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Overview of EX and EXT Shoreline Change Rate Calculation Methods

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Coastal erosion studies by the University of Hawaii Coastal Geology Group provide technical information on shoreline dynamics to help government agencies and the public manage coastal resources and avoid coastal hazards. It is vital to continually refine and improve models that provide statistically significant erosion hazard predictions to aid in the development of public policy. To further the goal of providing reliable erosion study results, we have developed the EX and EXT methods for calculating shoreline change rates (Frazer et al., in press; Genz et al., in press).

Coastal erosion studies by this group and others employ historical shoreline positions that are digitized from aerial photographs and survey charts (t-sheets) (Fletcher et al., 2003; National Academy of Sciences, 1990). Historical shorelines may be derived from several shoreline change reference features (SCRF's), such as, the vegetation line, high water line, or low water line. We utilize the low water line (indicated by the beach toe or base of the foreshore) as the SCRF for all photo and t-sheet years (Figure 1). A positional uncertainty is calculated for each historical shoreline based on observed fluctuations in the shoreline due to natural factors such as waves and tides. In addition, measurement uncertainties related to mapping the historical shoreline from an aerial photo or t-sheet are calculated. The historical shorelines are displayed together on a map for comparison and their relative distances are measured along shore-perpendicular transects spaced 20 m apart (Figure 2).

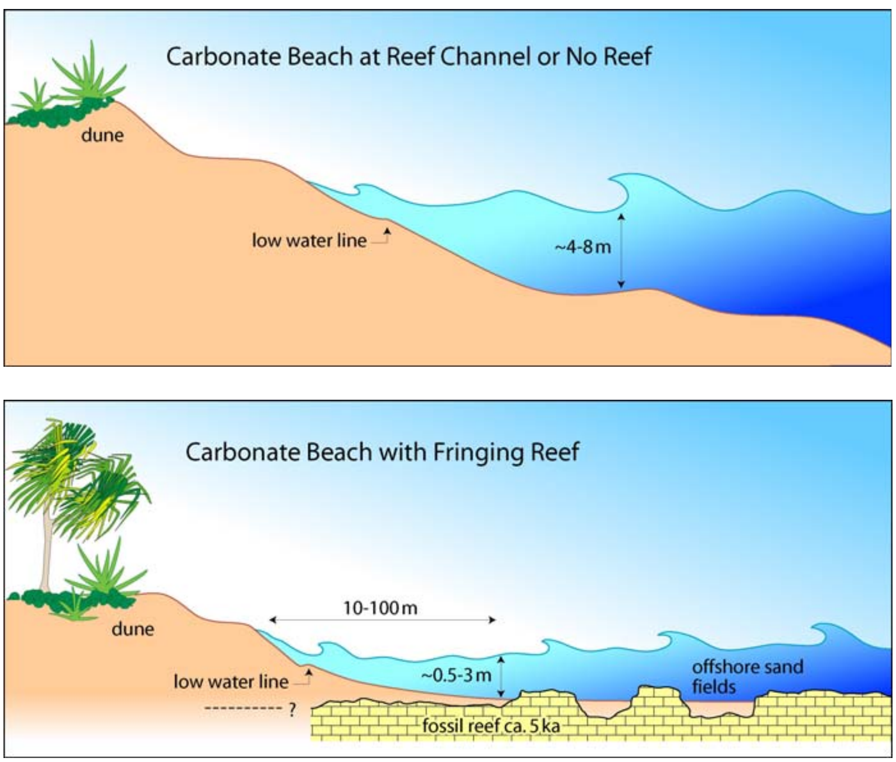

Figure 1. Typical cross-shore profiles of Hawaiian beaches. We utilize the low water line (indicated by the beach toe or base of the foreshore) as the shoreline change reference feature (SCRF).

Figure 2. Historical shorelines and shore-perpendicular transects (spaced 20 m) for measuring relative shoreline change (displayed on recent aerial photograph, with transect number).

Shoreline change rates are calculated from the time series of historical shoreline positions using a variety of statistical methods. In previous studies, our group and other coastal research groups (e.g., Theiler, et al., 2005) have utilized the single-transect (ST) method to calculate shoreline change rates. ST calculates a shoreline change rate and uncertainty at each shoreline transect using least squares (i.e., linear regression) to fit a trend line to the time series of historical shoreline positions. We employ weighted regression methods, which account for the uncertainty in each shoreline position when calculating a trend line (see: Fletcher et al., 2003; Genz et al., 2007). The slope of the line is the shoreline change rate (Figure 3).

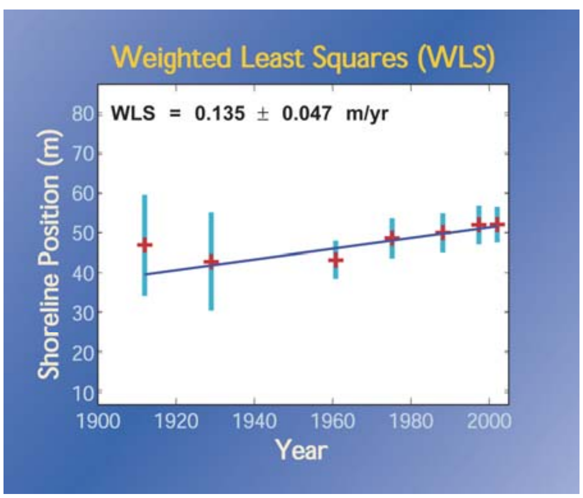

Figure 3. Example of single-transect (ST) rate calculation using weighted least squares. Each red cross is a historical shoreline position along a transect plotted in time and distance. Blue bars represent the positional and measurement uncertainty of each historical shoreline position. The slope of the line is the shoreline change rate.

EX and EXT Methods

Recent work by Frazer et al. (in press) identifies a number of problems with the ST method. First, ST is unparsimonious, i.e., it tends to over-fit the data by using more mathematical parameters than necessary to model the change at a beach because it models shoreline change independently at each transect. The principle of parsimony, when applied to mathematical modeling states that the simplest model (i.e., the model with the fewest mathematical parameters) that best fits the data is preferred. In theory, adjacent transects should tell a similar story about the change occurring at a beach because beach positions share sand along the shore. Thus, a model which utilizes data from adjacent transects will require fewer model parameters (a more parsimonious model) to calculate shoreline change rates. Second, ST produces many rates that are not statistically significant. For the purposes of our studies, an insignificant rate is one with a ± uncertainty that is greater than the rate, itself. In other words the rate is statistically indistinguishable from a rate of 0 m/yr. Third, short-term fluctuations in the beach due to seasonal and tidal changes (high complexity) and a lack of historical shoreline data (poor sampling) can mask the long-term trend when attempting to calculate a change rate from a single transect.

To address issues described above with the ST method, Frazer et al. (in press) developed the EX method for calculating shoreline change rates. The EX method is shown to produce more meaningful, i.e., statistically significant shoreline change rates than ST. EX also describes the shoreline change by avoiding over-fitting of the shoreline data. For example, if a hypothetical beach has 100 transects, ST treats each transect independently and a least squares trend is fit to each transect to describe the shoreline change. The hypothetical beach could, however, be modeled with a quadratic equation in the alongshore direction:

$$y(x,t) = y(x,t_1) + (t - t_1)(a + bx + cx^2) \qquad \text{Equation (1)}$$

where *y(x,t)* is the shoreline position at time (*t*) and transect location (*x*). The coefficients of this equation (*a*,*b*,*c*) are the least squares parameters, and $1, x, x^2$ are called ***basis functions***. Equation (1) fits a line at each transect (*x*) – the rate is changing linearly with time (*t*), while simultaneously fitting a quadratic in the alongshore direction – the rate is changing with transect location (*x*) in the shape of a parabola.

Similar to Equation (1), EX uses basis functions to model the rate in the alongshore direction, while still fitting a line at each transect. The basis functions in EX are called *eigenvectors*, which are calculated by first collecting the shoreline positions of all transects at each year into a matrix and then computing the principal components of this matrix. In Figure 4, there are 7 eigenvectors because there are 7 years of shoreline data (e.g., shorelines from 1912, 1960, etc.) (see also box 1 in Figure 5). The first eigenvector ($P_0(x)$) describes the pattern of the shoreline data with respect to transect location. This eigenvector typically contains the most information pertaining to the pattern in the shoreline data. Each successive eigenvector has additional, yet less, information of the pattern inherent in the shoreline data. In Figure 4, $P_6(x)$ contains very little additional information and is almost a flat line in the alongshore direction.

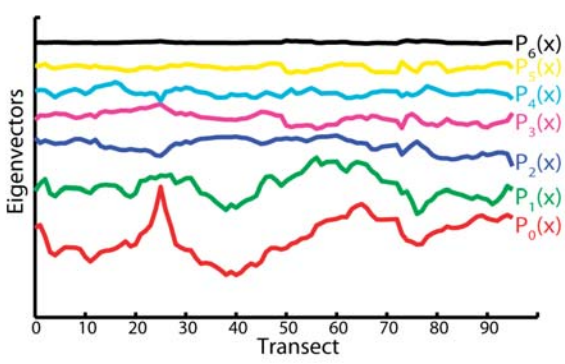

Figure 4. Eigenvectors calculated from historical shoreline data. The eigenvectors are used in various combinations to model the alongshore variation in shoreline change rates at a beach.

Once the eigenvectors are computed, different models of EX are tested to find the best EX model. To do this, we first start with the first eigenvector ($P_0(x)$) (see box 2 in Figure 5). We calculate the coefficient of the first eigenvector using least squares regression (see box 3 in Figure 5). We then compute an Information Criterion (IC) score of the model with only one eigenvector (see box 4 in Figure

5). An IC score (we use AICu) is an objective statistical criterion that balances the fit of the trend to the data with the number of parameters the model needed to fit the data. Models are 'rewarded' for improved

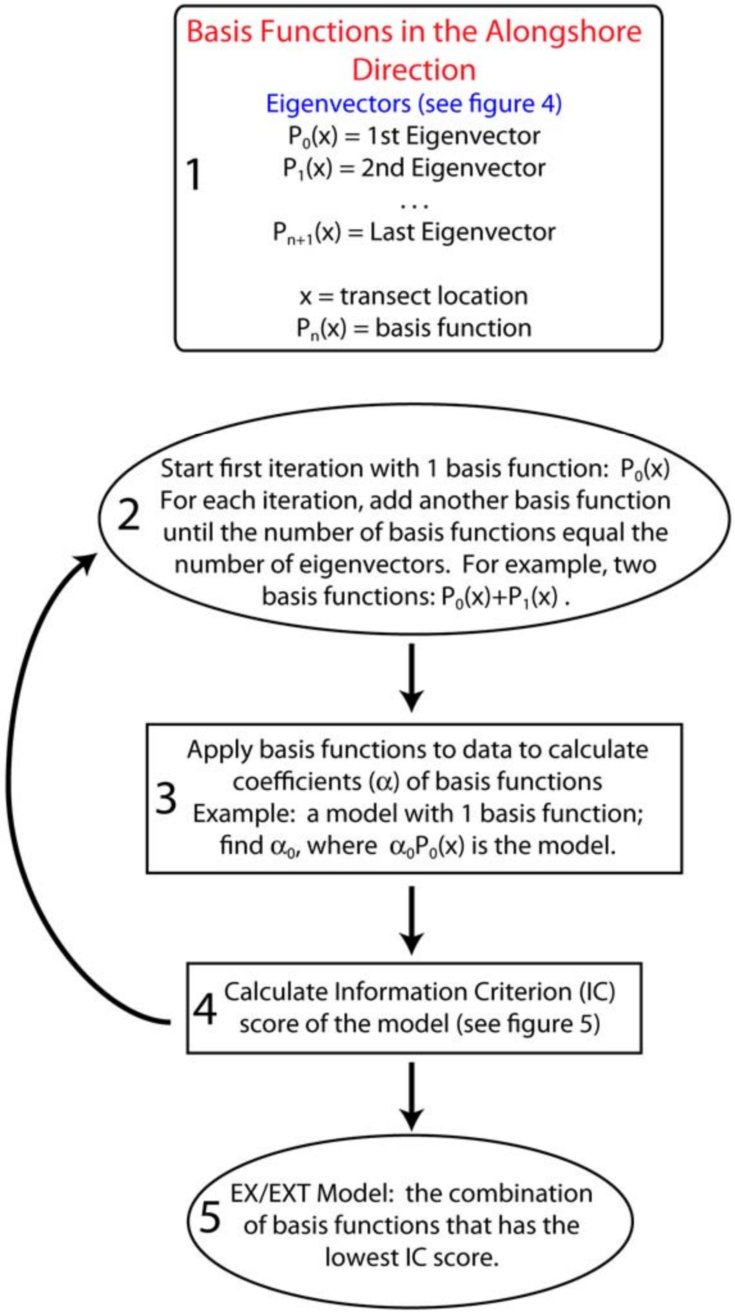

Figure 5. Flowchart describing the EX method.

fit to the data and 'penalized' for using additional parameters. The model with the lowest IC score is chosen as the favored (most parsimonious) shoreline change model. Once the IC score is computed, we add another eigenvector to the model ($P_0(x) + P_1(x)$) (box 2 in Figure 5). This is called a linear sum of the basis functions. We then repeat the process (boxes 3 and 4 in Figure 5), until all eigenvectors are combined. The EX model that has the lowest IC score is the best model for a particular beach (see box 5 in Figure 5). The resulting EX rates vary continuously in the alongshore direction (Figure 6) but are constant (linear) in time.

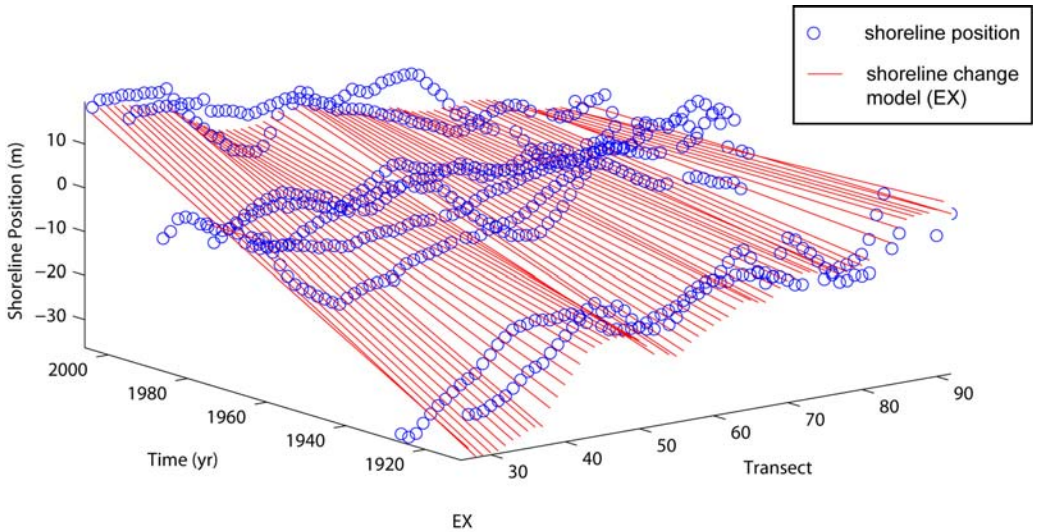

Figure 6. A shoreline change model calculated using EX method. The circles represent historical shoreline positions at each transect. The rates (slope in time and shoreline position) vary continuously in the alongshore direction but are constant (linear) in time.

An advancement of the EX method, called EXT, has been developed to model shoreline change with rates that vary along the shore and with time (Figure 7). Similar to EX, the eigenvectors are used to model the rate in the alongshore direction. However, EXT also simultaneously models the rate with a quadratic fit in the time domain (acceleration). Finding acceleration with shoreline data is an important advancement, as beaches may not erode or accrete at a constant (linear) rate as modeled by ST and EX. The EX and EXT methods often give meaningful, i.e., statistically significant, change rates for beaches where ST cannot. This yields more precise predictions of shoreline change.

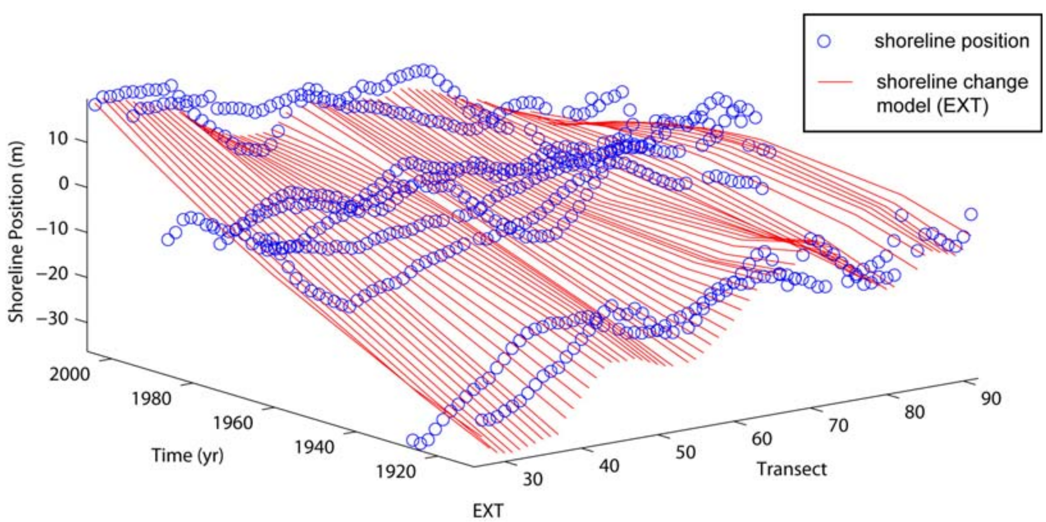

Figure 7. A shoreline change model calculated using EXT method. The circles represent historical shoreline positions at each transect. The rate (slope in shoreline position and time) varies continuously in the alongshore direction and time (acceleration).

Transect Plot

Individual transect plots illustrate the shoreline change model at each transect. For the EX and EXT rate calculation methods, the plots illustrate the shoreline change models sampled at an individual transect location. The horizontal axis is time (historical shoreline date) and the vertical axis is relative distance of the historical shorelines along the transect. The shoreline positions are depicted as red crosses and the shoreline change model is depicted with a blue line. The light blue bars represent the positional and measurement uncertainties of each shoreline position. A negative slope is equal to erosion and a positive slope is equal to accretion (Figure 8).

The EX method produces shoreline change rates which vary in the alongshore direction but are constant in time. Thus, EX is linear (constant rate) when viewed in the individual transect plots. If the EXT method identifies acceleration in the shoreline change rate, it will produce shoreline change models with rates that vary in the alongshore direction and with time. Thus, the EXT models may be non-linear when viewed in the individual transect plots. Examining the EXT model in a transect plot may provide a better illustration of how the rate changes with time, i.e. an erosion rate accelerating in time.

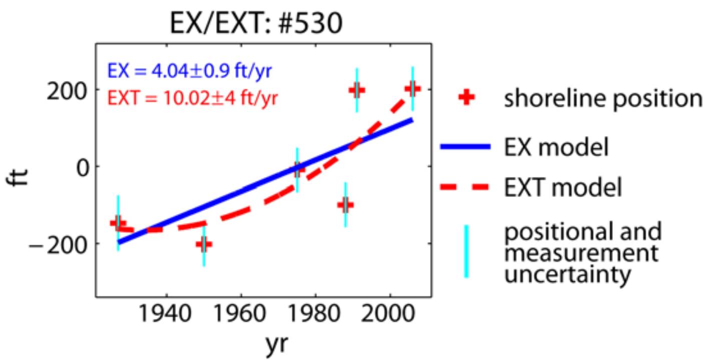

Figure 8. Individual transect plot of EX and EXT shoreline change model, showing accretion (positive rate). Historical shoreline time is shown on the horizontal axis and relative shoreline distance is shown on the vertical axis. Note: The EX rate is constant (linear) with time and the EXT rate varies with time (acceleration).

LITERATURE CITED

Fletcher, C.H., Rooney, J.J.B., Barbee, M., Lim, S.-C. and Richmond, B.M., 2003. Mapping Shoreline Change Using Digital Orthophotogrammetry on Maui, Hawaii. Journal of Coastal Research, Special Issue 38: 106-124.

Frazer, L.N., Genz, A.S. and Fletcher, C.H., in press. Toward Parsimony in Shoreline Change Prediction (I): New Methods. Journal of Coastal Research.

Genz, A.S., Fletcher, C.H., Dunn, R.A., Frazer, L.N. and Rooney, J.J., 2007. The Predictive Accuracy of Shoreline Change Rate Methods and Alongshore Beach Variation on Maui, Hawaii. Journal of Coastal Research, 23(1): 87-105.

Genz, A.S., Frazer, L.N. and Fletcher, C.H., in press. Toward Parsimony in Shoreline Change Prediction (II): Applying Statistical Methods to Real and Synthetic Data. Journal of Coastal Research.

National Academy of Sciences, 1990. Managing Coastal Erosion. National Research Council, Committee on Coastal Erosion Zone Management, National Academy Press, Washington D.C., 182 pp.

Thieler, E.R., Himmelstoss, E.A., Zichichi, J.L., and Miller, T.L., 2005. Digital Shoreline Analysis System (DSAS), http://woodshole.er.usgs.gov/project-pages/dsas/.