October 17, 2007

Overview of PX and PXT Shoreline Change Rate Methods

University of Hawaii Coastal Geology Group

Coastal erosion studies by the University of Hawaii Coastal Geology Group (UHCGG) provide technical information on shoreline dynamics to help government agencies and the public manage coastal resources and avoid coastal hazards. It is vital to continually refine and improve models that provide statistically significant erosion hazard predictions to aid in the development of public policy. To further the goal of providing reliable erosion study results, UHCGG has developed the PX and PXT methods for calculating shoreline erosion rates (Genz, *et al*., 2007; Genz, *et al*., in press; Frazer, *et al*., in press).

Coastal erosion studies by UHCGG and others employ historical shoreline positions that are digitized from aerial photographs and survey charts (t-sheets) (Fletcher, *et al*., 2003; National Academy of Sciences, 1991). Historical shorelines may be derived from several shoreline change reference features (SCRF's), such as, the vegetation line, high water line, or low water line. We utilize the low water line (beach toe or base of the foreshore) as the SCRF for all photo and t-sheet years (Fig. 1). A positional uncertainty is calculated for each historical shoreline based on observed fluctuations in the shoreline due to natural factors such as waves and tides. In addition, measurement uncertainties related to mapping the historical shoreline from an aerial photo or t-sheet are calculated. The historical shorelines are displayed together on a map for comparison and their relative distances are measured along shore-perpendicular transects spaced 20 m apart (Fig.2).

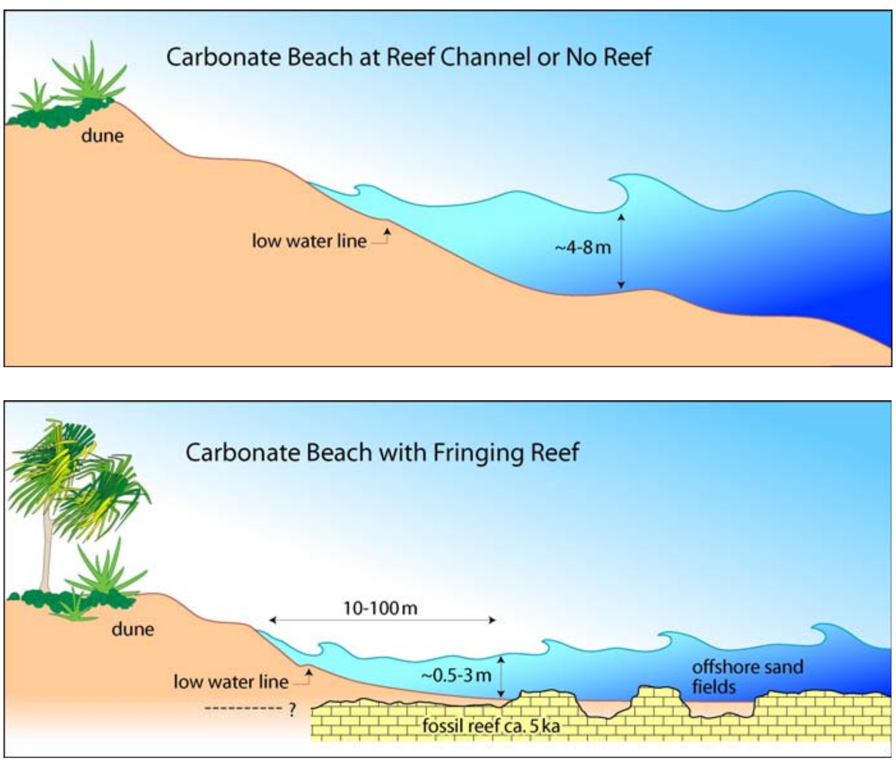

Figure 1: Typical cross-shore profiles of Hawaiian beaches. We utilize the low water line (beach toe or base of the foreshore) as the shoreline change reference feature (SCRF).

Figure 2: Historical shorelines and shore-perpendicular transects (spaced 20 m) for measuring relative shoreline change (displayed on recent aerial photograph, with transect number).

Shoreline change rates are calculated from the time series of historical shoreline positions with uncertainties using a variety of statistical methods. In previous studies, UHCGG and other coastal research groups have utilized the single-transect (S-T) method to calculate shoreline change rates. S-T calculates a shoreline change rate and uncertainty at each shoreline transect using linear regression to fit a trend line to the time series of historical shoreline positions. We employ weighted regression methods, which account for the uncertainty in each shoreline position when calculating a trend line (see: Genz, *et al*., 2007; Fletcher, *et al*., 2003). The slope of the line is the shoreline change rate (Fig. 3).

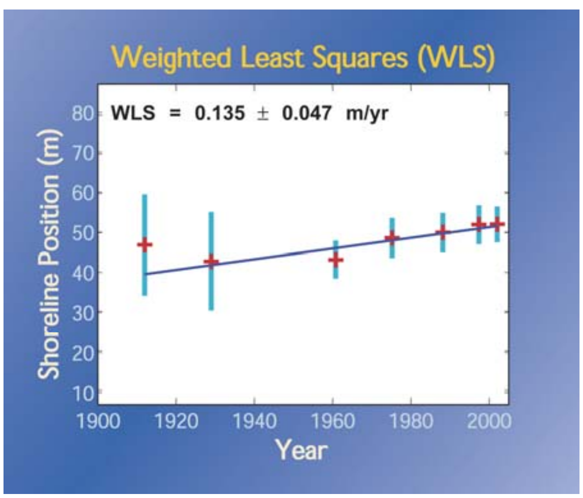

Figure 3: Example of Single Transect (S-T) rate calculation using weighted linear regression. Each red cross is a historical shoreline position along a transect plotted in time and distance. Blue bars represent the uncertainty of each historical shoreline position. The slope of the line is the shoreline change rate (erosion rate).

Recent work by UHCGG identifies a number of problems with the S-T method. First, S-T is unparsimonious, i.e., it tends to over-fit the data by using more mathematical parameters than necessary to model the change at a beach because it assumes adjacent transects are independent. In theory, adjacent transects should tell a similar story about the change occurring at a beach because beach positions share sand along the shore. Instead, S-T assumes each transect position is independent of other transects. That is, it treats the beach as if it were a set of 20m wide blocks that move independent of each other. Second, S-T produces many rates that are not statistically significant, i.e., the rates are statistically indistinguishable from a rate of 0.0 meters per year. Third, short-term fluctuations in the beach due to seasonal and tidal changes (high complexity) and a lack of historical shoreline data (poor sampling) can mask the long-term trend when attempting to calculate a change rate from a single transect.

To address these problems, UHCGG has developed the PX methods (Polynomial in distance X) for calculating shoreline change rates to produce more meaningful, i.e., statistically significant and defensible, shoreline change rates. PX combines data from all transects along a beach and models shoreline change for the entire length of beach using polynomial regression. The resulting shoreline change models produce rates that vary continuously in the alongshore direction (Fig. 4). These models employ information from the entire beach to model the rate at any one location.

An advancement of the PX method, called PXT (Polynomial in distance X and Time), has been developed to model shoreline change rates along the shore and with time (Figs 5 and 6). For sufficient data, PXT can find acceleration in the shoreline change rate – an important advance, as most beaches do not erode or accrete at a constant (linear) rate. Work by Genz, *et al*. and Frazer, *et al*. (in press) shows the PX and PXT methods often give meaningful, i.e., statistically defensible, change rates for beaches where S-T cannot. This yields more precise predictions of shoreline change.

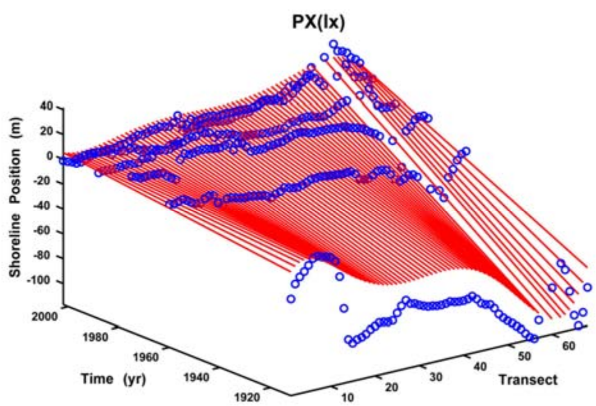

Figure 4: Example of 3-dimensional plot of shoreline change model calculated using PX method (no acceleration in change rates with time). The circles represent historical shoreline positions at each transect. The x-axis is transect number (along the shore), y-axis is time (historical shoreline date), and z-axis is normalized shoreline distance. The red lines represent samples of the change model at each transect. The rate varies continuously in the alongshore direction (x).

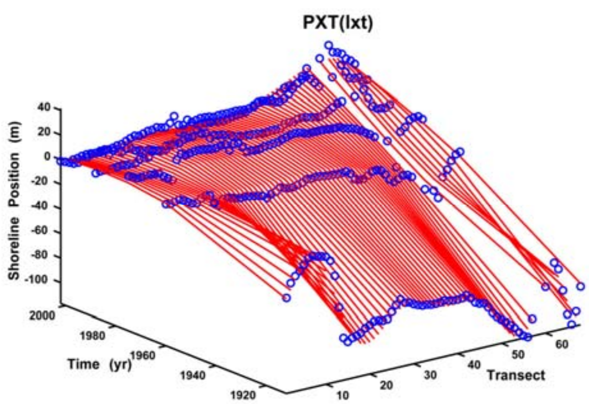

Figure 5: Example of 3-dimensional plot of shoreline change model calculated using PXT method (acceleration in change rates with time). The circles represent historical shoreline positions at each transect. The x-axis is transect number (along the shore), y-axis is time (historical shoreline date), and z-axis is normalized shoreline distance. The red lines represent samples of the change model at each transect. The rate varies continuously in the alongshore direction and time (acceleration).

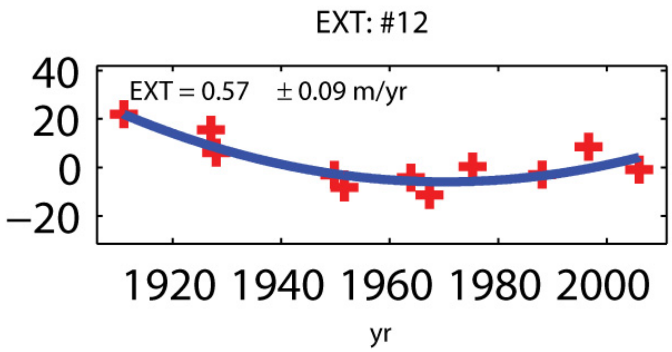

Figure 6: Plot of PXT (EXT) shoreline change model (blue) sampled at one transect showing acceleration in the erosion rate with time. The red crosses represent historical shoreline positions. The x-axis is time and the y-axis is normalized (relative) shoreline position. The reported shoreline change rate is the rate at the most recent shoreline time.

PX and PXT methods use three different models that employ basic mathematical functions called basis functions to describe shoreline change. The three types of basis functions are: Legendre polynomials, trigonometric functions (sines and cosines), and principal components (eigenvectors) of the beach data. For PX the beach models are respectively called LX, RX, and EX. For PXT, in which the rate of shoreline change varies through time (acceleration or deceleration), the models are referred to as LXT, RXT, and EXT. For a beach where acceleration or deceleration is identified, the reported rates and uncertainties are from the most recent shoreline time (the "present" rate) (Fig. 7). However, a rate may be calculated for any point in the time series.

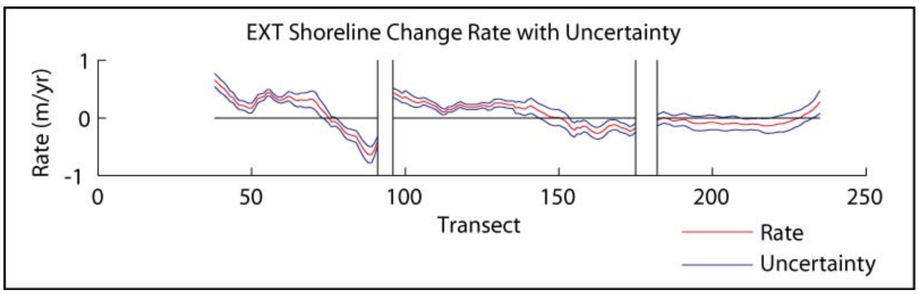

Figure 7: Sample plot of EXT shoreline change rates (present rates) with uncertainties (95 percentile) plotted in the alongshore direction. Gaps represent model boundaries at headlands, stream mouths, or other physical boundaries on the beach.

We use a statistical information criterion to find the simplest and best-fitting model for each of the individual PX and PXT methods. Rates and uncertainties are calculated for each of the basis function types in non-acceleration and acceleration change models (LX, RX, EX, LXT, RXT, and EXT) and S-T for comparison (Fig. 8). We then use the information criteria to identify the "best", i.e., most parsimonious, overall change model. All models are given a relative score based on the quality of fit with the shoreline data and the number of parameters used. The model with the lowest score is chosen as the best descriptor of shoreline change for a beach (Table 1). For each of the methods, information criteria may choose a model with no change (0.0 m/yr or "null model") if it is determined to be a better fit than a model with parameters (showing shoreline change). The null model may indicate that the beach is stable or that the uncertainties in the historical shoreline positions are too high to fit a change model.

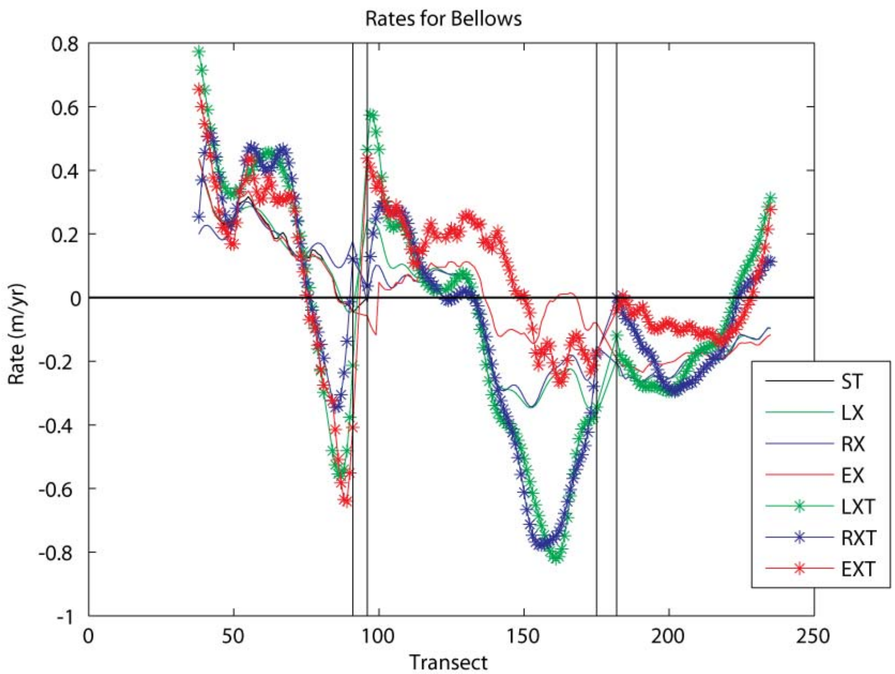

Figure 8: Sample plot (from Bellows Beach, Oahu) showing change rates calculated for all S-T, PX, and PXT methods. Note: Rates for transects 0-37 were not calculated because this area has no beach at present. The vertical lines indicate physical boundaries in the beach system.

Waimanalo,Oahu, Hawaii

Rate Calculation Method	Rate Parameters	Acceleration Parameters	criteria score
S-T(WLS)	0	n/a	7.93136
LX	27	n/a	-0.94086
RX	134	n/a	-0.91781
EX	7	n/a	-0.69226
LXT	27	23	-1.23152
RXT	134	32	-1.08274
EXT	7	11	-1.21547

Table 1: Example of parameters and statistical information criteria scores for S-T, PX and PXT shoreline change models. LXT, with the lowest criteria score, is chosen as the best change model for the beach.

LITERATURE CITED

FLETCHER, C. H.; ROONEY, J. J. B.; BARBEE, M.; LIM, S.-C., AND RICHMOND, B. M., 2003. Mapping Shoreline Change Using Digital Orthophotogrammetry on Maui, Hawaii. Journal of Coastal Research, 38, 106-124.

FRAZER, L.N.; GENZ, A.; FLETCHER, C. H., accepted. Toward Parsimony in Shoreline Change Prediction (I): New Methods. Journal of Coastal Research.

GENZ, A. S.; FLETCHER, C. H.; DUNN, R. A.; FRAZER, L. N.; AND ROONEY, J. J., 2007. The Predictive Accuracy of Shoreline Change Rate Methods and Alongshore Beach Variation on Maui, Hawaii. Journal of Coastal Research, 23(1), 87-105.

GENZ, A.; FRAZER, L.N.; FLETCHER, C. H., accepted. Toward Parsimony in Shoreline Change Prediction (II): New Methods. Journal of Coastal Research.

NATIONAL ACADEMY OF SCIENCES, 1990. Managing Coastal Erosion. Washington D.C.: National Research Council, Committee on Coastal Erosion Zone Management, National Academy Press.