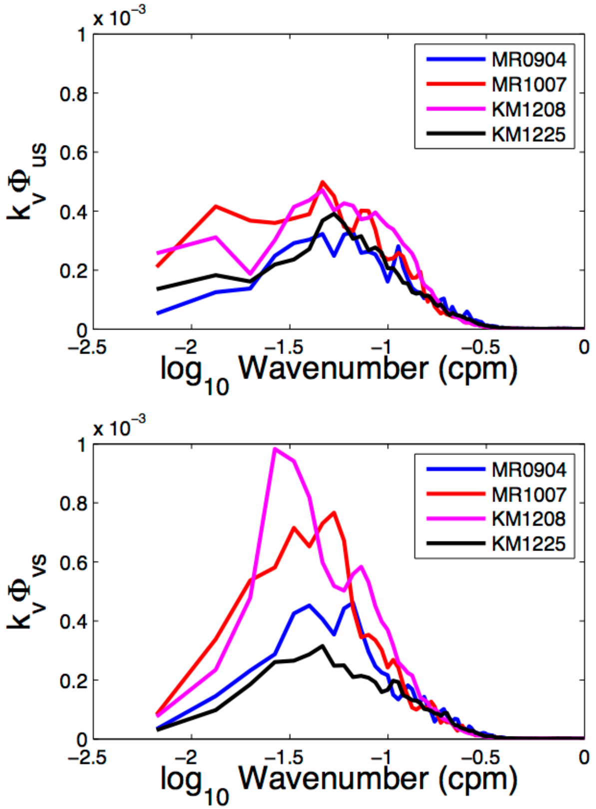
# Eq, 156E



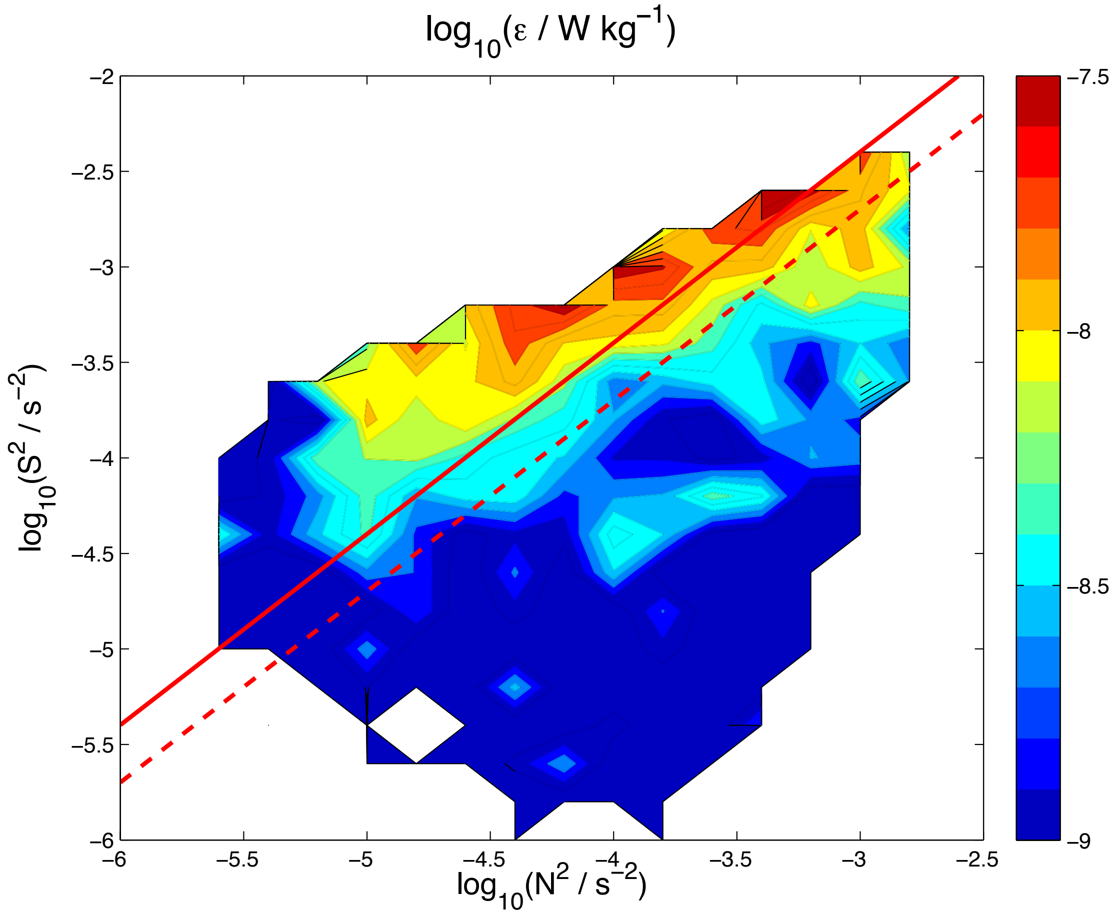




24hr time average

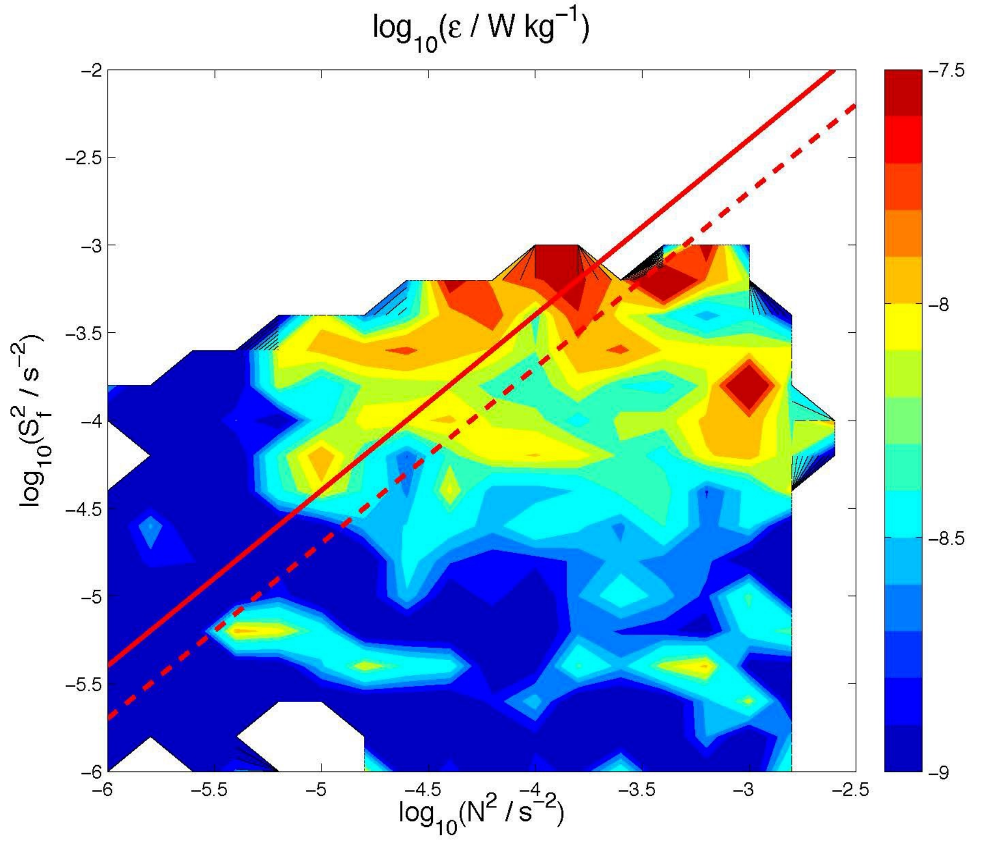


x 10^-3
MR0904
MR1007
KM1208
KM1225
k_v Φ_us
log10 Wavenumber (cpm)
x 10^-3
MR0904
MR1007
KM1208
KM1225
k_v Φ_vs
log10 Wavenumber (cpm)

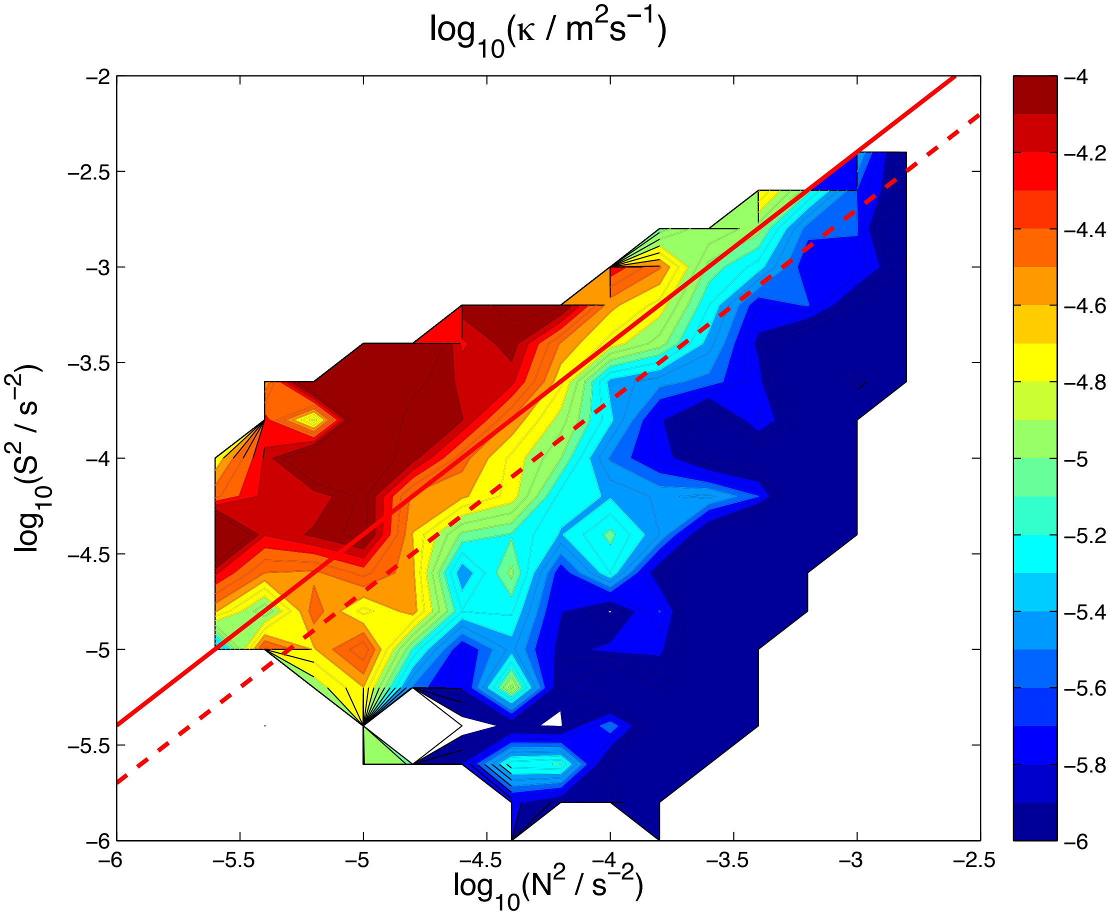


log10(ε / W kg−1)
log10(S2 / s−2)
log10(N2 / s−2)
−2
−2.5
−3
−3.5
−4
−4.5
−5
−5.5
−6
−5.5
−5
−4.5
−4
−3.5
−3
−2.5
−7.5
−8
−8.5
−9

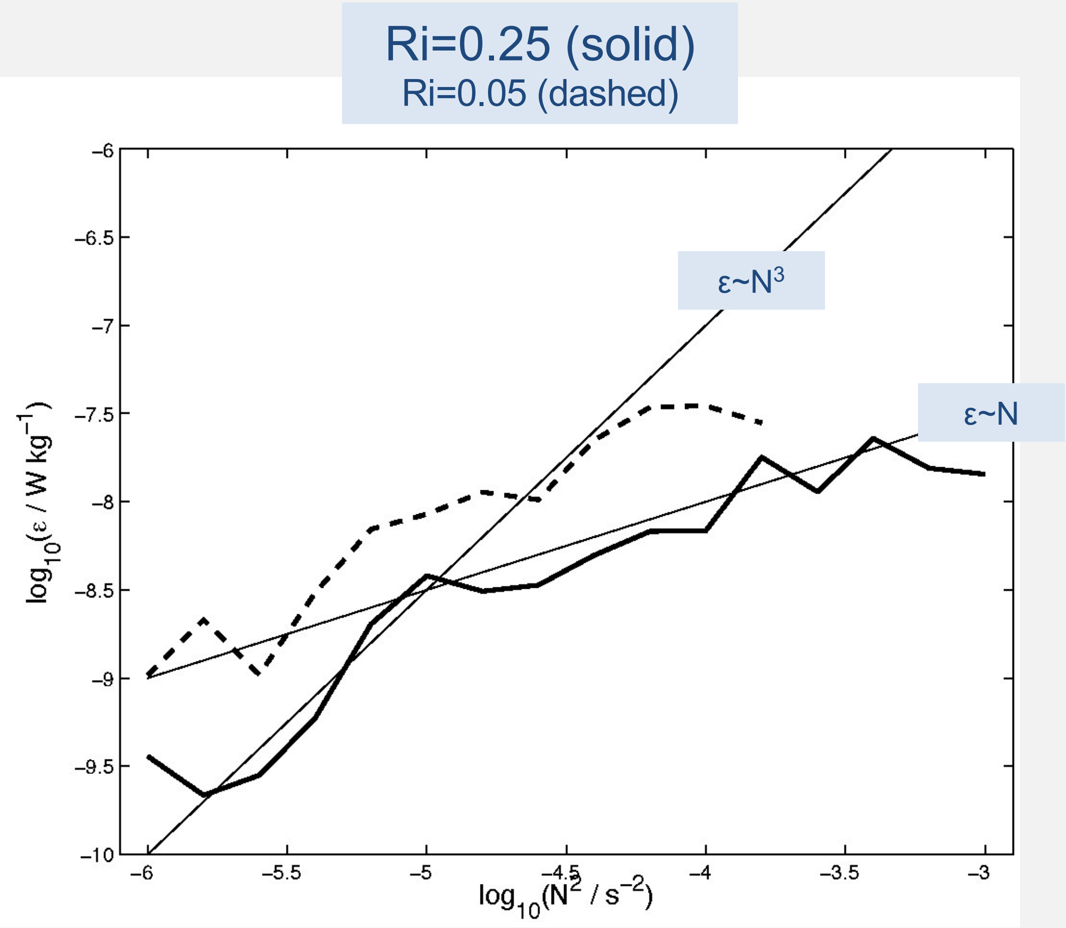
# Filtered



log10(ε / W kg−1)
−2
−2.5
−3
−3.5
−4
−4.5
−5
−5.5
−6
log10(S2f / s−2)
−6
−5.5
−5
−4.5
−4
−3.5
−3
−2.5
log10(N2 / s−2)
−7.5
−8
−8.5
−9




$$\kappa = \gamma \frac{\epsilon}{N^2}$$



Ri=0.25 (solid)
Ri=0.05 (dashed)
ε~N3
ε~N
log10(ε / W kg−1)
log10(N2 / s−2)
−6
−6.5
−7
−7.5
−8
−8.5
−9
−9.5
−10
−5.5
−5
−4.5
−4
−3.5
−3

The variation of ε ~N for constant Ri has implications for the scaling of the turbulence

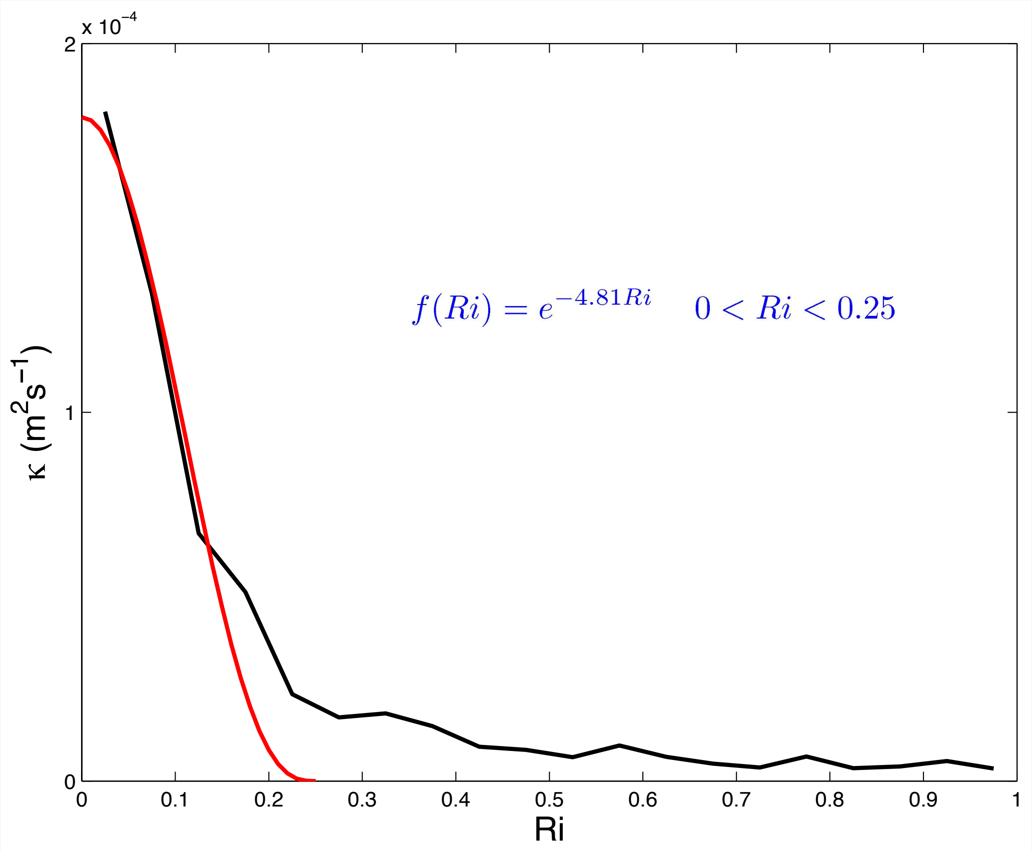
$$\epsilon = \ell_v^2 N^3 f(Ri)$$

then

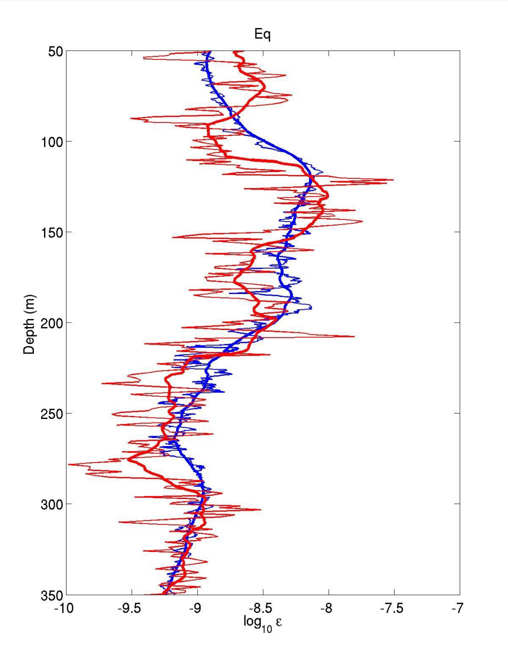
$$\ell_v = c\frac{u_t}{N}$$

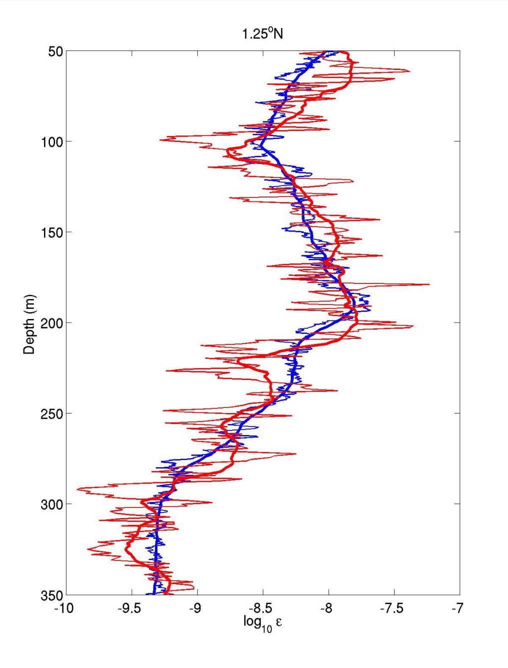
$$u_t \simeq 0.1\tilde{u}$$

$$\kappa = \frac{\gamma\epsilon}{N^2}$$


x 10^-4
2
1
0
κ (m^2s^-1)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Ri
$f(Ri) = e^{-4.81Ri} \quad 0 < Ri < 0.25$

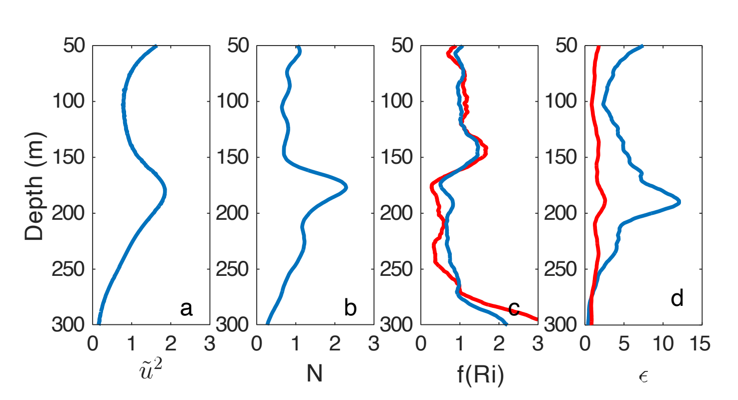
Eq, 156E




1N, 156E

Observed versus Estimated ε

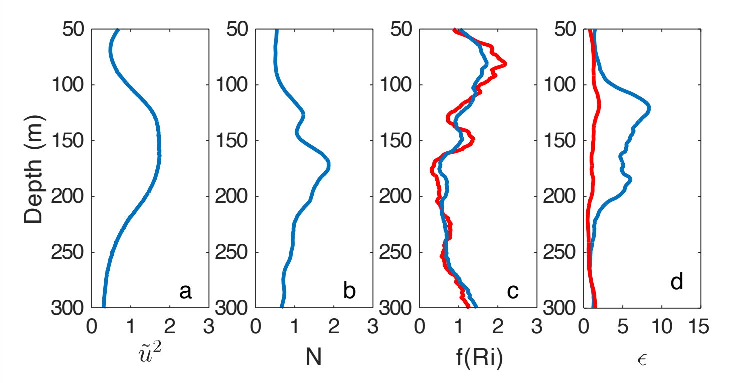


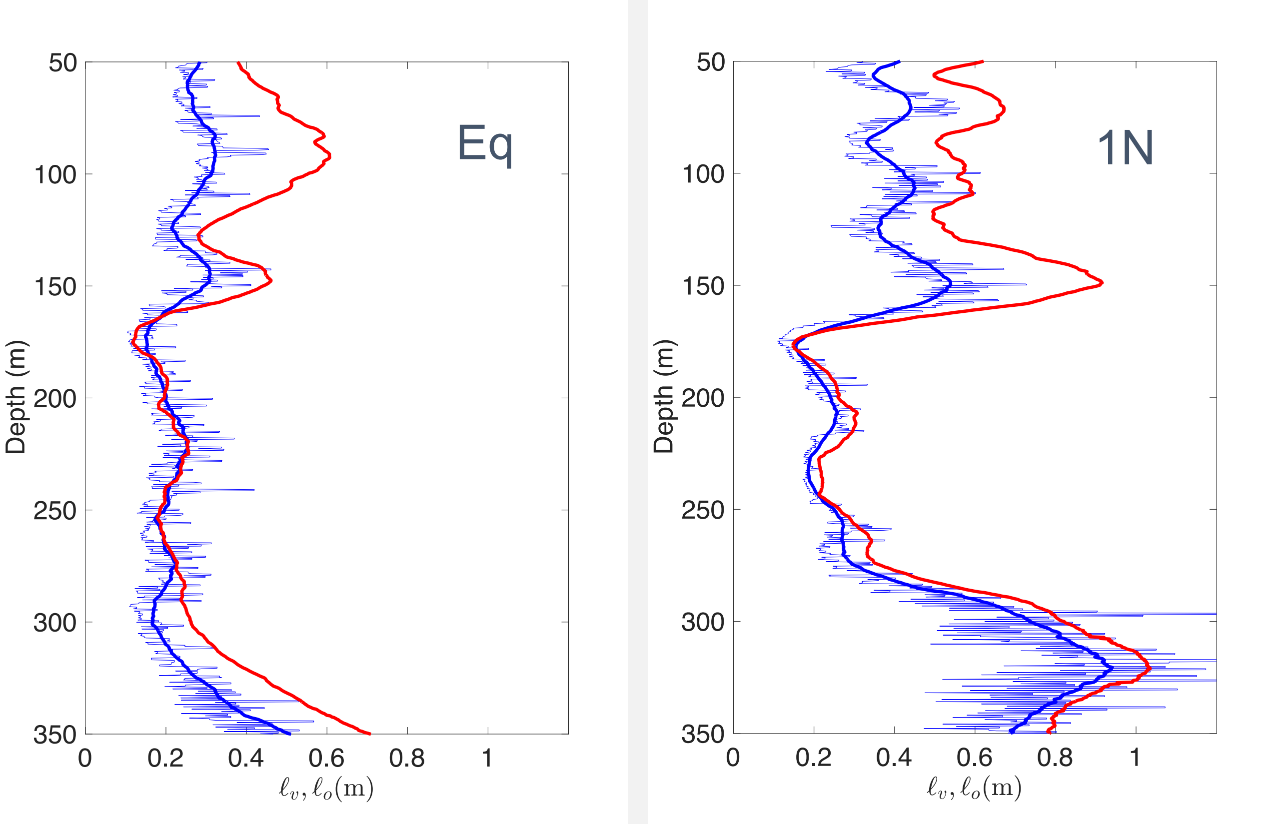

1N, 156E

# Deconstructed

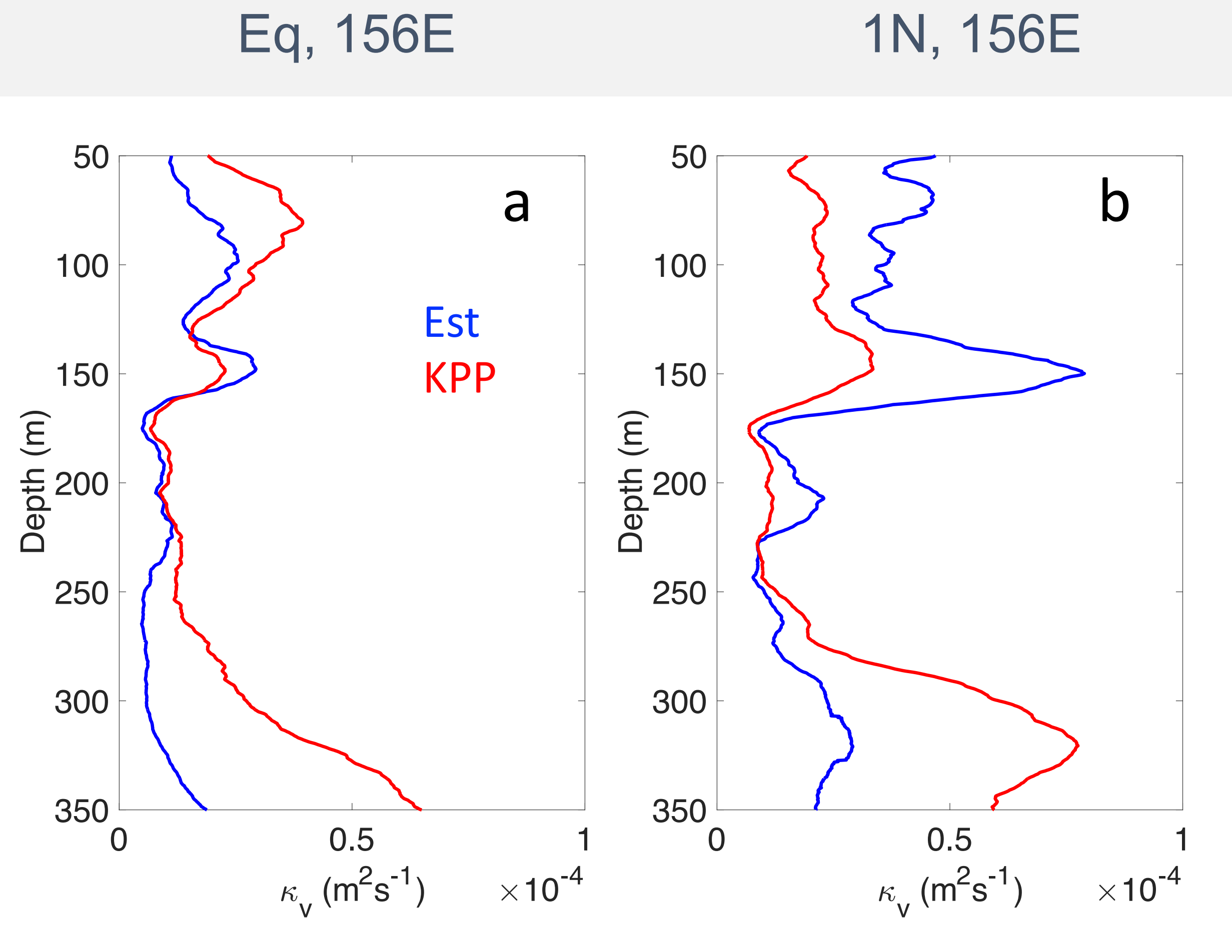
$$\epsilon \sim \tilde{u}^2 N f(Ri)$$

Eq, 156E






$$\ell_v = c\,0.1\,\frac{\tilde{u}}{N} \qquad \ell_o = \frac{\epsilon^{1/2}}{N^{3/2}}$$



Eq, 156E
1N, 156E
a
b
Est
KPP
Depth (m)
50
100
150
200
250
300
350
0
0.5
1
$\kappa_v$ ($m^2s^{-1}$)
$\times 10^{-4}$

- Important to resolve flow features generating turbulence
- Need to consider turbulent length scale
- What do you do if relevant scales are not resolved?

## Parameterization if $S^2$, $N^2$ **NOT** resolved

$$\kappa(\mathbf{x}, t) = \frac{\gamma}{N^2}\,\epsilon(S^2, N^2)$$

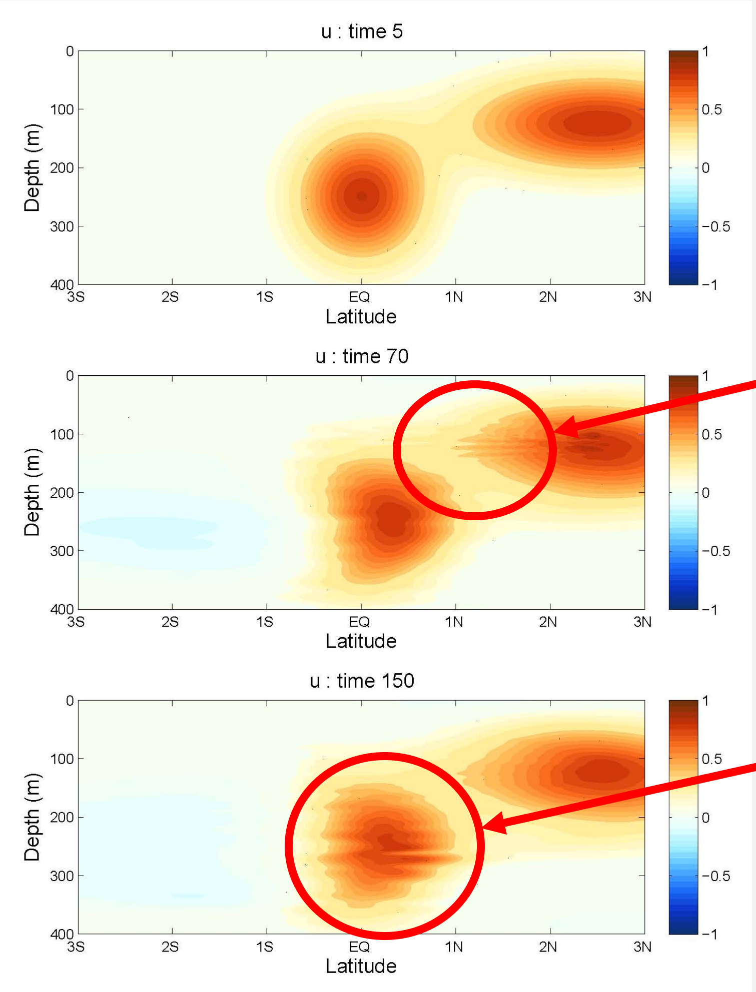
$$(S^2, N^2) \sim (\langle U \rangle, \langle N \rangle^2, F(x - x', t - t'), F_T \downarrow)$$

# Instabilities

Shear + Oscillatory flow = Inertial Instability and/or PSI

producing small vertical scales

Results from non linear model. Initial conditions: EUC and NECC with superimposed 20 cm $s^{-1}$ Yanai wave




Inertial instability

Day 70

PSI

Day 150

Model developed by Hidenori Aiki

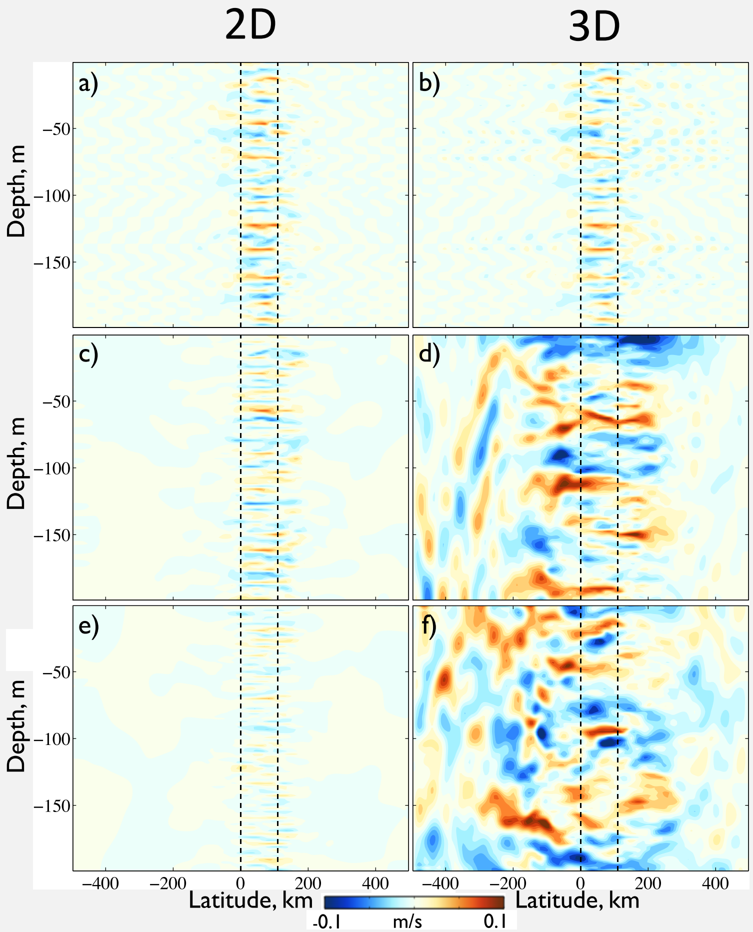



FIG. 2. Snapshots of the meridional component of velocity on days 40 (top), 240 (middle), and 480 (bottom row). Left panels: 2d; right panels: 3d at a given longitude. Vertical dashed lines mark the boundaries of the region of initial anomalous background vorticity.

Persistent presence of small vertical scale velocity features during three-dimensional equilibration of equatorial inertial instability

Natarov and Richards (2015)

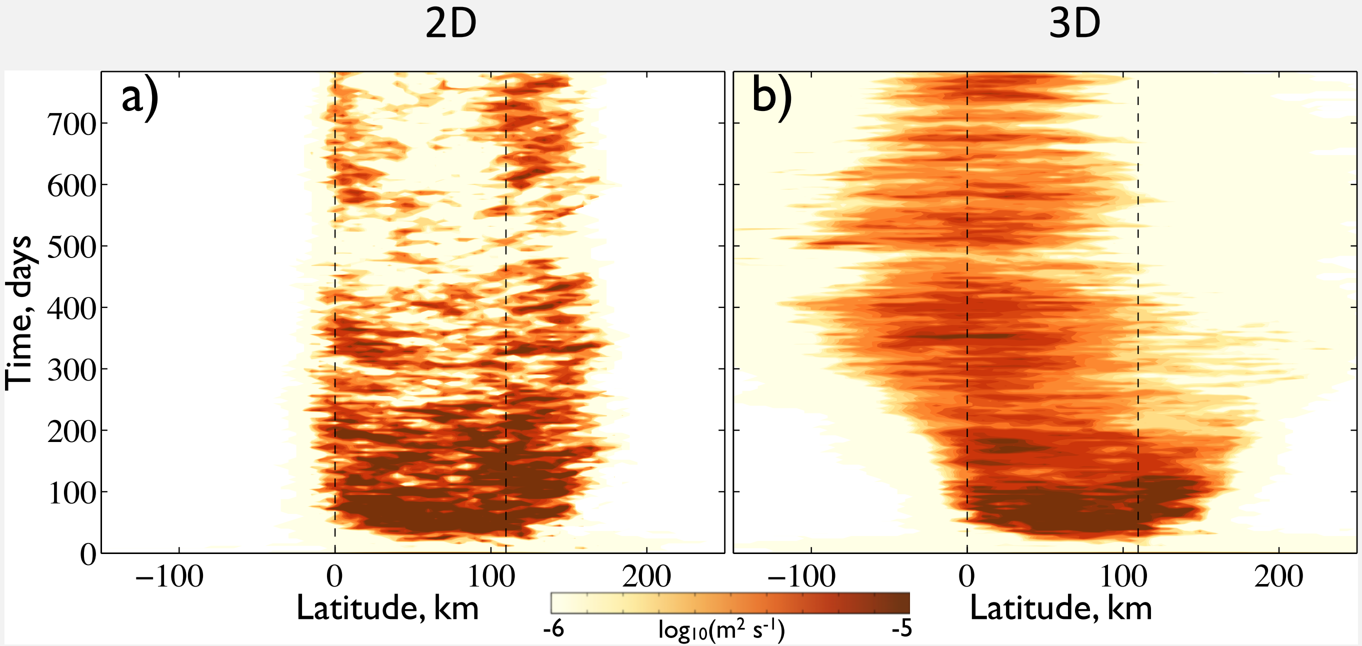


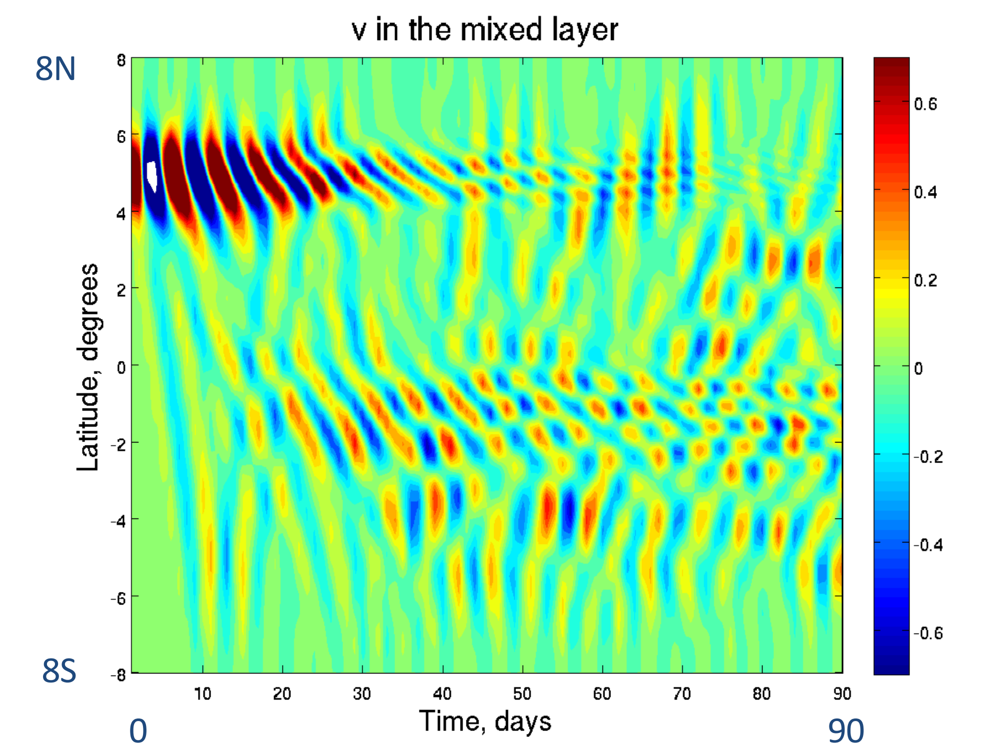

FIG. 10. Evolution of vertically and zonally averaged vertical eddy diffusion coefficient: (a) 2d, (b) 3d.

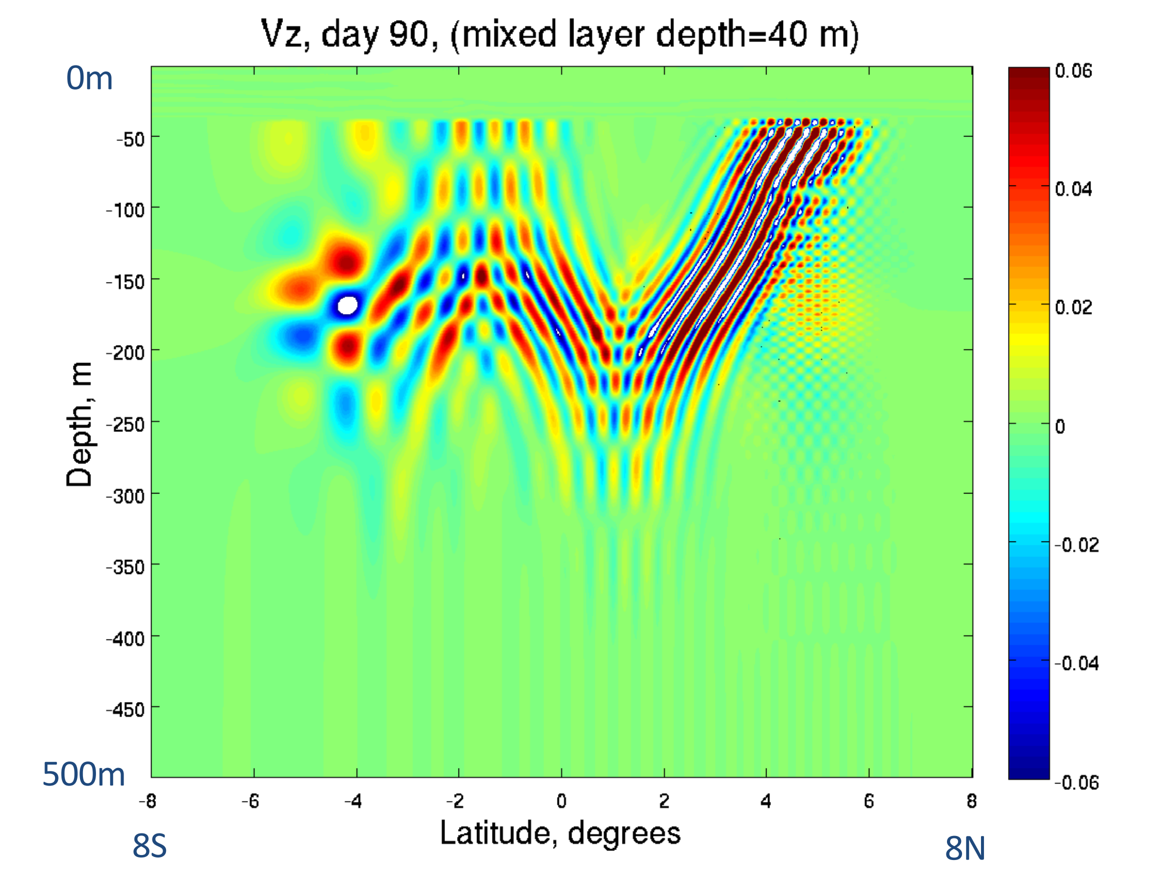
# Wind

Production of near-inertial oscillations

# Wind impulse applied to mixed layer at 5N

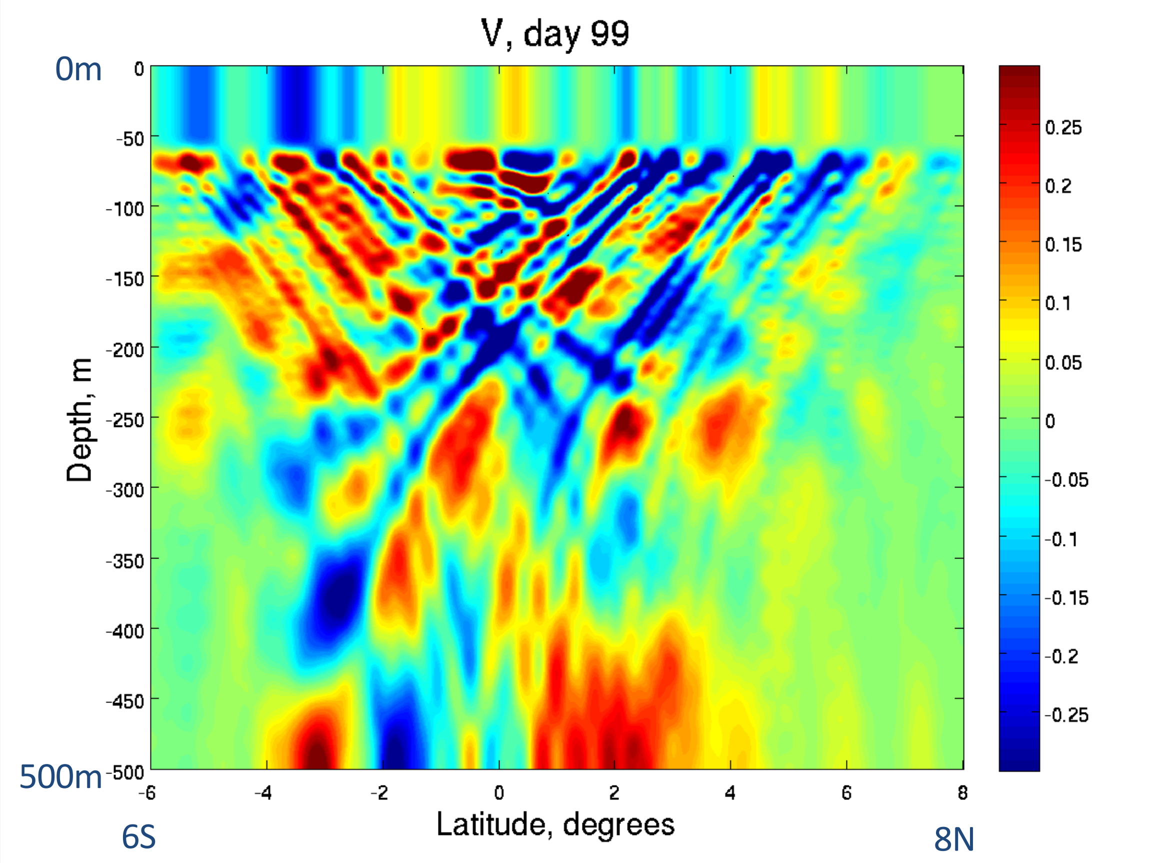
# (Linear continuously stratified model)





Vz, day 90, (mixed layer depth=40 m)
0m
500m
Depth, m
-50
-100
-150
-200
-250
-300
-350
-400
-450
-8
-6
-4
-2
0
2
4
6
8
Latitude, degrees
8S
8N
0.06
0.04
0.02
0
-0.02
-0.04
-0.06

# Linear model forced with QuikSCAT along 156E



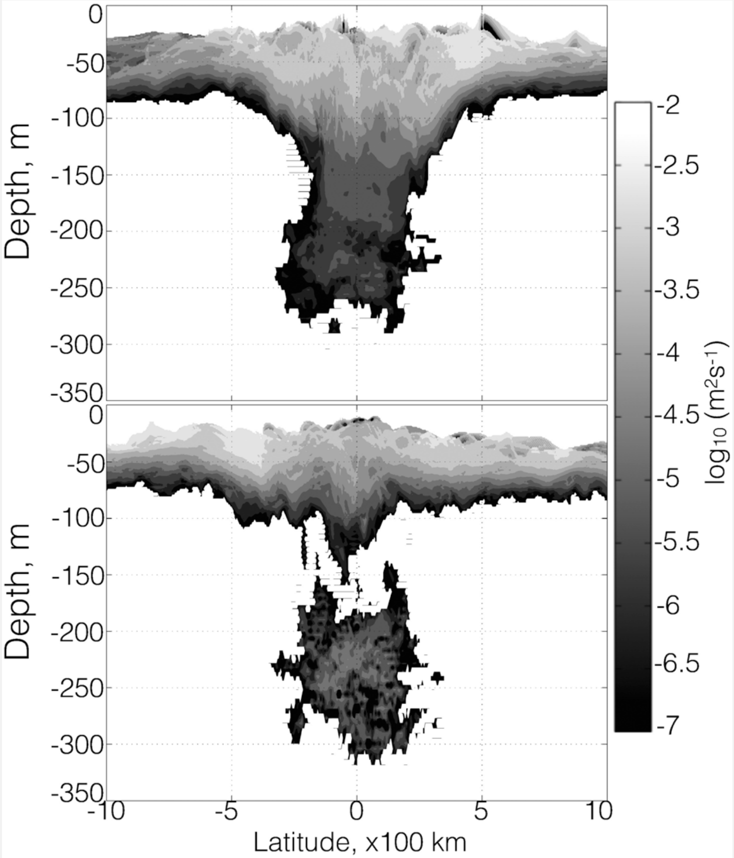



**Figure 4.** Base-10 logarithm of time-averaged vertical eddy viscosity coefficient below the mixed layer. (upper panel) Period I (equatorial winds). (lower panel) Period II (off-equatorial winds).

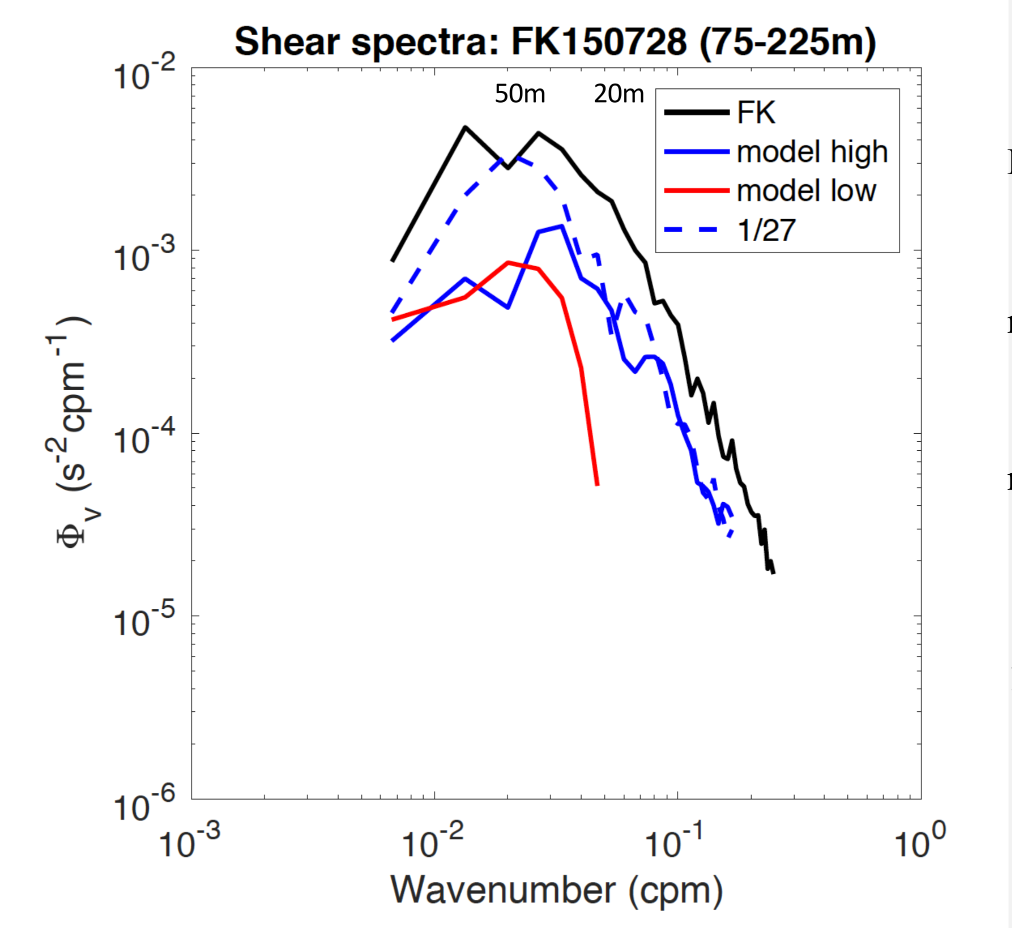
On-Eq winds

Off-Eq winds

Enhanced Energy Dissipation in the Equatorial Pycnocline by Wind-Induced Internal Wave Activity

Natarov and Richards 2019

Too hard? Well, try increasing ***vertical*** resolution



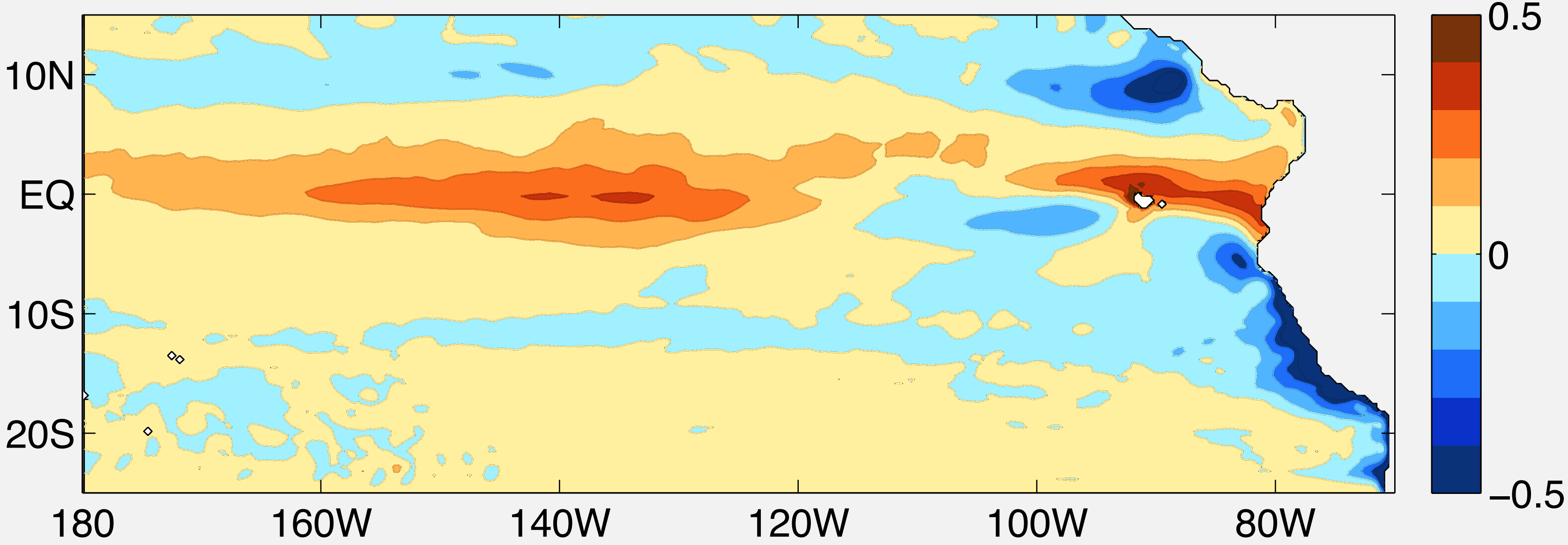

FK: R/V Falkor cruise,
August 2015 at 170ºW, 1ºN

model high: 1/3º horizonal,
3 m vertical (top 402 m)

model low: 1/3º horizonal,
5-37 m vertical (top ~400 m)
(16 m average)

1/27 (model): 1/27º horizonal,
3 m vertical (top 402 m)

# ΔSST




Model high – Model low