Kraus-Turner-Niiler Bulk Mixed Layers

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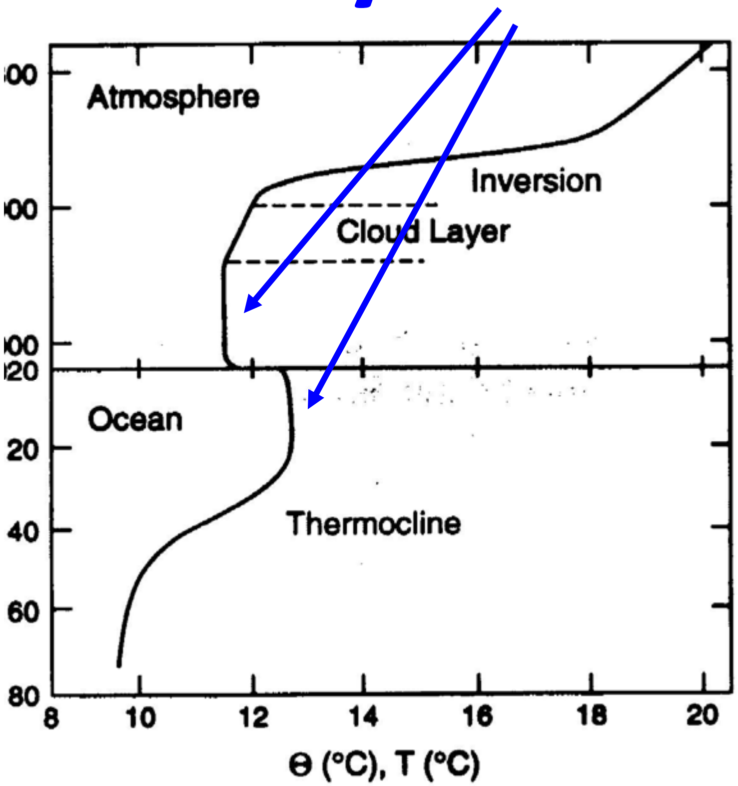
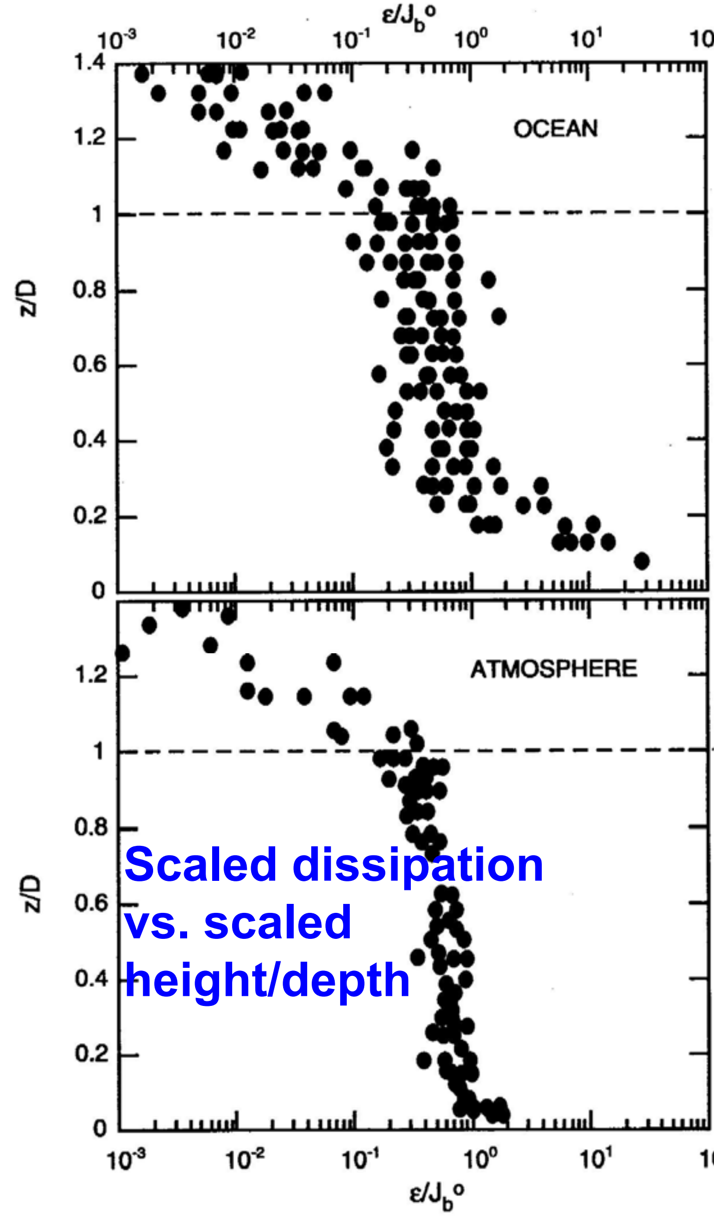
OCN/MET666

Fall 2012

More-or-less Well-mixed Layers



Kraus and Businger (1994, Ch. 6)



Scaled dissipation vs. scaled height/depth

Figure 2.2.2. Comparison of the ratio of the dissipation rate to the buoyancy flux as a function depth between the OML and the ABL under convective conditions (from Shay and Gregg, 1986).

J_bo is surface buoyancy flux

Bulk Mixed Layer Models

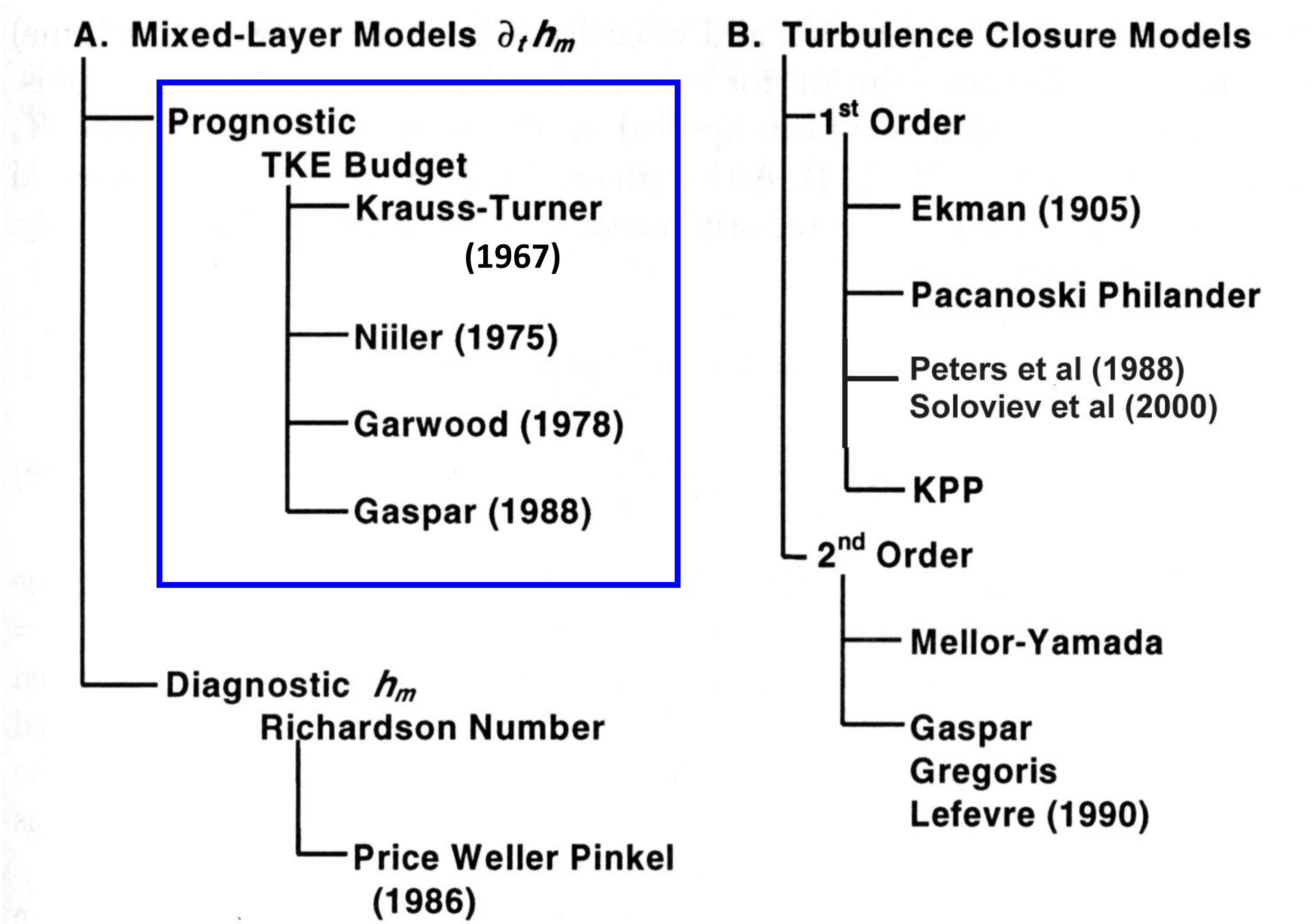


Figure 6. Classification of OBL mixing schemes into Mixed-Layer and Turbulence Closure models.

Basic Assumptions

- Horizontally homogenous (can be generalized)
- A vertically uniform layer exists
  - uniform scalar values
  - uniform shear (constant velocity "slab" layer sometimes assumed)
- Vertically-integrated budgets for scalars and momentum
- Layer depth is variable, determined by energy balance

A simple box model, with variable thickness

Caution: T≠ρ

Salt-stratified barrier layer

Lindstrom et al. (1987),

Godfrey and Lindstrom (1989),

Lukas and Lindstrom (1991),

Sprintall and Tomczak (1992)

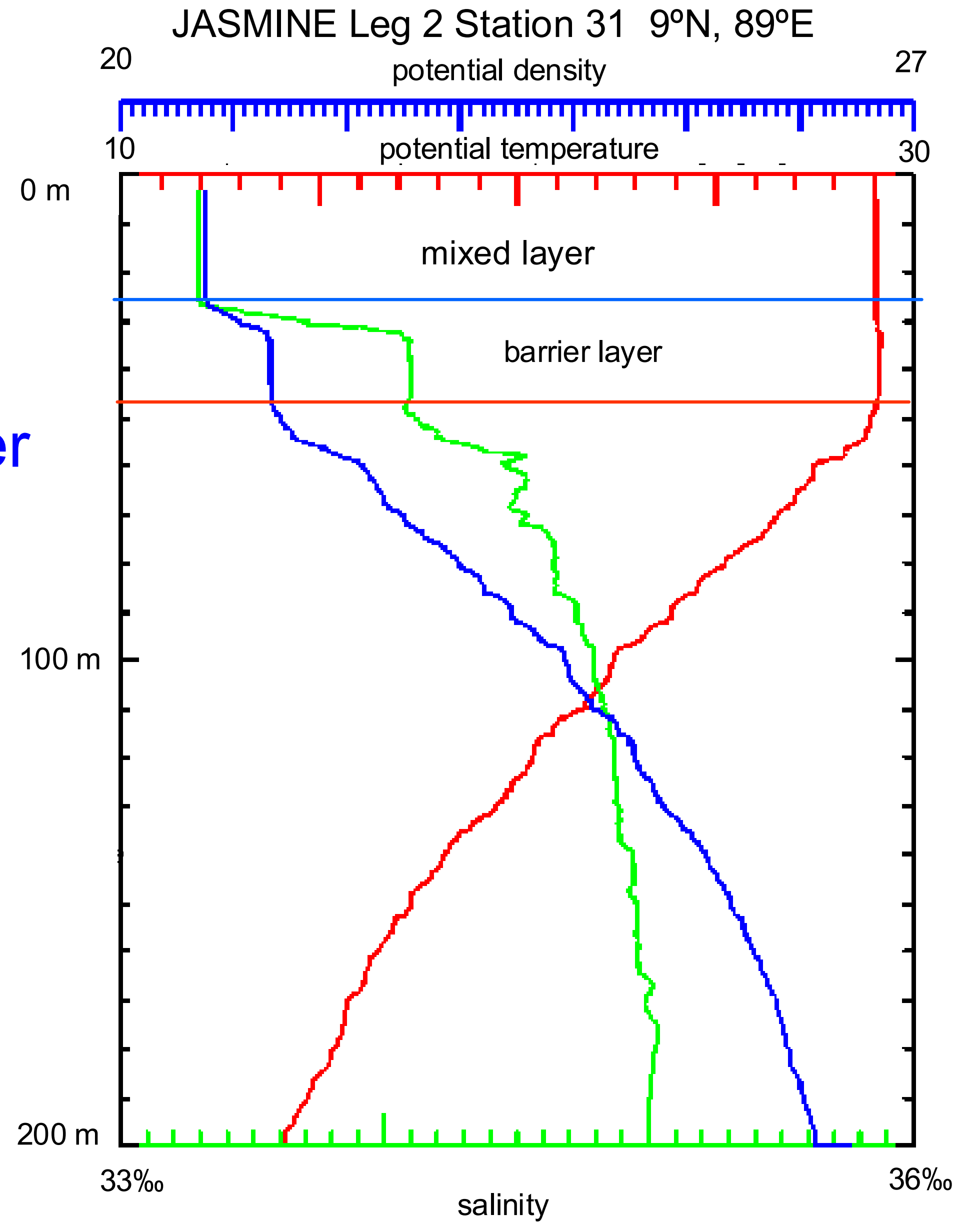
Ando and McPhaden (1997),

Vialard and Delecluse (1998),

Han (1999),

Masson et al. (2003),

and many more



Approach

- TKE budget for ML
- Potential Energy budget for ML
- ML thickness adjusts to balance these
- Assumptions
  - T, S, ρ, V are well-mixed
  - Gradients are discontinuous at ML bottom
  - turbulence is stationary
  - dissipation doesn't change temperature

Basic ML Equations

layer thickness

If horizontally homogeneous

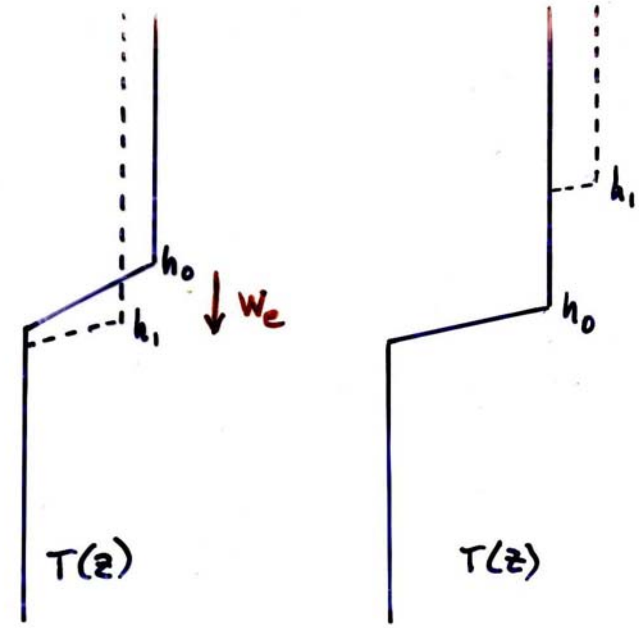
Overbar indicates layer average

$$\frac{\partial h}{\partial t} = -h\nabla_H \cdot \overline{\vec{V}} + w_e \equiv w_h + w_e$$

$\mathbf{w_e}$ is the turbulent entrainment velocity (recall entrainment occurs when "quiescent" region is incorporated into a turbulent region). Note that entrainment cannot be negative, though "detrainment" is sometimes used to label negative w_e.

$\mathbf{w_h}$ is the change in layer thickness due to 3-D processes such as Ekman pumping

ENTRAINMENT



Active entrainment
(deepening ML)

Shallowing ML
("detrainment")

$$W_e \equiv \frac{\partial h}{\partial t} \qquad \frac{\partial h}{\partial t} \geq 0$$

$$W_e \equiv 0 \qquad \frac{\partial h}{\partial t} < 0$$

W_e must be calculated from TKE equation

Entrainment is the expansion of turbulence into a nonturbulent region; the incorporation of formerly non-turbulent parcels of fluid into the region of active turbulence.

Basic ML Equations

buoyancy

Subscript r indicates reference state value

$$b = -g\,\frac{\rho - \rho_r}{\rho_r} \approx g\left[\alpha\left(T - T_r\right) - \beta\left(S - S_r\right)\right]$$

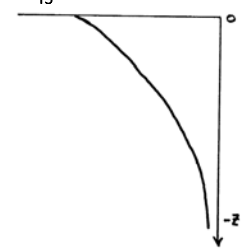
α and β are the thermal and haline expansion coefficients of expansion (used for linearizing the equation of state)

$$\alpha_r = -\frac{\partial\rho\left(T_r, S_r, 0\right)}{\partial T} \qquad \beta_r = \frac{\partial\rho\left(T_r, S_r, 0\right)}{\partial S}$$

Basic ML Equations

buoyancy

$$\frac{\partial b}{\partial t}+\frac{\partial \overline{w'b'}}{\partial z}=-\frac{g\alpha}{\rho_w c_p}\frac{\partial F_{is}}{\partial z}\equiv\frac{\partial S_b}{\partial z}$$



$F_{is}(z)$ is the penetrating shortwave radiation (sign convention positive upward; F_{is} is always negative). $\partial S_b/\partial z$ is the buoyancy source due to penetrating shortwave radiation

Integrating from the surface to the ML depth:

downward SW so $S_b(0)=(-)$

$$h\frac{\partial \overline{b}}{\partial t}=-\overline{w'b'}\Big|_0+\overline{w'b'}\Big|_h-S_b(0)=-B-w_e\Delta b$$

B typically ≥ 0

B is buoyancy flux from surface (sen. + LW + SW + LH). $S_b(0)$ is buoyancy flux at surface due to penetrating SW. Note that this formulation assumes all shortwave radiation is absorbed in the ML (not a good assumption in general for climate …)

Basic ML Equations

temperature and salinity

$$\frac{\partial T}{\partial t} + \frac{\partial \overline{w'T'}}{\partial z} = -\frac{1}{\rho c_p}\frac{\partial F_{is}}{\partial z}$$

$$\frac{\partial S}{\partial t} + \frac{\partial \overline{w'S'}}{\partial z} = 0$$

Integrating from the surface to the ML depth

$$h\frac{\partial \overline{T}}{\partial t} = -\overline{w'T'}\Big|_0 + \overline{w'T'}\Big|_h - F_{is}(0) \equiv -\frac{Q_n}{\rho_r c_p} - w_e \Delta T$$

$$h\frac{\partial \overline{S}}{\partial t} = -\overline{w'S'}\Big|_0 + \overline{w'S'}\Big|_h \equiv S_r(E-P) - w_e \Delta S$$

$S_r(E-P)$ is sometimes called the virtual salt flux, even though salt is not exchanged during evaporation and precipitation. (Salt is exchanged through spray production and evaporation.)

Again, note that this assumes all shortwave radiation is absorbed in the ML …

Basic ML Equations

momentum

$$\frac{\partial \vec{V}}{\partial t} + f\vec{k} \times \vec{V} + \frac{\partial \overline{w'\vec{V}'}}{\partial z} = 0$$

Assumes no horizontal gradients of velocity and pressure

$$\frac{\partial w}{\partial z} = -\nabla_H \cdot \vec{V}$$

$\vec{k}$ is unit vertical vector

Time-varying Ekman flow if

$$\frac{\partial \overline{w'\vec{V}'}}{\partial z} = A_V \frac{\partial^2 \vec{V}}{\partial z^2}$$

Basic ML Equations

momentum

Integrate vertically as for other variables, assuming w=0 at the sea surface

$$\frac{\partial \overline{\vec{V}}}{\partial t} + f\vec{k} \times \overline{\vec{V}} = -\overline{w'\vec{V}'}\Big|_0 + \overline{w'\vec{V}'}\Big|_h$$

$$w_h = -\nabla_H \cdot \overline{\vec{V}}$$

Basic ML Equations
surface fluxes – heat and freshwater

positive fluxes are upward

$$Q_n = F_{is} + F_{nlw} + Q_H + Q_E \equiv \rho_r c_p \overline{w'T'}\big|_0 + F_{is}$$

$$F_{fw} = E - P = \frac{1}{\rho_r L_v} Q_E - P \equiv \frac{1}{S_r} \overline{w'S'}\big|_0$$

Q_n is the net surface heat flux
Q_H is the sensible heat flux
Q_E is the latent heat flux
F_{is} is the penetrating shortwave radiation
F_{nlw} is the net longwave radiation (usually outgoing)
F_{fw} is the net freshwater flux
E is the evaporation from the surface (proportional to latent heat flux)
P is the rainfall rate
L_v is latent heat of vaporization

Basic ML Equations

surface fluxes – buoyancy

positive fluxes are upward

$$B = \overline{w'b'}\Big|_0 + S_b(0) = \frac{g}{\rho}\left[\frac{\alpha}{c_p}Q_n + \beta S_r(E-P)\right]$$

$$h\frac{\partial \overline{b}}{\partial t} = -B - w_e \Delta b$$

Basic ML Equations
surface fluxes – momentum

$$\frac{\tau}{\rho_r} = \overline{w'\vec{V}'}\Big|_0 = u_{*\,w}^2 \cong \frac{\rho_a}{\rho_r} C_d U_{10}^2$$

Note friction velocity in water much smaller than in air, while τ is continuous across the interface.

Basic ML Equations

entrainment fluxes

heat

$$\overline{w'T'}\Big|_h = -w_e\left(T - T_{h^+}\right) \equiv -w_e \Delta T$$

salt

$$\overline{w'S'}\Big|_h = -w_e\left(S - S_{h^+}\right) \equiv -w_e \Delta S$$

buoyancy

$$\overline{w'b'}\Big|_h = -w_e\left(b - b_{h^+}\right) \equiv -w_e \Delta b$$

Basic ML Equations

entrainment fluxes

momentum flux

$$\left.\overline{w'\vec{V}'}\right|_h = -w_e \Delta\overline{\vec{V}} + C\overline{\vec{V}}\left|\overline{\vec{V}}\right|$$

C – drag coefficient for ML moving over ocean interior

Basic ML Equations

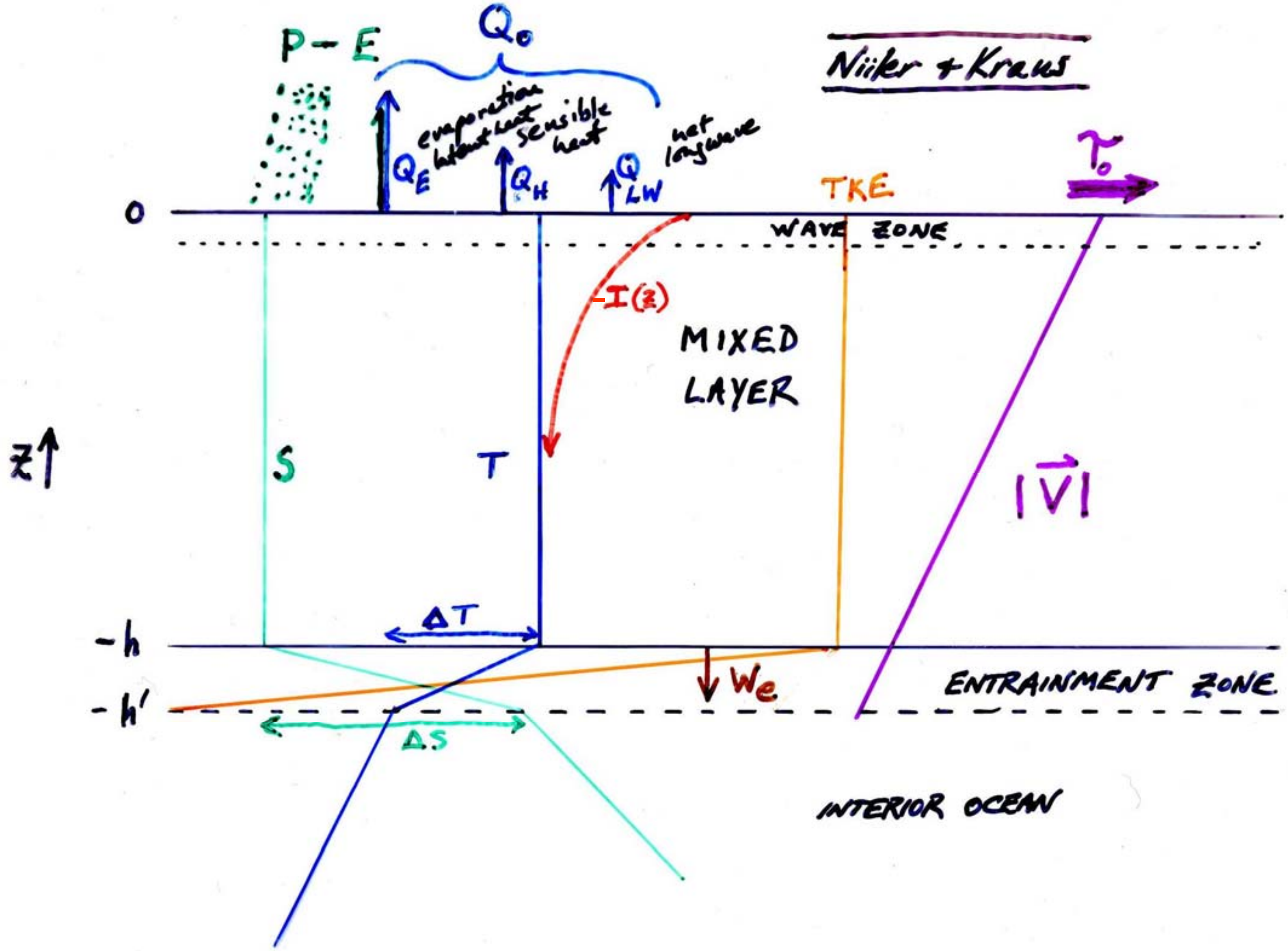
layer evolution

$$\frac{\partial \overline{T}}{\partial t} = -\frac{1}{h}\left[\frac{Q_n}{\rho_r c_p} + w_e \Delta T\right]$$

$$\frac{\partial \overline{S}}{\partial t} = \frac{1}{h}\left[S_r\left(E - P\right) - w_e \Delta S\right]$$

Note that the rate of change of T and S is proportional to 1/h. Shallow ML is more sensitive to surface and entrainment fluxes!

Schematic ML Model



Variables

u, v – horizontal momentum

T, S → b

h

We have equations for the variables above. But, we don't yet have an equation for

w_e – entrainment velocity

This will depend on the budget for TKE=½q^2

Niiler and Kraus (1977)

R_0 – net surface flux of radiant energy (per unit area)
B_0 – net surface buoyancy flux

I_0 – surface flux of penetrating shortwave radiation
J_0 – surface buoyancy flux due to penetrating radiation

$$I(z) = I_0 e^{z/\gamma}$$

$$\frac{\partial I(z)}{\partial z} = \frac{I_0}{\gamma} e^{z/\gamma}$$

$$J_0 = \frac{g\alpha}{\rho_r c_p} I_0 = S_b(0)$$

R_0-I_0 – net longwave radiation at sea surface

Note that positive values are upward fluxes

T_0, S_0 are ML averages

Basic ML Equations

TKE

$$\frac{\partial TKE}{\partial t} = SP + BP + Tr + Pr - D$$

$$\frac{1}{2}\frac{\partial q^2}{\partial t} = -\overline{w'\vec{V}'}\frac{\partial \overline{\vec{V}}}{\partial z} + \overline{w'b'} - \frac{1}{2}\frac{\partial}{\partial z}\left[\overline{w'\left(w'^2 + \vec{V}'^2\right)} + \frac{\overline{w'p'}}{\rho}\right] - \varepsilon$$

triple products

Set = 0

Reynolds Stress

$$\frac{\partial \overline{w'\vec{V}'}}{\partial t} = -\overline{w'^2}\frac{\partial \overline{\vec{V}}}{\partial z} - \frac{\partial \overline{w'^2\vec{V}'}}{\partial z} - \left(\overline{\vec{V}'\frac{\partial p'}{\partial z}} + \overline{w'\nabla_H p'}\right)$$

Integral TKE Budget

Integrating from the bottom of the ML to the surface

$$\frac{\partial}{\partial t}\int \bar{e}\,dz = -\int \overline{w'\vec{V}'}\frac{\partial \overline{\vec{V}}}{\partial z}dz + \frac{1}{\rho_r}\int \overline{w'b'}dz - \frac{1}{\rho_r}\overline{w'p'}\Big|_h - \int \varepsilon dz$$

Note that the turbulent transport term disappears because no transport into atmosphere or below mixed layer.

The second to last term represents the radiation of internal gravity waves from the base of the mixed layer, and we'll neglect this as it is small compared to dissipation.

We'll also neglect the storage term, assuming that the turbulence is nearly stationary.

$$0 = SP + BP - D$$

Basic ML Equations

TKE boundary conditions

BC: wind stress = rate of work done by wind stress

$$\left[\frac{1}{2}\overline{w'\left(w'^2+\vec{V}'^2\right)}+\frac{\overline{w'p'}}{\rho}\right]_0 = m_1 u_*$$

BC: mechanical energy flux needed to energize non-turbulent fluid entrained into the ML

$$\left[\frac{1}{2}\overline{w'\left(w'^2+\vec{V}'^2\right)}+\frac{\overline{w'p'}}{\rho}\right]_h = \frac{1}{2} w_e q^2$$

Potential Energy

h → h + Δh $\Delta PE = h\Delta b\Delta h/2$

$\Delta PE/\Delta t = h\Delta b\Delta h/2\Delta t = w_e h\Delta b/2$

In the entrainment zone

$$\frac{\partial PE}{\partial t} = \frac{\partial TKE_w}{\partial t} + \frac{\partial TKE_b}{\partial t}$$

$$w_e h\Delta b \big/ 2 = mu_*^3 + \frac{h}{2}\left[\frac{B - |B|}{2} + n\frac{B + |B|}{2}\right]$$

TKE

Wind-driven mixing

$$\frac{\partial TKE}{\partial t} = m u_*^3$$

Buoyancy-driven mixing (only for B>0)

$$\overline{w'b'} = B\left(1 + z/h\right)$$

Linear decay in ML →

$$\int_{-h}^{0} \overline{w'b'}\, dz = hB/2$$

Some fraction, (1-n), is lost to dissipation in the ML, so only n is left for production of TKE at base of ML and entrainment. n=1 is the limiting case for convective plumes reaching the ML bottom without attenuation.

Basic ML Equations
shear production

Assuming SP is occurring only at the surface and at the base of the ML

$$SP = \overline{u'w'}\frac{\partial \overline{\vec{V}}}{\partial z} = SP\big|_0 - SP\big|_h \approx m u_*^3 - \frac{1}{2} w_e \left(\Delta \vec{V}\right)^2$$

Basic ML Equations

turbulent buoyancy flux

$$\overline{w'b'} = B_0 + \frac{z}{h}\left(B_0 + w_e \Delta b + \frac{g\alpha}{\rho c_p} I_0 \right) + \frac{g\alpha}{\rho c_p} I_0 \left(1 - e^{z/\gamma} \right)$$

z=0, → B; z=-h, → $-w_e\Delta b-(g\alpha/\rho c_p)e^{-\gamma h}$

INTEGRATED TKE

$$\underset{A}{\tfrac{1}{2} W_e (g^2} + \underset{B}{c_i^2} - \underset{C}{\vec{V}^2}) = \underset{D}{m_1 u^{*3}} + \underset{E}{\tfrac{1}{2} h B_0} + \underset{F}{\left(\tfrac{h}{2} - \tfrac{1}{\gamma}\right) J_0} - \underset{G}{C_d |\vec{V}|^3} - \underset{H}{\int_{-h}^{0} \epsilon \, dz}$$

$$c_i^2 = \Delta b h \qquad J_0 = \frac{g \alpha}{\rho c_p} I_0$$

~~A~~ rate of energy used to make entrained parcels turbulent

B rate of work done in lifting and mixing parcels

C rate of change of mean kinetic energy of ML by mixing at base of ML

D rate of wind work

E rate of change of potential energy due to air-sea fluxes

F rate of change of potential energy due to penetrating radiation

~~G~~ rate of work done by internal wave radiation stress

H rate of turbulent dissipation

Basic ML Equations
dissipation

From dimensional analysis, $\varepsilon = f(u_*^3)$

Niiler and Kraus (1977) set

$$\overline{D} = \int_{-h}^{0} \varepsilon dz = (m_1 - m) u_*^3 + \frac{(1-s)}{2} w_e \overline{\vec{V}}^2 + \frac{(1-n)h}{2} \frac{B_0 + |B_0|}{2}$$

Non-zero only when buoyancy flux destabilizes surface

Garwood (1977) argued for

$$\varepsilon = f\left(\frac{u_*^3}{h}\right)$$

B C D E F

$$W_e\left(c_i^2 - s\,\vec{\bar{V}}^2\right) = 2\,m\,u^{*3} + \frac{h}{2}\left[(1+n)B_0 - (1-n)|B_0|\right] + \left(h - \frac{2}{8}\right)J_0$$

dissipation of convectively generated turbulence

$n=0$ local dissipation

$n=1$ no local dissipation

Niiler and Kraus (1977)

radiative heating with no wind
calm, sunny day (like nocturnal PBL)

No wind → $u_* = 0$;
Q_H, Q_E=0? If no free convection/gustiness …

a) $R_0 - I_0 < 0$ → $B_0 < 0$ with $u_* = 0$ ➔ $h=0$

$R_0 = I_0$ + net LW

b) $R_0 - I_0 > 0$ (surface heat loss with bulk warming)

$$h = \frac{2}{\gamma}\frac{J_0}{nB_0 + J_0} \approx \frac{2}{\gamma}\frac{I_0}{n(R_0 - I_0) + I_0}$$

n=1 (nonlocal dissipation)

$$h = \frac{2}{\gamma}\frac{I_0}{R_0}$$

n=0 (local dissipation)

$$h = \frac{2}{\gamma}$$

R_0 – net surface flux of radiant energy (per unit area)
B_0 – net surface buoyancy flux

I_0 – surface flux of penetrating shortwave radiation
J_0 – surface buoyancy flux due to penetrating radiation

Note that positive values are upward fluxes

Niiler and Kraus (1977)

radiative heating with no wind
calm, sunny day (like nocturnal PBL)

No wind → $u_*=0$;
Q_H, Q_E=0? If no free convection/gustiness …

$R_0=I_0$ + net LW < 0

$$\frac{\partial T_0}{\partial t} = -\frac{R_0\gamma}{\rho c_p}\frac{n(R_0 - I_0) + I_0}{2I_0} = -\frac{R_0}{\rho c_p h}$$

outgoing LW=f(T_0)! → R_0-I_0↑ → negative feedback

I_0 will change also – more convection for higher T_0 →I_0↑ (less shortwave)

Niiler and Kraus (1977)

radiative cooling with no wind

calm nighttime (like daytime convective PBL)

I_0=0, R_0 = net LW (+)

$$w_e c_i^2 = \frac{h}{2}\left[(1+n)B_0 - (1-n)|B_0|\right]; \quad c_i^2 = \Delta b h$$

$$w_e = \frac{nhB_0}{c_i^2} = \frac{nB_0}{\Delta b}$$

$$\frac{\partial h}{\partial t} = \frac{n}{\rho c_p}\frac{R_0}{\Delta T}$$

Heating depends on n:

$$\frac{\partial T_0}{\partial t} = -\left[\frac{1+n}{2\rho c_p}\frac{R_0}{h}\right]$$

n=0 all dissipation of convective turbulence in ML – no entrainment cooling
n=1 no dissipation occurs in ML → energy available for entrainment

Niiler and Kraus (1977)

increasing stability with steady wind

Assume $$w_e = \frac{\partial h}{\partial t} = 0$$

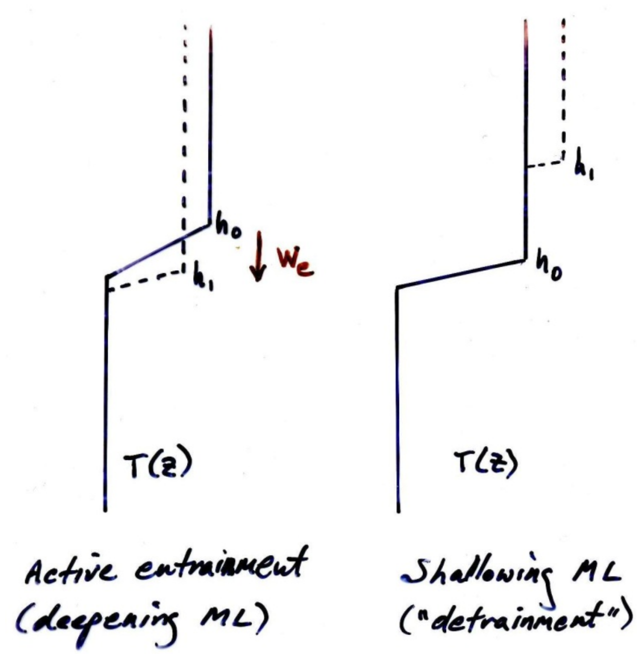
with radiative heating $$h = \frac{-2mu_*^3 + 2\, J_0 / \gamma}{B_0 + J_0}$$

At night, J_0=0 $$h = -\frac{2mu_*^3}{B_0} = mL$$

L = Monin-Obukhov length scale

Issues

- With cycling of ML (diurnal, seasonal) what are the entrainment temperature, salinity and buoyancy values?
  - ΔT=?
  - ΔS=?
  - Δb=?



- What is the influence of upwelling, $w_h \neq 0$?

How to deal with cyclical entrainment heat flux into slab mixed layer?

What is the entrainment temperature?

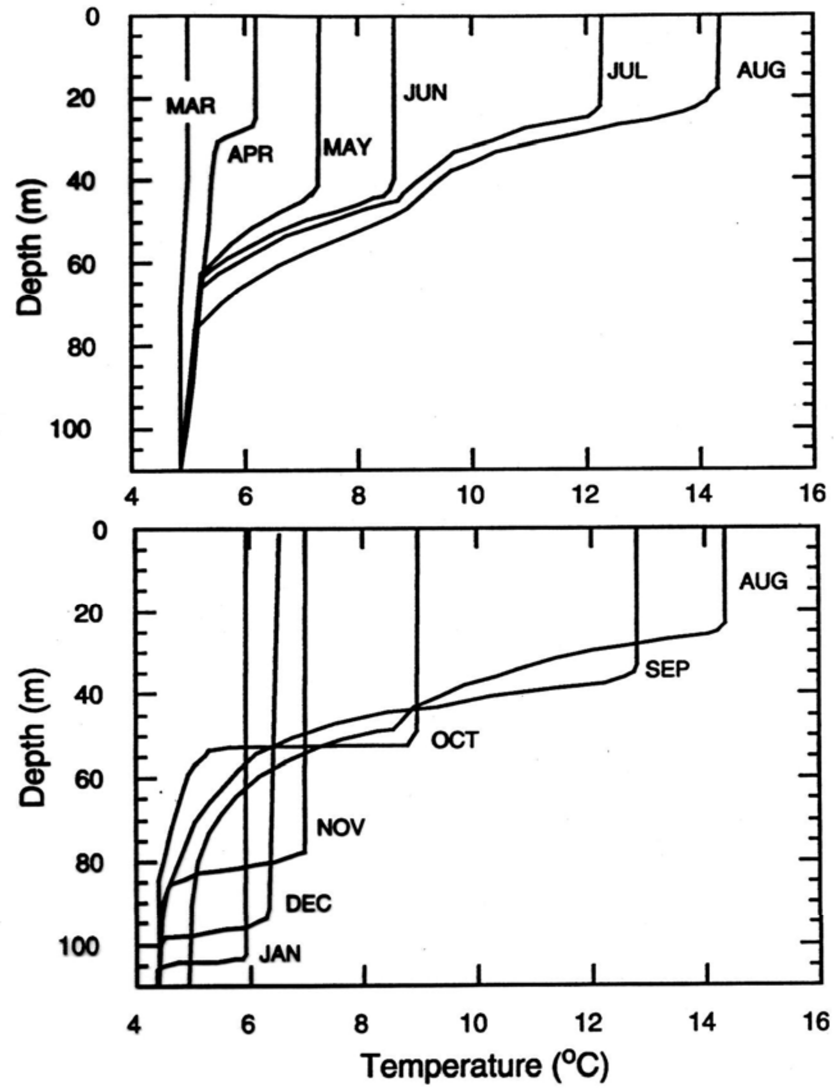
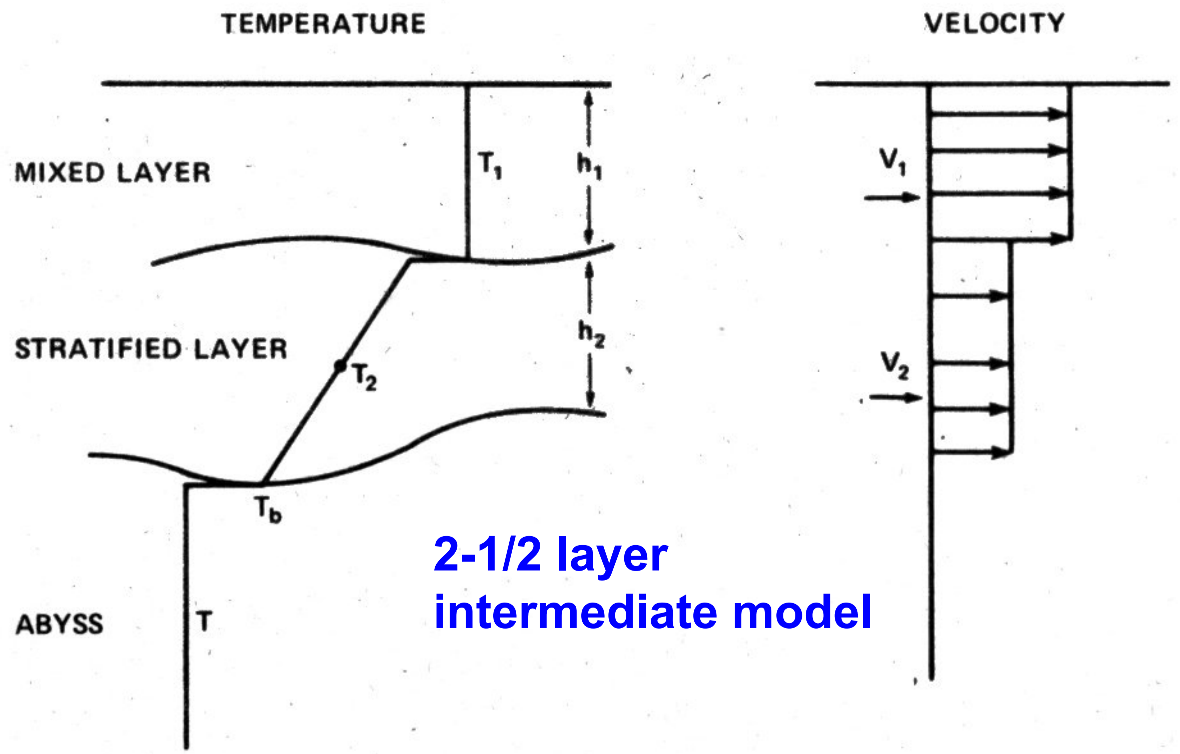


Figure 2.6.1 Seasonal cycle of temperature evolution in the midlatitude upper ocean.

What is the Entrainment Temperature?

Schopf and Cane (1983, J. Phys. Oceanogr., 13, 917-935)



2-1/2 layer
intermediate model

FIG. 1. Vertical structure of temperature and velocity assumed in the model equations. T_k, h_k and $\mathbf{V}_k$ vary with time, latitude and longitude.

Energetics

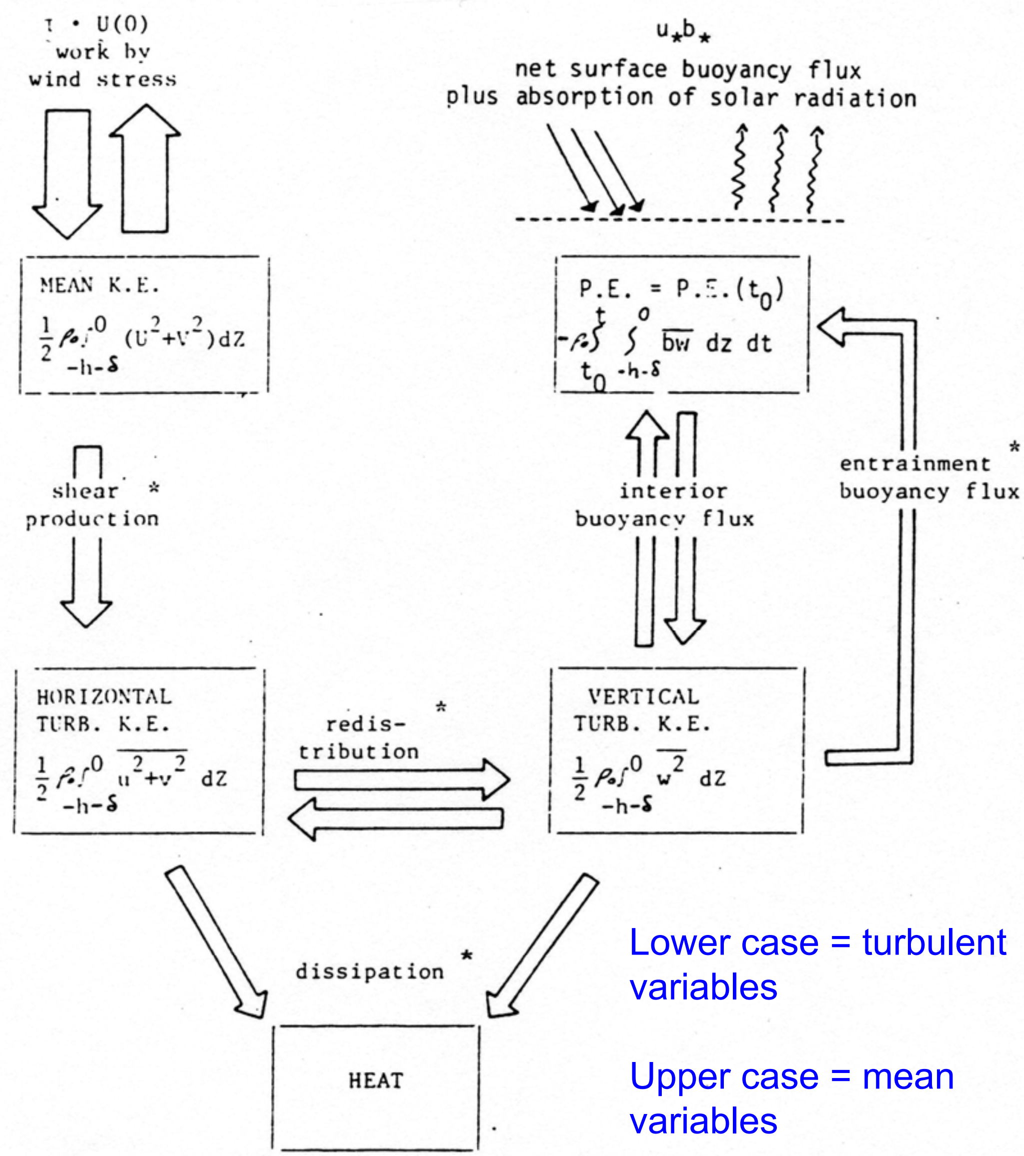


FIG. 2. Mechanical energy budget for the ocean mixed layer. Asterisks indicate those processes that must be parameterized to close the system of equations.

Lower case = turbulent variables

Upper case = mean variables

Garwood (1977)

Energetics- what do we ignore?

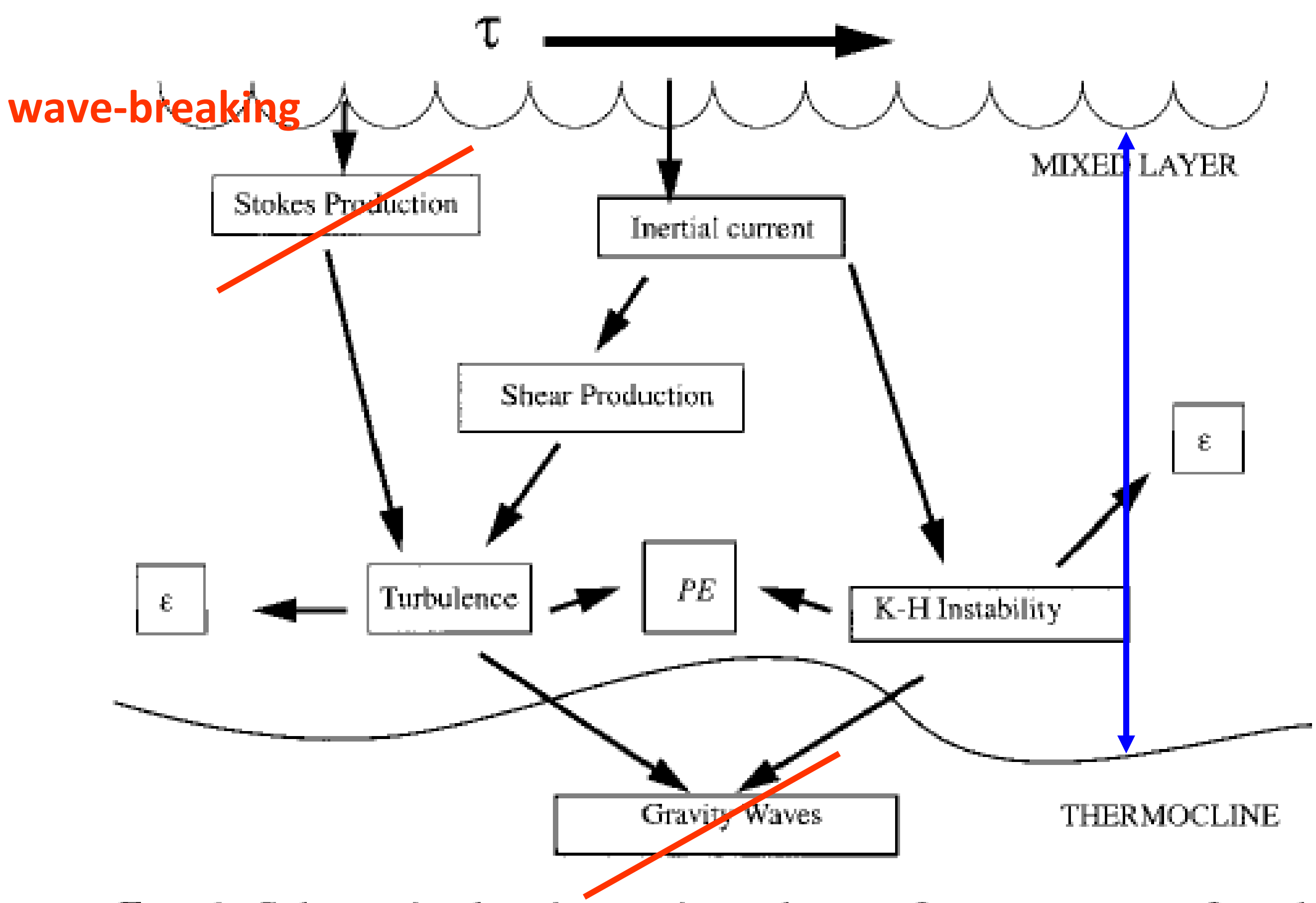


FIG. 1. Schematic showing main pathways for energy transferred to the upper ocean via the surface wind stress.

Skyllingstad et al. (2000, JPO)

Inertial Currents

$$\overline{\vec{V}}^{\,2} \equiv \frac{2u_*^4}{h^2 f^2}\left(1-\cos ft\right) = \frac{\Delta bh}{Ri_*}$$

$$Ri_* = \left(\frac{c_i}{\vec{V}}\right)^2; \quad c_i = \sqrt{\Delta bh}$$

$$w_e\left(Ri_* - s\right)\overline{\vec{V}}^{\,2} = 2m_*^3 + nhB_0$$

$$h = \left[Ri_* \frac{2u_*^4}{\Delta bf^2}\left(1-\cos ft\right)\right]^{\frac{1}{3}}$$

(1-s) is fraction of shear production at ML base that is dissipated

$$t_f = \frac{\pi}{f}$$

half an inertial period

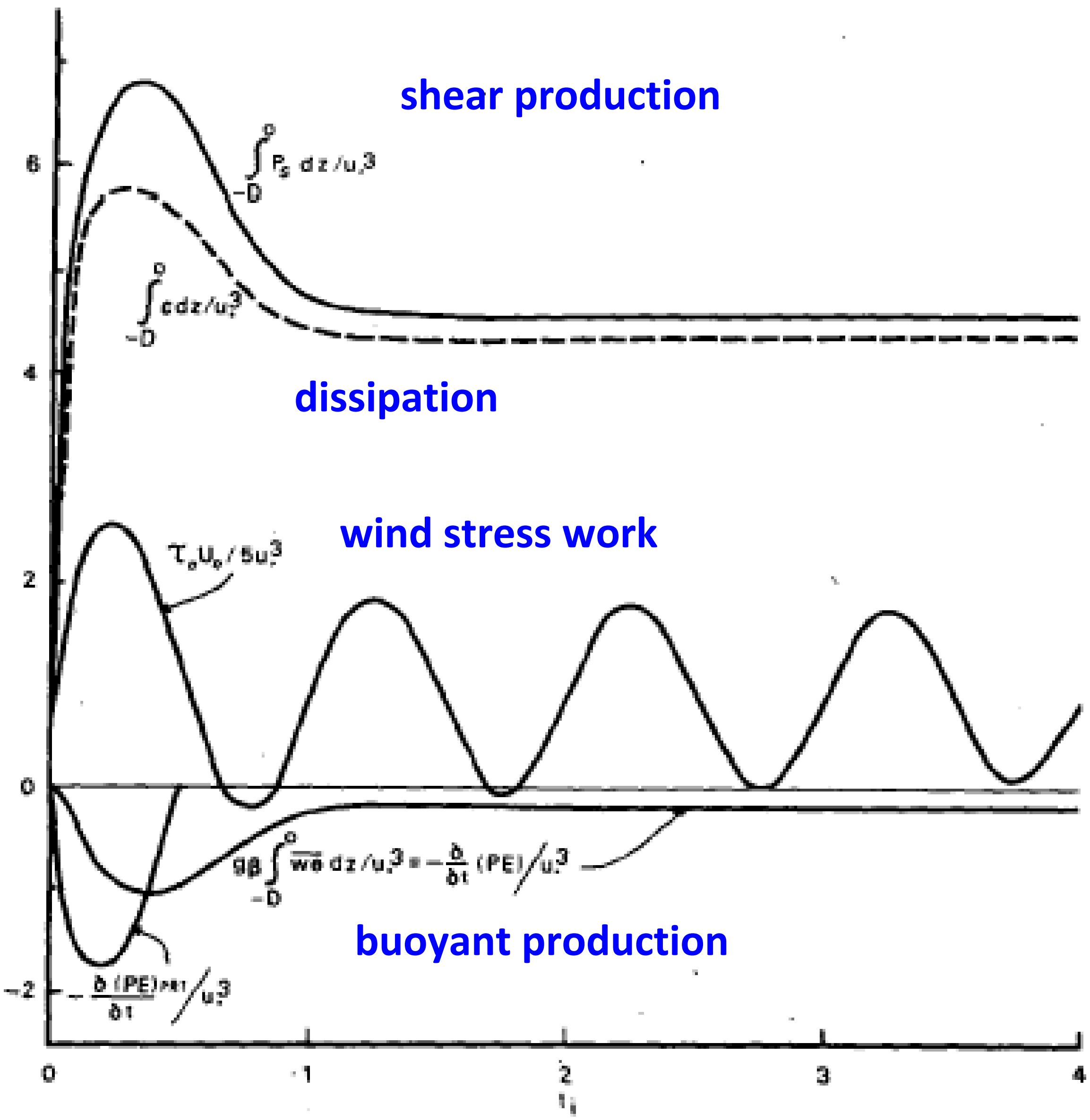


FIG. 6. Depth integrals of shear production P_s, dissipation ϵ, buoyant production $g\beta\overline{w\theta}$ of present and PRT models, and one-fifth of work done by surface wind stress.

Kundu (1980, JPO)

Inertial Resonance

wind stress and currents rotate together

90 phase difference between velocity components, clockwise (NH) rotation and exponential increase in speed

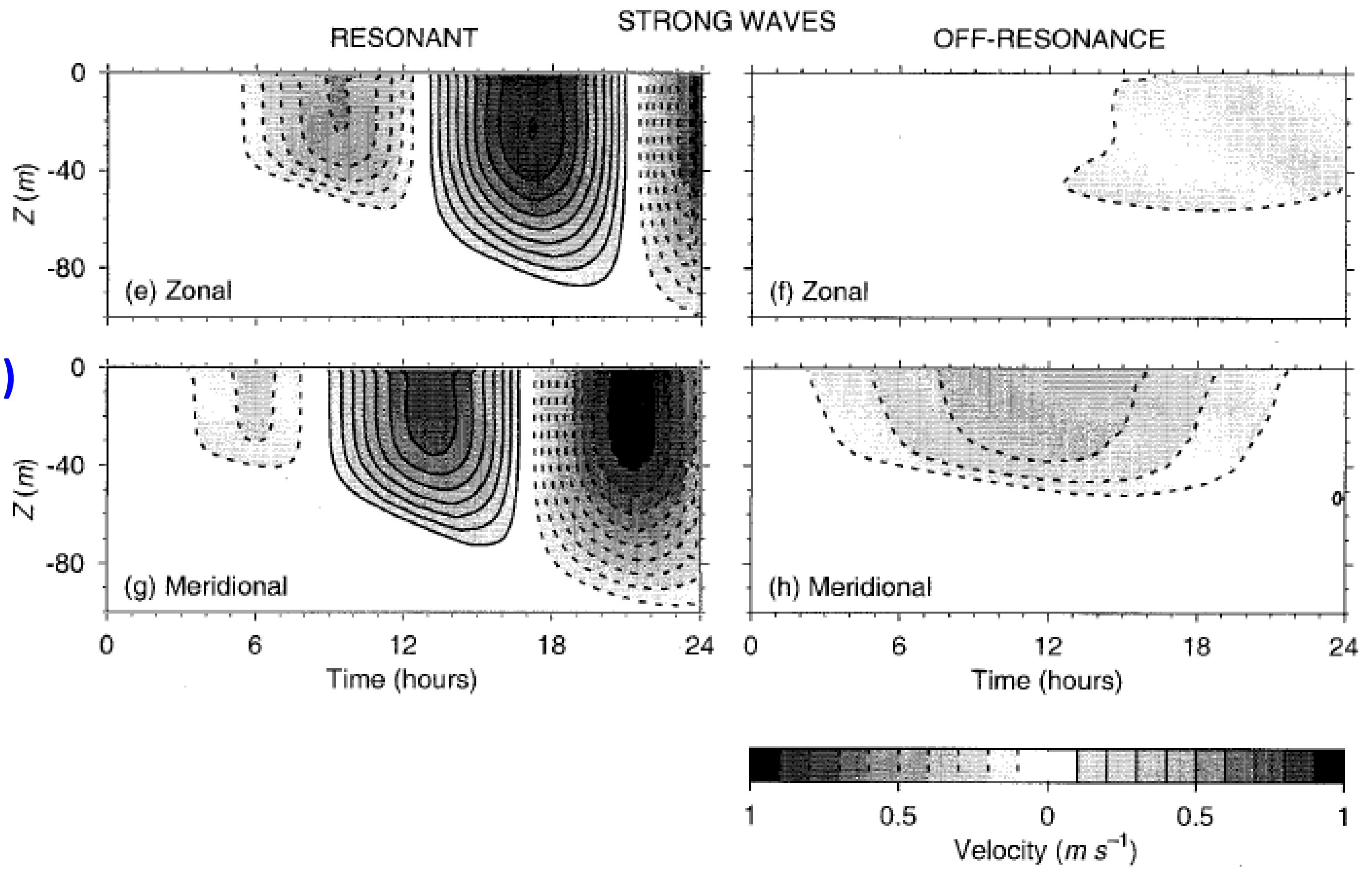


FIG. 2. Horizontally averaged zonal velocity for weak waves with (a) resonant and (b) off-resonance winds. Horizontally averaged meridional velocity for weak waves with (c) resonant and (d) off-resonance winds. Horizontally averaged zonal velocity for strong waves with (e) resonant and (f) off-resonance winds. Horizontally averaged meridional velocity for strong waves with (g) resonant and (h) off-resonance winds.

Inertial Resonance

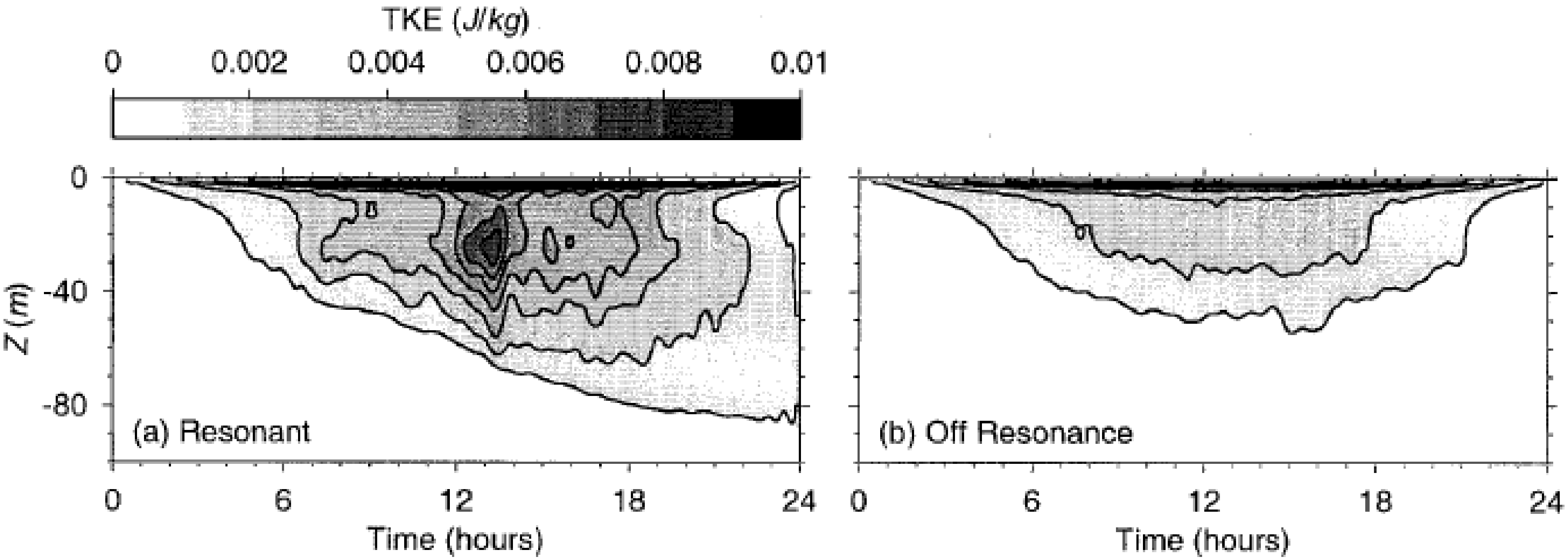
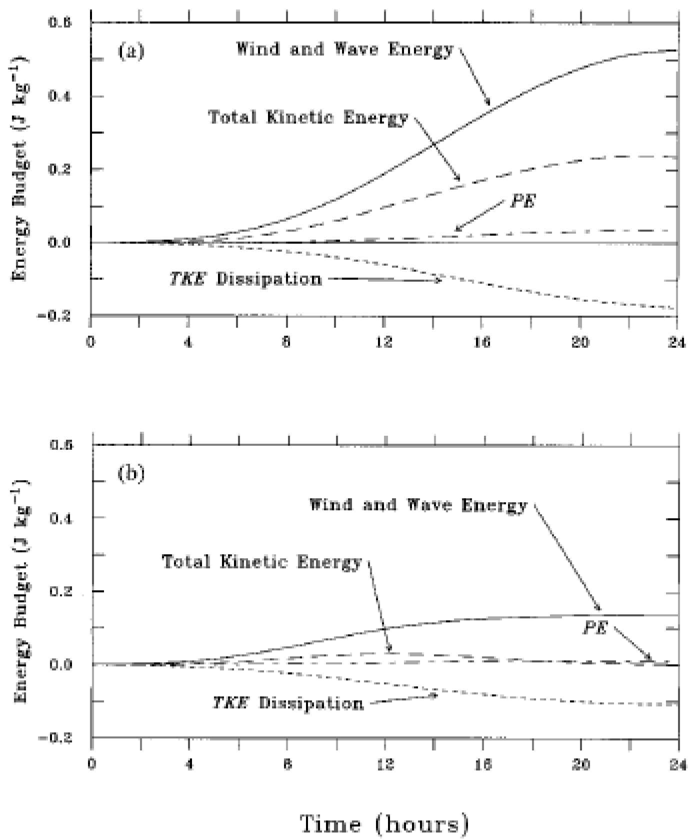


FIG. 6. Time–depth section of the specific turbulent kinetic energy for (a) resonant and (b) off-resonance cases.

Difference in max TKE and ML depth

Inertial Resonance

impact on energy budgets



resonant

off-resonant

FIG. 11. Cumulative vertically integrated input wind energy, total kinetic energy (KE), PE, and turbulence dissipation rate for (a) resonant and (b) off-resonance cases.

Have to include
all significant sources of energy

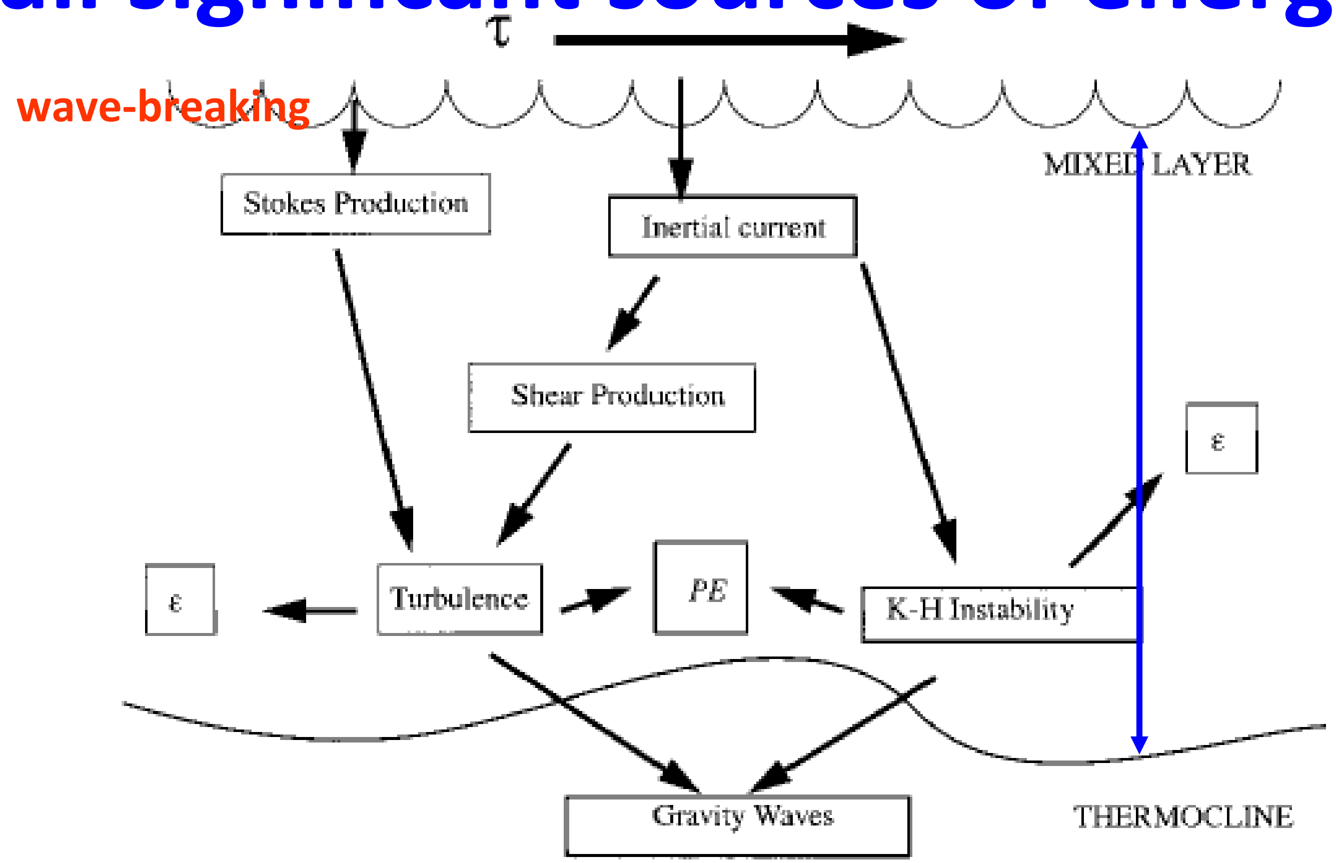
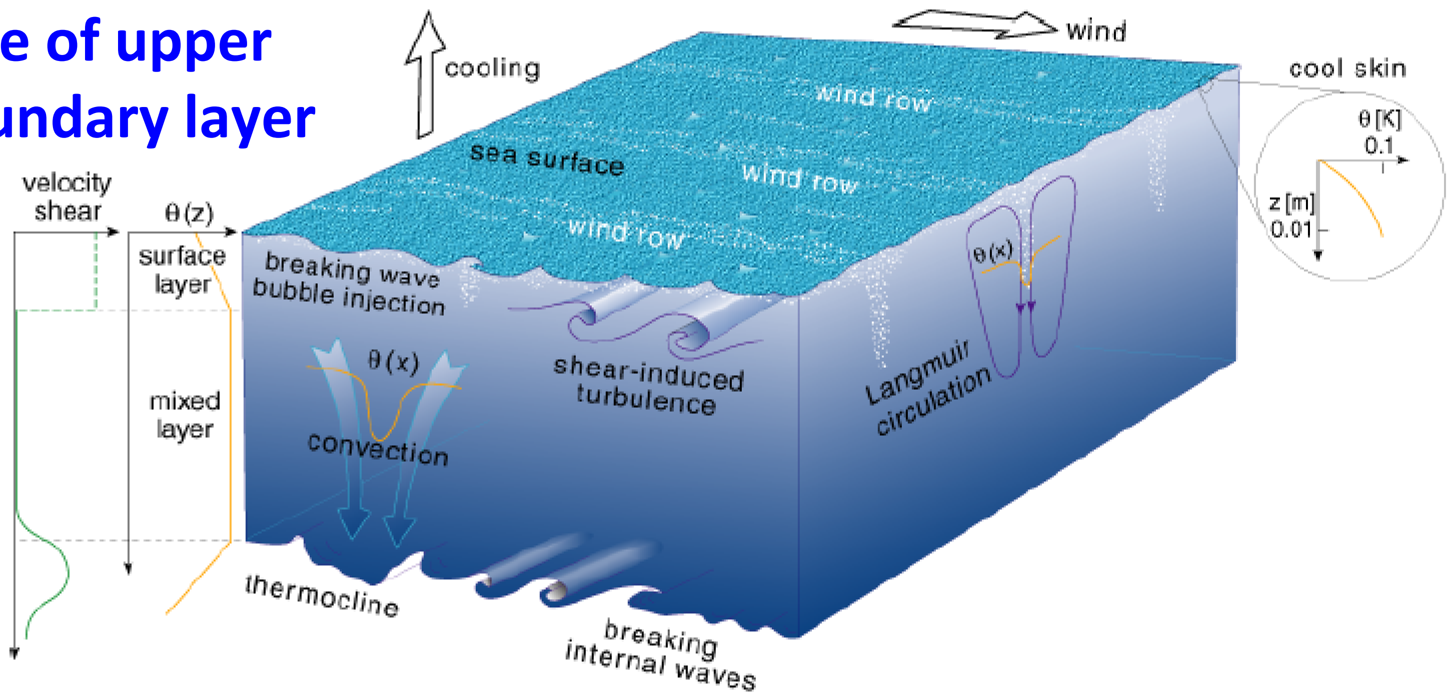


FIG. 1. Schematic showing main pathways for energy transferred to the upper ocean via the surface wind stress.

Skyllingstad et al. (2000, JPO)

Structure of upper ocean boundary layer



Moum and Smyth (2000)

Figure 2 Schematic showing processes that have been identified by a wide range of observational techniques as important contributors to mixing the upper ocean in association with surface cooling and winds. The temperature (θ) profiles shown here have the adiabatic temperature (that due to compression of fluid parcels with depth) removed; this is termed potential temperature. The profile of velocity shear (vertical gradient of horizontal velocity) indicates no shear in the mixed layer and non-zero shear above. The form of the shear in the surface layer is a current area of research. Shear-induced turbulence near the surface may be responsible for *temperature ramps* observed from highly-resolved horizontal measurements. *Convective plumes* and *Langmuir circulations* both act to redistribute fluid parcels vertically; during *convection*, they tend to move cool fluid downward. *Wind-driven shear* concentrated at the mixed layer base (thermocline) may be sufficient to allow *instabilities* to grow, from which *internal gravity waves* propagate and *turbulence* is generated. At the surface, *breaking waves* inject bubbles and highly energetic turbulence beneath the sea surface and disrupt the ocean's *cool skin*, clearing a pathway for more rapid heat transfer into the ocean.

Bulk Mixed Layer Models

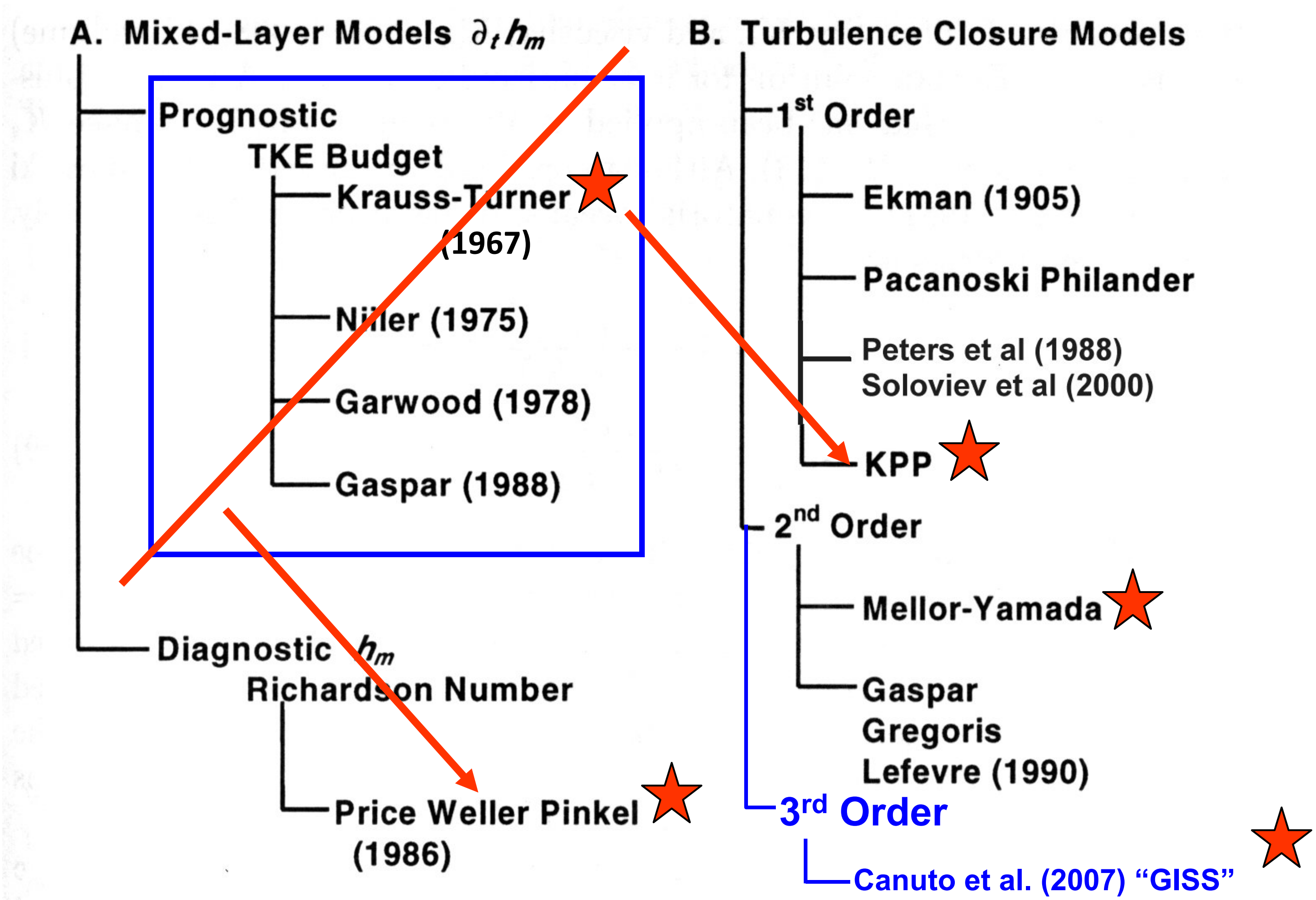


Figure 6. Classification of OBL mixing schemes into Mixed-Layer and Turbulence Closure models.

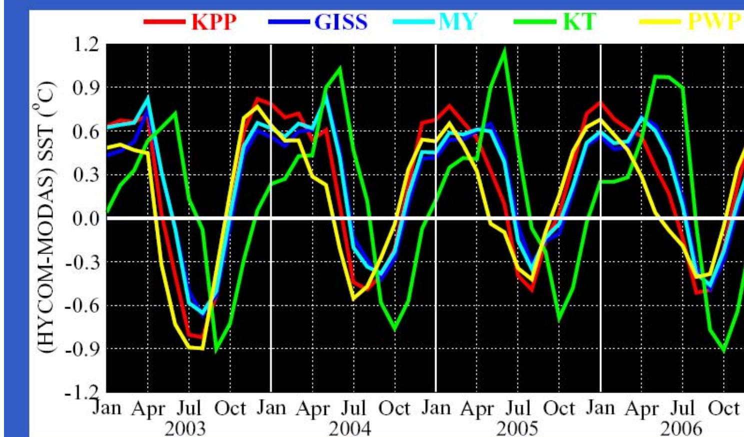
Performance of Mixed Layer Models in the Mediterranean Sea

A. Birol Kara, Alan J. Wallcraft and Harley E. Hurlburt   2008, unpubl. rept.

Basin–Averaged RMS SST Difference (°C) by Year

	2003	2004	2005	2006	2003-2006
KPP	0.86	0.78	0.79	0.78	0.81
GISS	0.79	0.76	0.76	0.76	0.78
MY	0.83	0.75	0.74	0.75	0.78
KT	0.88	0.88	0.90	0.96	0.91
PWP	0.90	0.73	0.74	0.73	0.79

Same forcing via NOGAPS through bulk formulas



Annual mean bias is ≈ 0.1°C from all models.

The vertical mixing models in HYCOM

KPP → Large et al. (1994)

GISS → Canuto et al. (2002)

MY2.5 → Mellor and Yamada (1982)

KT → Kraus and Turner (1967)

PWP → Price et al. (1986)

Summary and Conclusions

• All mixed layer models perform similarly:

→ validation against MODAS SST

→ 0.8°C RMS difference

• Upwelling from each model varies.

• Changes in the net surface heat flux

→ 50 Wm^{-2} difference from KT